Superconducting phase transitions induced by chemical potential in (2+1)-dimensional four-fermion quantum field theory

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Article

The presentation is based on the paper of K.G. Klimenko, R.N. Zhokhov, V.Ch. Zhukovsky "Superconducting phase transitions induced by chemical potential in (2+1)-dimensional four-fermion quantum field theory".

Why it can be interesting

Interest for investigation of (2+1)-models with four-fermion interactions of the Gross-Neveu (GN) type is explained by more simple structure of QFT in two-, rather than in three spatial dimensions. It is much easier to investigate qualitatively such real physical phenomena as dynamical symmetry breaking, and to model phase diagrams of real quantum chromodynamics (QCD) etc. in the framework of (2+1)-Gross-Neveu (GN)type models.There another motivation for studying these models .The fact that there are many condensed matter systems which, firstly, have a (quasi-)planar structure and, secondly, their excitation spectrum is described adequately by relativistic Dirac-like equation rather than by Schrödinger one. Among these systems are the high-T_c cuprate and iron superconductors , the one-atom thick layer of carbon atoms, or graphene, etc. Thus, many properties of such condensed matter systems can be explained in the framework of (2+1) GN-type models.

Model and its Lagrangian

The Lagrangian of the model is

$$L = \sum_{k=1}^{N} \bar{\psi}_k \left(\gamma^\nu i\partial_\nu + \mu\gamma^0\right)\psi_k + \frac{G_1}{N}\left(\sum_{k=1}^{N} \bar{\psi}_k\psi_k\right)^2 + \tag{1}$$

$$\frac{G_2}{N}\left(\sum_{k=1}^{N} \psi_k^T C\psi_k\right)\left(\sum_{j=1}^{N} \bar{\psi}_j C\bar{\psi}_j^T\right)$$

where μ is the fermion number chemical potential. ψ_k ($k = 1, ..., N$) is a fundamental multiplet of $O(N)$ group. Moreover, each field ψ_k is a four-component Dirac spinor. The quantities γ^ν ($\nu = 0, 1, 2$) are matrices in the 4-dimensional spinor space. Moreover, $C \equiv \gamma^2$ is the charge conjugation matrix.

Lagrangian

Clearly, the Lagrangian L is invariant under transformations from the internal auxiliary $O(N)$ group, which is introduced here in order to make it possible to perform all the calculations in the framework of the nonperturbative large-N expansion method. Physically more interesting is that the model (1) is invariant under the discrete chiral transformation, $\psi_k \to \gamma^5 \psi_k$ and $U(1)$ fermion number group, $\psi_k \to \exp(i\alpha)\psi_k$ $(k = 1, ..., N)$, responsible for the fermion number conservation or, equivalently, for the electric charge conservation law in the system under consideration.

Algebra of the γ^ν-matrices

The algebra of the γ^ν-matrices as well as their particular representation are.: $\gamma^\mu = diag(\tilde{\gamma}^\mu, -\tilde{\gamma}^\mu)$, where $\tilde{\gamma}^\mu$

$$\tilde{\gamma}^0 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tilde{\gamma}^1 = i\sigma_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \tilde{\gamma}^2 = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

In addition to the Dirac matrices $\gamma^\mu \quad (\mu = 0, 1, 2)$ there exist two other matrices γ^3, γ^5 which anticommute with all $\gamma^\mu \quad (\mu = 0, 1, 2)$ and with themselves

$$\gamma^3 = \begin{pmatrix} 0\,, & I \\ I\,, & 0 \end{pmatrix}, \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = i\begin{pmatrix} 0\,, & -I \\ I\,, & 0 \end{pmatrix}, \qquad (3)$$

with I being the unit 2×2 matrix.

Linearized Lagrangian

The linearized version of Lagrangian that contains auxiliary bosonic fields $\sigma(x)$, $\Delta(x)$ and $\Delta^*(x)$ has the following form

$$\mathcal{L} = -\frac{N\sigma^2}{4G_1} - \frac{N}{4G_2}\Delta^*\Delta \tag{4}$$

$$+\sum_{k=1}^{N}\left[\bar{\psi}_k\left(\gamma^\nu i\partial_\nu + \mu\gamma^0 - \sigma\right)\psi_k - \frac{\Delta^*}{2}\psi_k^T C\psi_k - \frac{\Delta}{2}\bar{\psi}_k C\bar{\psi}_k^T\right].$$

Euler-Lagrange equations of motion for bosonic fields

Euler-Lagrange equations of motion for bosonic fields which take the form

$$\sigma(x) = -2\frac{G_1}{N}\sum_{k=1}^{N}\bar{\psi}_k\psi_k, \quad \Delta(x) = -2\frac{G_2}{N}\sum_{k=1}^{N}\psi_k^T C\psi_k, \tag{5}$$

$$\Delta^*(x) = -2\frac{G_2}{N}\sum_{k=1}^{N}\bar{\psi}_k C\bar{\psi}_k^T.$$

Thermodynamic potential

We study the phase structure of the four-fermion model (1) starting from the equivalent semi-bosonized Lagrangian. In the leading order of the large-N approximation, the effective action $\mathcal{S}_{\rm eff}(\sigma,\Delta,\Delta^*)$ of the considered model is expressed by means of the path integral over fermion fields

$$\exp(i\mathcal{S}_{\rm eff}(\sigma,\Delta,\Delta^*)) = \int\prod_{l=1}^{N}[d\bar\psi_l][d\psi_l]\exp\Big(i\int\mathcal{L}\,d^3x\Big),$$

where

$$\mathcal{S}_{\rm eff}(\sigma,\Delta,\Delta^*) = -\int d^3x\left[\frac{N}{4G_1}\sigma^2(x)+\frac{N}{4G_2}\Delta(x)\Delta^*(x)\right]+\widetilde{\mathcal{S}}_{\rm eff}.$$

Thermodynamic potential

$\widetilde{\mathcal{S}}_{\text{eff}}$ is given by:

$$\exp(i\widetilde{\mathcal{S}}_{\text{eff}}) = \int \prod_{l=1}^{N} [d\bar{\psi}_l][d\psi_l] \tag{6}$$

$$\exp\left\{ i \int \sum_{k=1}^{N} \left[\bar{\psi}_k (\gamma^\nu i\partial_\nu + \mu\gamma^0 - \sigma)\psi_k - \frac{\Delta^*}{2} \psi_k^T C \psi_k - \frac{\Delta}{2} \bar{\psi}_k C \bar{\psi}_k^T \right] d^3x \right\}.$$

Thermodynamic potential

For simplicity, we suppose that the above mentioned ground state expectation values do not depend on space-time coordinates, i.e.

$$\langle\sigma(x)\rangle \equiv M, \quad \langle\Delta(x)\rangle \equiv \Delta, \quad \langle\Delta^*(x)\rangle \equiv \Delta^*, \tag{7}$$

M, Δ, Δ^* are coordinates of the global minimum point of the thermodynamic potential (TDP) $\Omega(M, \Delta, \Delta^*)$. In the leading order of the large-N expansion it is defined by the following expression:

$$\int d^3x\Omega(M, \Delta, \Delta^*) =$$

$$= -\frac{1}{N}\mathcal{S}_{\rm eff}\{\sigma(x), \Delta(x), \Delta^*(x)\}\Big|_{\sigma(x)=M, \Delta(x)=\Delta, \Delta^*(x)=\Delta^*},$$

Thermodynamic potential

$$\int d^3x\Omega(M,\Delta,\Delta^*) = \int d^3x\left(\frac{M^2}{4G_1}+\frac{\Delta\Delta^*}{4G_2}\right)+ \qquad (8)$$

$$\frac{i}{N}\ln\left(\int\prod_{l=1}^{N}[d\bar{\psi}_l][d\psi_l]e^{i\int\left[\bar{\psi}_k D\psi_k-\frac{\Delta^*}{2}\psi_k^T C\psi_k-\frac{\Delta}{2}\bar{\psi}_k C\bar{\psi}_k^T\right]d^3x}\right),$$

where $D=\gamma^\nu i\partial_\nu+\mu\gamma^0-M$.

Thermodynamic potential

We suppose that $\Delta = \Delta^* \equiv \Delta$, where Δ is already a real quantity.

$$\int [dq] \exp\Big(i \int d^3x \Big[-\frac{1}{2} q^T A q + \eta^T q \Big] \Big) = \tag{9}$$

$$(\det(A))^{1/2} \exp\Big(-\frac{i}{2} \int d^3x \Big[\eta^T A^{-1} \eta \Big] \Big),$$

where A is an antisymmetric operator

Thermodynamic potential

We obtain the following expression for the zero temperature, $T = 0$, TDP of the GN model:

$$\Omega(M,\Delta) = \frac{M^2}{4G_1} + \frac{\Delta^2}{4G_2} + i\int \frac{d^3p}{(2\pi)^3} \ln\left[(p_0^2 - (\mathcal{E}_\Delta^+)^2)(p_0^2 - (\mathcal{E}_\Delta^-)^2)\right], \quad (10)$$

where $(\mathcal{E}_\Delta^\pm)^2 = E^2 + \mu^2 + \Delta^2 \pm 2\sqrt{M^2\Delta^2 + \mu^2 E^2}$
and $E = \sqrt{M^2 + |\vec{p}|^2}$.
we suppose that $\mu \geq 0$, $M \geq 0$

Regularized expression for TDP

We regularize the zero temperature TDP by cutting momenta, i.e. we suppose that $|p_1| < \Lambda$, $|p_2| < \Lambda$. As a result we have the following regularized expression (which is finite at finite values of Λ):

$$\Omega^{reg}(M,\Delta) =$$

$$= \frac{M^2}{4G_1} + \frac{\Delta^2}{4G_2} - \frac{1}{\pi^2}\int_0^\Lambda dp_1 \int_0^\Lambda dp_2 \left(\mathcal{E}_\Delta^+ + \mathcal{E}_\Delta^-\right) = \tag{11}$$

$$\begin{aligned} = \quad & M^2\left[\frac{1}{4G_1} - \frac{2\Lambda\ln(1+\sqrt{2})}{\pi^2}\right] + \Delta^2\left[\frac{1}{4G_2} - \frac{2\Lambda\ln(1+\sqrt{2})}{\pi^2}\right] \\ - \quad & \frac{2\Lambda^3(\sqrt{2}+\ln(1+\sqrt{2}))}{3\pi^2} + \mathcal{O}(\Lambda^0), \end{aligned} \tag{12}$$

Regularization

We suppose that the bare coupling constants G_1 and G_2 depends on the cutoff parameter Λ in such a way that in the limit $\Lambda \to \infty$ one obtains a finite expressions in the square brackets . Clearly, to fulfil this requirement it is sufficient to require that

$$\frac{1}{4G_1} \equiv \frac{1}{4G_1(\Lambda)} = \frac{2\Lambda \ln(1+\sqrt{2})}{\pi^2} + \frac{1}{2\pi g_1} \tag{13}$$

$$\frac{1}{4G_2} \equiv \frac{1}{4G_2(\Lambda)} = \frac{2\Lambda \ln(1+\sqrt{2})}{\pi^2} + \frac{1}{2\pi g_2},$$

where $g_{1,2}$ are finite and Λ are independent model parameters with dimensionality of inverse mass.

Renormalized TDP.

$$\Omega^{ren}(M,\Delta) = \tag{14}$$

$$= \lim_{\Lambda\to\infty}\left\{\Omega^{reg}(M,\Delta)\Big|_{G_1=G_1(\Lambda),G_2=G_2(\Lambda)} + \frac{2\Lambda^3(\sqrt{2}+\ln(1+\sqrt{2}))}{3\pi^2}\right\}.$$

Expression for the renormalized TDP at $\mu = 0$, $\mu \neq 0$.

In vacuum, i.e. at $\mu = 0$, TDP is usually called effective potential.

$$V(M,\Delta) \equiv \Omega^{ren}(M,\Delta)\Big|_{\mu=0} = \tag{15}$$

$$= \frac{M^2}{2\pi g_1} + \frac{\Delta^2}{2\pi g_2} + \frac{(M+\Delta)^3}{6\pi} + \frac{|M-\Delta|^3}{6\pi}.$$

Expression for the renormalized TDP at $\mu \neq 0$.

$$12\pi\Omega^{ren}(M,\Delta) =$$

$$\begin{aligned} = \;& \frac{6M^2}{g_1} + \frac{6\Delta^2}{g_2} + 2\left(M + \sqrt{\mu^2+\Delta^2}\right)^3 + 2\left|M - \sqrt{\mu^2+\Delta^2}\right|^3 \\ - \;& 3t_+\left(M + \sqrt{\mu^2+\Delta^2}\right) + 3t_-\left|M - \sqrt{\mu^2+\Delta^2}\right| \\ - \;& \frac{3(\mu^2-M^2)\Delta^2}{\mu} \ln\left|\frac{t_+ + \mu(M+\sqrt{\mu^2+\Delta^2})}{t_- + \mu|M-\sqrt{\mu^2+\Delta^2}|}\right|, \end{aligned} \tag{16}$$

where $t_\pm = M\sqrt{\mu^2+\Delta^2} \pm \mu^2$.

Phase structure of the model at $T = 0$

The coordinates of the global minimum point (M_0, Δ_0) of the TDP $\Omega^{ren}(M, \Delta)$ define the ground state expectation values of auxiliary fields $\sigma(x)$ and $\Delta(x)$. Namely, $M_0 = \langle \sigma(x) \rangle$ and $\Delta_0 = \langle \Delta(x) \rangle$. The quantities M_0 and Δ_0 are usually called order parameters, or gaps, because they are responsible for the phase structure of the model or, in other words, for the properties of the model ground state. Moreover, the gap M_0 is equal to the dynamical mass of one-fermionic excitations of the ground state. As a rule, gaps depend on model parameters as well as on various external factors. In our consideration the gaps M_0 and Δ_0 are certain functions of the free model parameters g_1 and g_2 and such external factors as chemical potential μ and temperature T.

Phase structure of the model in vacuum

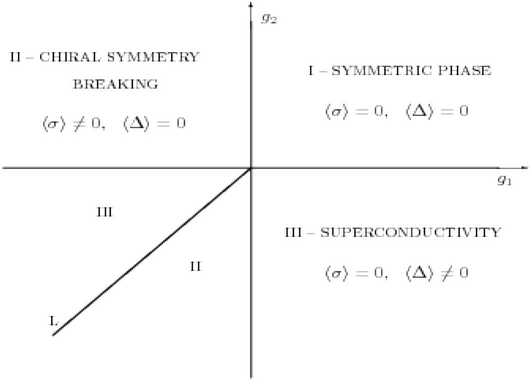


Рис.: The (g_1, g_2)-phase portrait of the model at $\mu = 0$. The shorthands I, II and III denote the symmetric, the chiral symmetry breaking and the superconducting phases, respectively. In the phase II $\langle\sigma\rangle = -1/g_1$. In the phase III $\langle\Delta\rangle = -1/g_2$. On the curve L$\equiv \{(g_1, g_2) : g_1 = g_2\}$, where $g_{1,2} < 0$, there is a coexistence of the phases II and III.

Phase structure of the model in vacuum

The plane (g_1, g_2) is divided into several areas. In each area one of the phases I, II or III is implemented. In the phase I, i.e. at $g_1 > 0$ and $g_2 > 0$, the global minimum of the effective potential $V(M, \Delta)$ is arranged at the origin. So in this case we have $M_0 = \langle\sigma(x)\rangle = 0$ and $\Delta_0 = \langle\Delta(x)\rangle = 0$. As a result, in the phase I both discrete chiral and continuous electromagnetic $U(1)$ symmetries remain intact and fermions are massless. Due to this reason the phase I is called symmetric. In the phase II, which is allowed only for $g_1 < 0$, at the global minimum point (M_0, Δ_0) the relations $M_0 = -1/g_1$ and $\Delta_0 = 0$ are valid. So in this phase chiral symmetry is spontaneously broken down and fermions acquire dynamically the mass M_0. Finally, in the superconducting phase III, where $g_2 < 0$, we have the following values for the gaps $M_0 = 0$ and $\Delta_0 = -1/g_2$.

Consideration of the chemical potential

Numerical and analytical investigations of the TDP show that its minimum points are of the form $(M \neq 0, \Delta = 0)$, $(M = 0, \Delta \neq 0)$ or $(M = 0, \Delta = 0)$ only. So to study the properties of the global minimum point it is enough to consider its reductions on the M- and Δ-axes, where the TDP becomes

$$
\begin{aligned}
12\pi\Omega^{ren}(M,\Delta)\Big|_{\Delta=0} &= \frac{6M^2}{g_1} + 2\,(M+\mu)^3 + 2\,|M-\mu|^3 \\
&- 3\mu\,(M+\mu)^2 + 3\mu(M-\mu)\,|M-\mu|\,, \qquad (17) \\
12\pi\Omega^{ren}(M,\Delta)\Big|_{M=0} &= \frac{6\Delta^2}{g_2} + 4(\mu^2+\Delta^2)^{3/2} - 6\mu^2\sqrt{\mu^2+\Delta^2} \\
&- 3\mu\Delta^2 \ln\left(\frac{(\mu+\sqrt{\mu^2+\Delta^2})^2}{\Delta^2}\right), \qquad (18)
\end{aligned}
$$

Particle density

we study the behavior of a particle density n in different phases when μ varies,as well

$$n = -\frac{\partial \Omega^{ren}(M, \Delta)}{\partial \mu}\Big|_{M=M_0, \Delta=\Delta_0}. \quad (19)$$

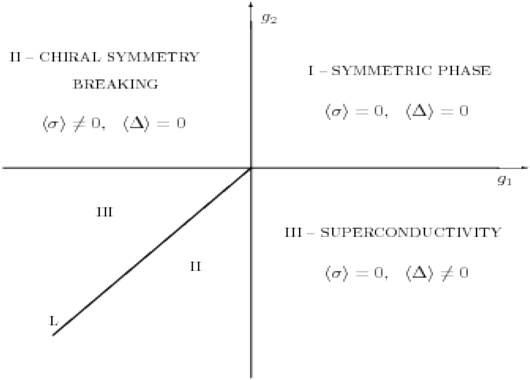
expressions for the particle density in the chiral symmetry broken II and superconducting III phases:

$$n\Big|_{\text{phase II}} = \frac{1}{2\pi}(\mu^2 - M_0^2)\theta(\mu - M_0), \quad (20)$$

$$n\Big|_{\text{phase III}} = \frac{1}{2\pi}\left[\mu\sqrt{\mu^2 + \Delta_0^2} + \Delta_0^2 \ln \frac{\mu + \sqrt{\mu^2 + \Delta_0^2}}{\Delta_0}\right] \quad (21)$$

where $\theta(x)$ is the Heaviside step-function.

Phase structure of the model in vacuum



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Phase structure of the model at nonzero μ. The cas: $g_1 < 0$.

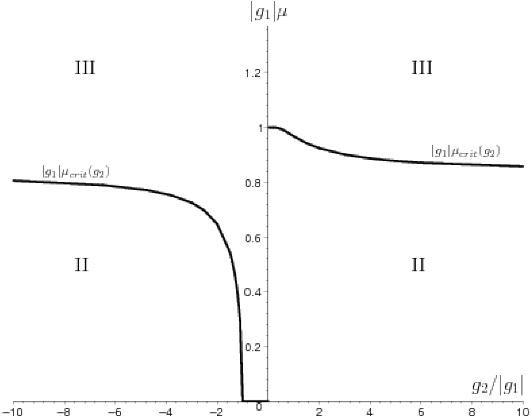

Рис.: The (μ, g_2)-phase portrait of the model and critical chemical potential $\mu_{crit}(g_2)$ vs g_2 at arbitrary fixed $g_1 < 0$. At each point $\mu = \mu_{crit}(g_2) \neq 0$ there is a first order phase transition from the chiral symmetry breaking phase II to the superconducting phase III.

Superconducting gap.The cas: $g_1 < 0$.

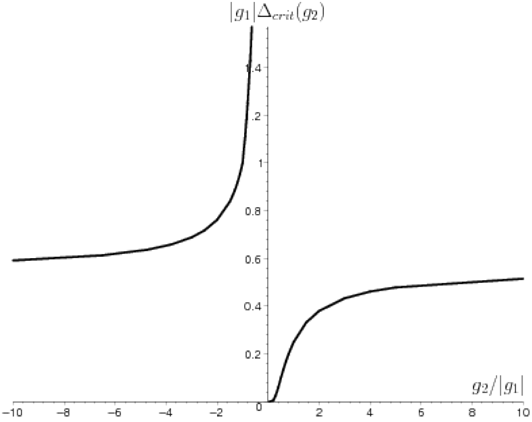

Рис.: Superconducting gap $\Delta_0 = \Delta_{crit}(g_2)$ vs g_2 which is generated at the critical point, i.e. at $\mu = \mu_{crit}(g_2)$, at arbitrary fixed $g_1 < 0$.

Particle density .

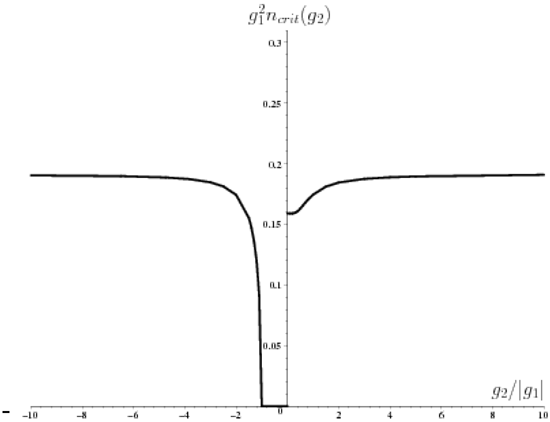

Рис.: Particle density $n = n_{crit}(g_2)$ vs g_2 which is generated at the critical point, i.e. at $\mu = \mu_{crit}(g_2)$, at arbitrary fixed $g_1 < 0$. At $\mu < \mu_{crit}(g_2)$ the particle density n is equal to zero.

Superconducting gap. The cas: $g_1 < 0$ and $g_2 = 0.5|g_1|$.

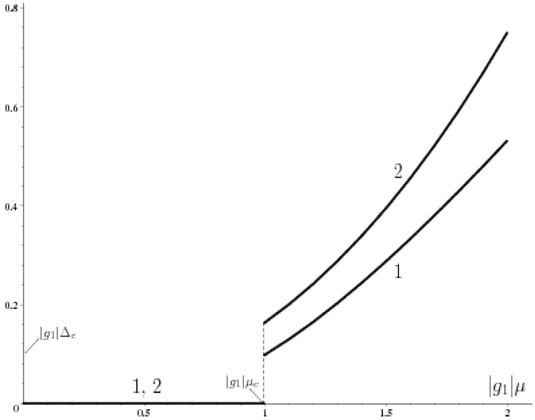

Рис.: Superconducting gap Δ_0 and particle density n vs μ at fixed $g_1 < 0$ and $g_2 = 0.5|g_1|$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|\Delta_0$ and $|g_1|^2 n$, correspondingly.

Superconducting gap. The cas: $g_1 < 0$.

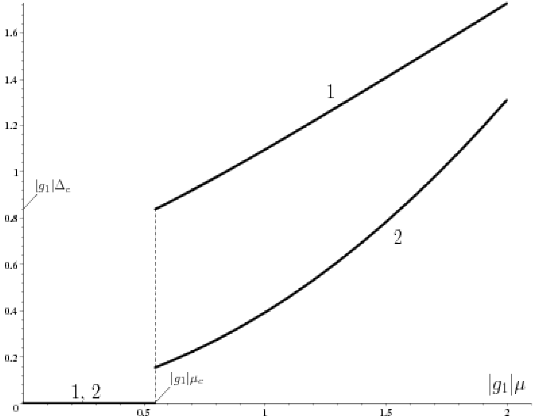

Рис.: Superconducting gap Δ_0 and particle density n vs μ at fixed $g_1 < 0$ and $g_2 = -1.5|g_1|$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|\Delta_0$ and $|g_1|^2 n$, respectively.

Superconducting gap Δ_0.

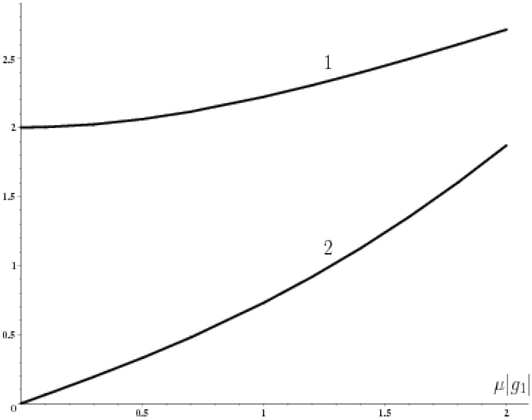

Рис.: Superconducting gap Δ_0 and particle density n vs μ at arbitrary fixed g_1 (both at $g_1 < 0$ and $g_1 > 0$) as well as at $g_2 = -0.5|g_1|$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|\Delta_0$ and $|g_1|^2 n$, respectively.

Superconducting gap. The cas: $g_1 > 0$.

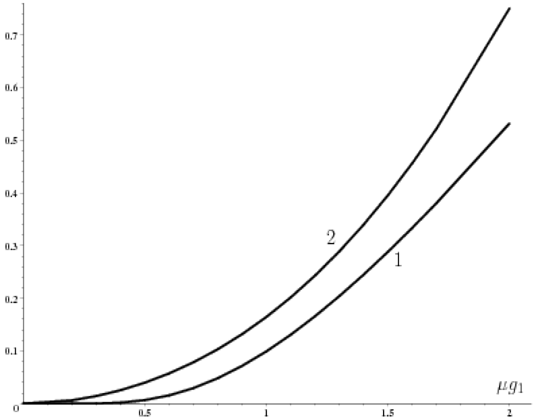

Рис.: Superconducting gap Δ_0 and particle density n vs μ at arbitrary fixed $g_1 > 0$ as well as at $g_2 = 0.5 g_1$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|\Delta_0$ and $|g_1|^2 n$, respectively.

Superconducting gap.The cas: $g_1 > 0$.

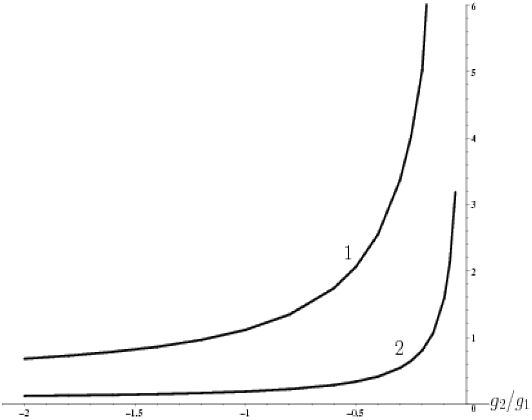

Рис.: Superconducting gap Δ_0 and particle density n vs $g_2 < 0$ at arbitrary fixed $g_1 > 0$ and $\mu = 0.5/g_1$. Curves 1 and 2 are the plots of the dimensionless quantities $g_1\Delta_0$ and $g_1^2 n$, respectively.

Superconducting gap Δ_0.

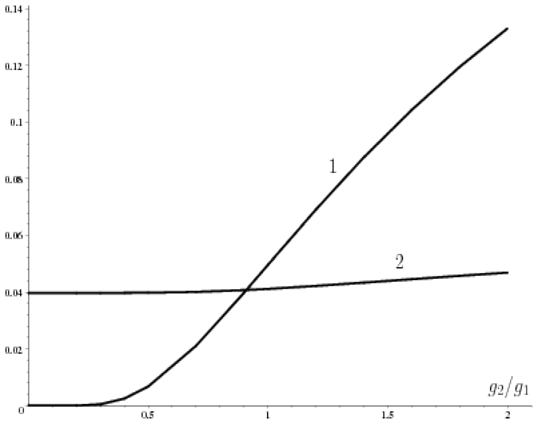

Рис.: Superconducting gap Δ_0 and particle density n vs $g_2 > 0$ at arbitrary fixed $g_1 > 0$ and $\mu = 0.5/g_1$. Curves 1 and 2 are the plots of the dimensionless quantities $g_1\Delta_0$ and $g_1^2 n$, respectively.

Finite temperature

In order to get the corresponding (unrenormalized) thermodynamic potential $\Omega_T(M, \Delta)$ at finite temperature one can simply start from the expression for the TDP at zero temperature and perform the following standard replacements:

$$\int_{-\infty}^{\infty} \frac{dp_0}{2\pi}(\cdots) \to iT \sum_{n=-\infty}^{\infty} (\cdots), \qquad p_0 \to p_{0n} \equiv i\omega_n \equiv i\pi T(2n+1), \quad$$

i.e. the p_0-integration should be replaced by the summation over Matsubara frequencies ω_n.

Finite temperature

Summing over Matsubara frequencies in the obtained expression , one can find for the TDP

$$\Omega_T(M,\Delta) = \frac{M^2}{4G_1} + \frac{\Delta^2}{4G_2} \tag{23}$$

$$-\int_{-\infty}^{\infty} \frac{d^2p}{(2\pi)^2} \left(\mathcal{E}_\Delta^+ + \mathcal{E}_\Delta^-\right) - 2T \int_{-\infty}^{\infty} \frac{d^2p}{(2\pi)^2} \ln\left(\left[1+e^{-\beta\mathcal{E}_\Delta^+}\right]\left[1+e^{-\beta\mathcal{E}_\Delta^-}\right]\right),$$

where $\beta = 1/T$. Clearly, only the first integral in this expression (which is the same as in the zero temperature case) is responsible for ultraviolet divergency of the whole TDP . So, regularizing the TDP in the way as it was done in (11) for zero temperature TDP and then replacing $G_{1,2} \to G_{1,2}(\Lambda)$), we can obtain in the limit $\Lambda \to \infty$ a finite expression denoted as $\Omega_T^{ren}(M,\Delta)$,

Finite temperature

$$\Omega_T^{ren}(M,\Delta)\Big|_{\Delta=0} = \Omega_{T=0}^{ren}(M,\Delta)\Big|_{\Delta=0} - \tag{24}$$

$$-2T\int_{-\infty}^{\infty}\frac{d^2p}{(2\pi)^2}\ln\left(\left[1+e^{-\beta(E+\mu)}\right]\left[1+e^{-\beta|E-\mu|}\right]\right) =$$

$$= \frac{M^2}{2\pi g_1}+\frac{M^3}{3\pi}-2T\int_{-\infty}^{\infty}\frac{d^2p}{(2\pi)^2}\ln\left(\left[1+e^{-\beta(E+\mu)}\right]\left[1+e^{-\beta(E-\mu)}\right]\right),$$

$$\Omega_T^{ren}(M,\Delta)\Big|_{M=0} = \Omega_{T=0}^{ren}(M,\Delta)\Big|_{M=0} - \tag{25}$$

$$-2T\int_{-\infty}^{\infty}\frac{d^2p}{(2\pi)^2}\ln\left(\left[1+e^{-\beta E_\Delta^+}\right]\left[1+e^{-\beta E_\Delta^-}\right]\right),$$

where $E=\sqrt{|\vec{p}|^2+M^2}$, $(E_\Delta^\pm)^2=(|\vec{p}|\pm\mu)^2+\Delta^2$

Phase structure of the model in vacuum

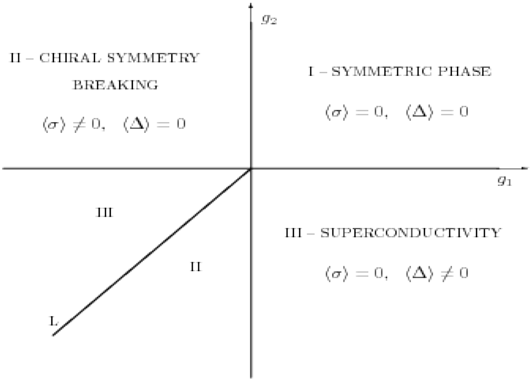

Рис.: The (g_1, g_2)-phase portrait of the model at $\mu = 0$. The shorthands I, II and III denote the symmetric, the chiral symmetry breaking and the superconducting phases, respectively. In the phase II $\langle\sigma\rangle = -1/g_1$. In the phase III $\langle\Delta\rangle = -1/g_2$. On the curve L$\equiv \{(g_1, g_2) : g_1 = g_2\}$, where $g_{1,2} < 0$, there is a coexistence of the phases II and III.

Phase structure of the model at finite temperature and μ .The case of $g_2 = -0.5|g_1|$, $g_1 < 0$ and $g_1 > 0$

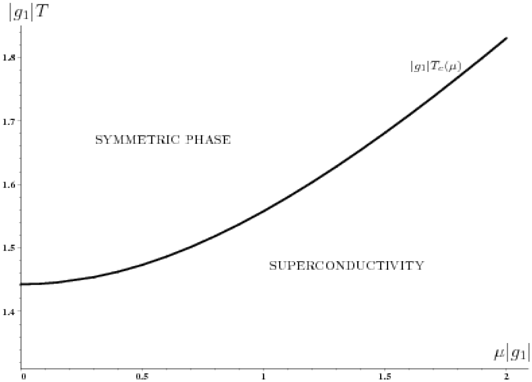

Рис.: (μ, T)-phase diagram of the model at $g_2 = -0.5|g_1|$ and arbitrary fixed g_1 both at $g_1 < 0$ and $g_1 > 0$.

Phase structure of the model at finite temperature and μ .The case of $g_1 > 0$ and at $g_2 = 0.5g_1$

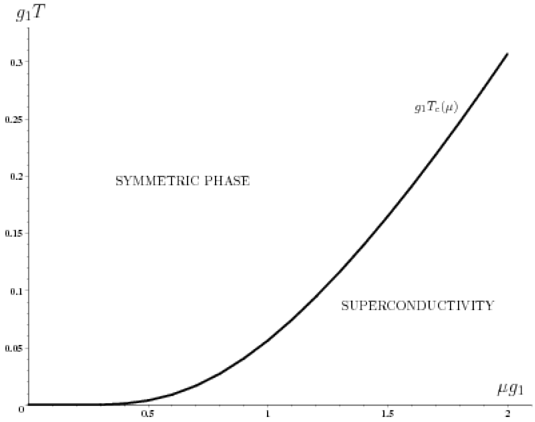

Рис.: (μ, T)-phase diagram of the model at arbitrary fixed $g_1 > 0$ and at $g_2 = 0.5g_1$.

Phase structure of the model at finite temperature and μ .The case of $g_2 = -1.5|g_1|$ and $g_1 < 0$

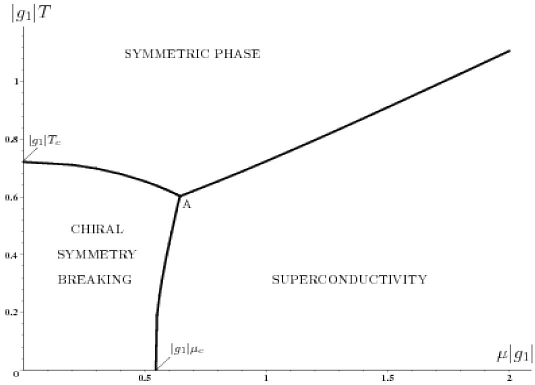

Рис.: (μ, T)-phase diagram of the model at $g_2 = -1.5|g_1|$ and arbitrary fixed $g_1 < 0$.The coordinates of the tricritical point A are the following ones, $|g_1|\mu_A \approx 0.645$ and $|g_1|T_A \approx 0.602$. Moreover, $|g_1|\mu_c \approx 0.545$ and $|g_1|T_c = 1/(2\ln 2) \approx 0.721$.

Phase structure of the model at finite temperature and μ . The case of $g_2 = 0.5|g_1|$ and $g_1 < 0$

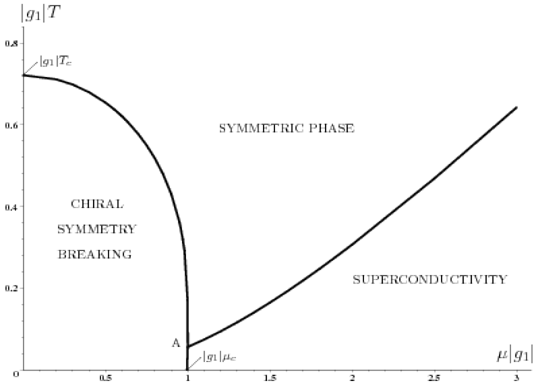

Рис.: (μ, T)-phase diagram of the model at $g_2 = 0.5|g_1|$ and arbitrary fixed $g_1 < 0$. The coordinates of the tricritical point A are the following ones, $|g_1|\mu_A \approx 0.999$ and $|g_1|T_A \approx 0.056$. Moreover, $|g_1|\mu_c \approx 0.995$ and $|g_1|T_c = 1/(2\ln 2) \approx 0.721$.

Conclusions.

The case $T = 0$, $\mu = 0$.
In this case the phase portrait is presented in terms of the finite coupling constants g_1 and g_2.
The case $T = 0$, $\mu \neq 0$.
At $T = 0$ and at growing chemical potential the system is transformed into a superconducting state.
The case $T > 0$, $\mu \neq 0$.
At fixed μ and increasing temperature the symmetric phase is restored. However, at arbitrary fixed T, growth of the chemical potential leads to appearing of superconductivity in the system at arbitrary relations between coupling constants g_1 and g_2.

-

The fact that chemical potential induces superconductivity phenomenon is the main result of our paper.

System in external magnetic field.

We investigate the influence of an external magnetic field $\vec{B}$ on the (2+1)-dimensional GN–type model with two massless fermions $\psi_1(x)$ and $\psi_2(x)$ belonging to a reducible 4-component spinor representation of the (2+1)-dimensional Lorentz group (the spinor fields $\psi_1(x)$ and $\psi_2(x)$ are introduced for quasiparticles (electrons) with spin projections 1/2 and -1/2 on the direction of the magnetic field $\vec{B}$, respectively). The model describes low-energy dynamics of quasiparticles both in the fermion-antifermion (or chiral) and fermion-fermion (or Cooper pairing) channels. External magnetic field is parallel to the system plane, i.e. $\vec{B} = \vec{B}_{\parallel}$.

$$L = \sum_{k=1}^{2} \bar{\psi}_k \left[\gamma^\rho i \partial_\rho + \mu \gamma^0 - \nu (-1)^k \gamma^0 \right] \psi_k + G_1 \left(\sum_{k=1}^{2} \bar{\psi}_k \psi_k \right)^2 +$$

$$+ G_2 \left(\sum_{k=1}^{2} \psi_k^T C \psi_k \right) \left(\sum_{j=1}^{2} \bar{\psi}_j C \bar{\psi}_j^T \right)$$

Superconducting gap Δ_0 and particle density n. The case of $g_1 > 0$, $g_2 = 0.5g_1$, $\mu = 0.5/g_1$

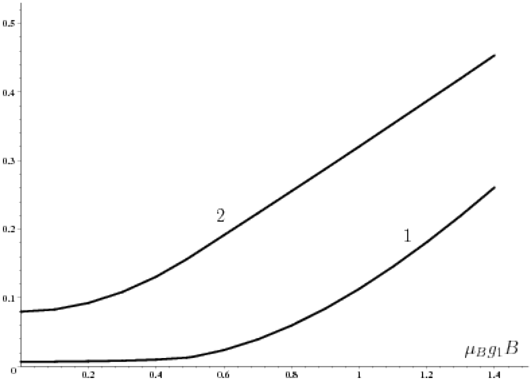

Рис.: *The case $\mu \neq 0$:* Superconducting gap Δ_0 and particle density n vs B at arbitrary fixed $g_1 > 0$ as well as at $g_2 = 0.5g_1$ and $\mu = 0.5/g_1$. Curves 1 and 2 are the plots of the dimensionless quantities $g_1\Delta_0$ and $g_1^2 n$, respectively.

Magnetization m and magnetic susceptibility χ. The case of $g_1 > 0$, $g_2 = 0.5g_1$, $\mu = 0.5/g_1$.

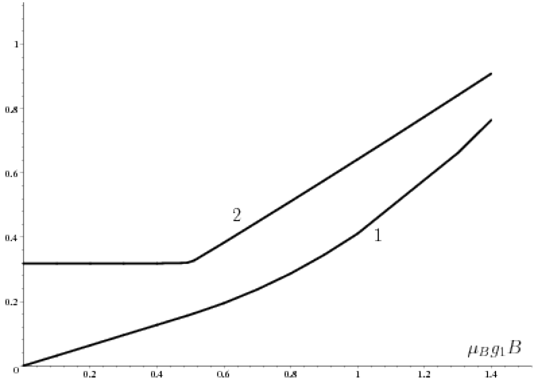

Рис.: *The case $\mu \neq 0$:* Magnetization m and magnetic susceptibility χ vs B at arbitrary fixed $g_1 > 0$ as well as at $g_2 = 0.5g_1$ and $\mu = 0.5/g_1$. Curves 1 and 2 are the plots of the dimensionless quantities $g_1^2 m/\mu_B$ and $g_1\chi/\mu_B^2$, respectively.

The (μ, B)-phase portrait of the model. The case of $g_1 < 0$, $g_2 = -1.5|g_1|$.

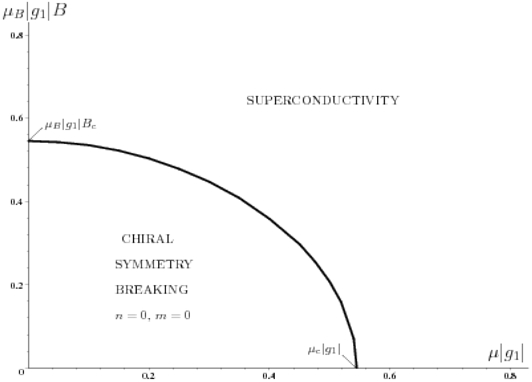

Рис.: *The case $\mu \neq 0$:* The (μ, B)-phase portrait of the model at arbitrary fixed $g_1 < 0$ as well as at $g_2 = -1.5|g_1|$.

The (μ, B)-phase portrait of the model. The case of $g_1 < 0$, $g_2 = 0.5|g_1|$.

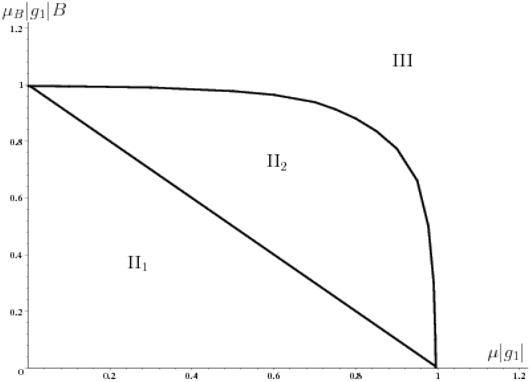

Рис.: *The case $\mu \neq 0$:* The (μ, B)-phase portrait of the model at arbitrary fixed $g_1 < 0$ as well as at $g_2 = 0.5|g_1|$. Here II_1 and II_2 denote the chiral symmetry breaking phases with $n = 0$, $m = 0$ and $n \neq 0$, $m \neq 0$, respectively. The notation III stands for the superconducting phase.

The gaps M_0 and Δ_0 . The case of $g_1 < 0$, $g_2 = 0.5|g_1|$, $\mu = 0.7/|g_1|$.

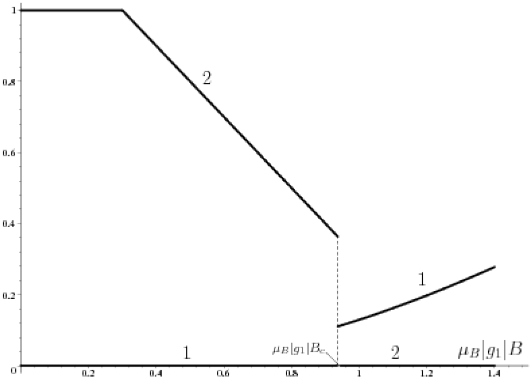

Рис.: *The case $\mu \neq 0$:* The gaps M_0 and Δ_0 vs B at arbitrary fixed $g_1 < 0$ as well as at $g_2 = 0.5|g_1|$ and $\mu = 0.7/|g_1|$. Curves 1 and 2 are the plots of the dimensionless quantities $|g_1|M_0$ and $|g_1|\Delta_0$, respectively. Here $\mu_B|g_1|B_c \approx 0.937$.

Particle density n, magnetization m and magnetic susceptibility χ. The case of $g_1 < 0$, $g_2 = 0.5|g_1|$, $\mu = 0.7/|g_1|$

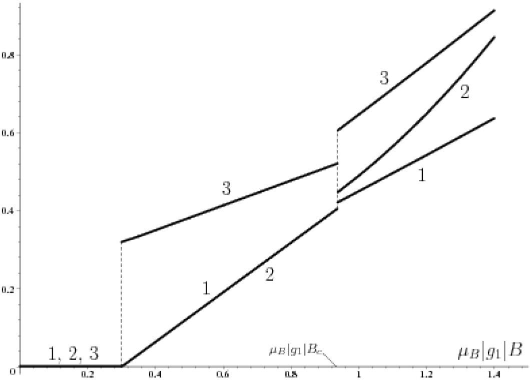

Рис.: *The case* $\mu \neq 0$*:* Particle density n, magnetization m and magnetic susceptibility χ vs B at arbitrary fixed $g_1 < 0$ as well as at $g_2 = 0.5|g_1|$ and $\mu = 0.7/|g_1|$. Curves 1, 2 and 3 are the plots of the dimensionless quantities $g_1^2 n$, $g_1^2 m/\mu_B$ and $|g_1|\chi/\mu_B^2$, respectively. Here $\mu_B|g_1|B_c \approx 0.937$.

Article

These results are from the paper of K.G. Klimenko, R.N. Zhokhov, V.Ch. Zhukovsky "Superconductivity phenomenon induced by external in-plane magnetic field in (2+1)-dimensional Gross–Neveu type model".

Спасибо за внимание.