Pseudo-Goldstone nature of inflaton in a fundament to solving of cosmological constant problem

Valery Kiselev & Ja.Balitsky

MIPT (Moscow) & IHEP (Protvino), Russia

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Report sections

1. pseudo-Goldstone + pseudo-Galilean boson $\equiv pG^2$
   - Global symmetry
   - Galilean symmetry & couplings
   - Symmetry breaking by gravity
2. Matching to the inflaton
   - The effective potential induced by gravitational loops
   - Expansion in inverse energy scale
   - Transformations of fields
   - Scales & charge
3. Suppression of cosmological constant
4. Conclusion

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Non-gravitational dynamics

Irrespective value of initial point of energy count:
vacuum energy density $\rho_\Lambda = \Lambda^4$

$$\Lambda \mapsto \Lambda + \Lambda_u. \tag{1}$$

Nambu–Goldstone boson: the global shift symmetry

$$\phi(x) \mapsto \phi(x) + f_G\, u, \tag{2}$$

spontaneously broken by setting ρ_Λ

Interaction (explicitly second order in derivatives of ϕ):

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + b_e\,\partial_\mu\phi\, j^\mu + \frac{1}{2}\, b_T\,\partial_\mu\phi\,\partial_\nu\phi\, T^{\mu\nu} + \phi\cdot\text{Const.}\ldots \tag{3}$$

Galilean solution

No sources: symmetry

$$\phi(x) \mapsto \phi(x) + f_G k_\mu x^\mu + \phi_0, \tag{4}$$

conserved if

$$\nabla_\mu j^\mu = 0, \qquad \nabla_\mu T^{\mu\nu} = 0. \tag{5}$$

Couplings to the electromagnetic current and energy-momentum tensor

$$b_e = \frac{\varkappa}{M}, \qquad \varkappa = \pm 1, 0,$$

M sets the scale of dimensional expansion for the interaction operators,

$$b_T \sim \frac{1}{M^4}.$$

Symmetry breaking by gravity: bare term

Break off under the conservation of Galilean symmetry: uniquely!!!

$$\mathcal{L}^{\rm int} = -\frac{1}{2}\, b_\xi\, \phi\, \mathcal{R}, \tag{6}$$

New globally symmetric interaction

$$\mathcal{L}_{SG} = \frac{1}{2}\, b_G\, \partial_\mu\phi\, \partial_\nu\phi \left(\mathcal{R}^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}\mathcal{R}\right),$$

due to Bianchi identity $\nabla_\mu \left(\mathcal{R}^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}\mathcal{R}\right) \equiv 0$. The coupling constant b_G scales as

$$b_G \sim \frac{1}{M^2}.$$

Hence, $M \sim$ Planckian mass.

Gravitational loops

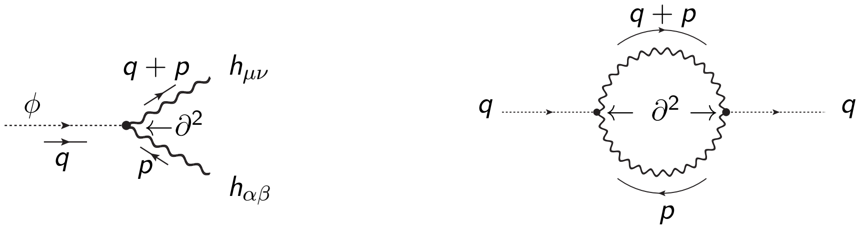

Figure: The graviton coupling to the scalar pseudo-Goldstone boson (left) and the graviton loop for the quadratic term of scalar field (right, momenta are labeled).

Quadratic term of effective action

$$\Gamma[\phi] = \frac{1}{2}\,\phi^2 \Gamma^{(2)}[q],$$

Quadratic term of effective action

effective mass m

$$m^2 = -\Gamma^{(2)}[0] = c_2(1 - c_2')\,\xi^2\,\frac{\Lambda_c^4}{m_{\rm Pl}^2}, \qquad c_2 = -\frac{39}{64\pi^2}, \tag{7}$$

correction to the kinetic term

$$\frac{\partial^2 \Gamma^{(2)}[0]}{\partial q^\mu \partial q^\nu} = \eta_{\mu\nu}\,k_2(1 - k_2')\,\xi^2\,\frac{\Lambda_c^2}{m_{\rm Pl}^2}, \qquad k_2 = \frac{147}{48\pi^2}. \tag{8}$$

The correction to the kinetic term can be of the order of unit:

$$\xi^2 \sim \frac{m_{\rm Pl}^2}{\Lambda_c^2}, \qquad \text{expect } \xi \gg 1. \tag{9}$$

The mass scale of pseudo-Goldstone boson is of the order of loop cut-off,

$$m \sim \Lambda_c. \tag{10}$$

Higher terms

The effective potential:

$$V_{\rm eff} = \sum_{n=2j} \frac{c_n}{n!}\,(1 - c_n')\,\phi^n \Lambda_n^4\,\frac{\xi^n}{m_{\rm Pl}^n}. \tag{11}$$

The kinetic term:

$$K_{\rm eff} = (\partial_\mu \phi)^2 \sum_{n=2j} \frac{k_n}{n!}\,(1 - k_n')\,\phi^{n-2} \Lambda_n^2\,\frac{\xi^n}{m_{\rm Pl}^n}, \tag{12}$$

The actual expansion of $K_{\rm eff}$ in ϕ corresponds to the expansion in powers of <u>inverse Planck mass</u> $m_{\rm Pl}$ only if the cut-off scales range as

$$\Lambda_n^2 \sim m_{\rm Pl}^2\,\frac{1}{\xi^n}, \qquad \Lambda_n^2 \sim \Lambda_c^2\,\frac{1}{\xi^{n-2}}, \qquad \text{at } \Lambda_c = \Lambda_2. \tag{13}$$

Suppression

Therefore, the effective potential takes the form

$$V_{\text{eff}} = \sum_{n=2j} \frac{c_n}{n!} \left(1 - \tilde{c}'_n\right) \frac{\phi^n}{m^n_{\text{Pl}}} \Lambda^2_c m^2_{\text{Pl}} \frac{1}{\xi^{n-2}} \tag{14}$$

Thus, the higher orders of potential at $n > 2$ are suppressed as $\xi^{2-n} \ll 1$, at least. *We conclude that the leading term of potential is reduced to the quadratic or mass term, indeed.*

From the Jordan frame to the Einstein frame

$$g_{\mu\nu} \mapsto \frac{1}{\Omega(\phi)}\, g_{\mu\nu}, \qquad \Omega(\phi) = 1 + b_\xi \phi,$$

At large couplings $\xi \gg 1$, the potential of inflaton χ,

$$V_E \approx \frac{m^2 m_{\rm Pl}^2}{2\,\xi^2}\left(1 - \mathrm{e}^{-\frac{\chi}{m_{\rm Pl}}\frac{\sqrt{6}}{3}}\right)^2, \quad \text{plateau } V_E^{\rm inf} = \Lambda_{\rm inf}^4 \sim \frac{m_{\rm Pl}^4}{\xi^4}, \tag{15}$$

at $\xi \sim 300$ we get $\Lambda_{\rm inf} \sim 10^{15-16}$ GeV, while the inflaton mass

$$m_{\rm inf}^2 = \frac{2m^2}{3\,\xi^2}, \qquad m_{\rm inf} \sim 10^{13}\ \text{GeV}, \tag{16}$$

in agreement with the current phenomenology of Universe inflation [Starobinsky model].

Fundamental scale and charge

Bare action

$$\mathcal{L}_G \sim -\frac{1}{2}\,h_\perp\,\partial^2 h_\perp,\quad \mathcal{L}_\phi \sim -\frac{1}{2}\,\phi\,\partial^2\phi,\quad \mathcal{L}_{\rm int} \sim -\frac{1}{2}\,h_\perp\,\partial^2 h_\perp\cdot\frac{\phi}{\Lambda_c}$$

No Planckian mass!

Gravitational interaction with matter:

$$\mathcal{L}_G = -\frac{\alpha}{4\pi\Lambda_c}\,h_{\mu\nu}T^{\mu\nu},\qquad \frac{1}{m_{\rm Pl}} = \frac{\alpha}{4\pi\Lambda_c},\qquad \xi^{-1}\sim\frac{\alpha}{4\pi}$$

The coupling constant $\alpha \sim \frac{1}{25} \longleftarrow \alpha_{\rm GUT}$? Expansions in α:

$$K_{\rm eff} \sim \frac{1}{2}(\partial_\mu\phi)^2\sum\left(\frac{\phi}{\Lambda_c}\right)^n\alpha^n,\qquad V_{\rm eff}\sim\Lambda_c^2\phi^2\sum\left(\frac{\phi}{\Lambda_c}\right)^n\alpha^{2n}.$$

Universal cut off ≡ fine tuned renormalization

Observable quantity

$$K_{\text{eff}} = (\partial_\mu \phi)^2 \sum_{n>2} \frac{k_n}{n!} \boxed{(1 - k'_n)\,\Lambda_n^2\,\frac{\xi^n}{m_{\text{Pl}}^n}}\,\phi^{n-2}, \tag{17}$$

Universality:

$$\Lambda_n \mapsto \Lambda_c, \qquad \xi \sim \frac{m_{\text{Pl}}}{\Lambda_c} \qquad \Rightarrow \qquad \boxed{\frac{\bullet}{\circ}} \mapsto \frac{\alpha^{n-2}}{\Lambda_c^{n-2}}$$

if

$$k'_n \sim 1 \mapsto k'_n = 1 + \delta k'_n, \qquad \delta k'_n \sim \alpha^{n-2}$$

The origin of fine tuning ≡ the matching to a ultimate unified, finite (!) or renormalizable theory of gravity and gauge interactions

Cosmological constant problem

The problem by Zel'dovich:

- Classical vacuum: empty space-time $\equiv$ Minkowsky space-time
- Quantum vacuum: $\Rightarrow \left\{ \begin{array}{l} \text{zero-point fluctuations} \\ \text{condensates} \end{array} \right.$

Zero-point fluctuations

$$\rho_\Lambda \sim \int^\Lambda k \mathrm{d}^3 k \sim \Lambda^4$$

$\Lambda \sim$ scale of particle interaction, say, $\Lambda \sim m_{\mathrm{Pl}} \sim 10^{18}$ GeV
Really, $\Lambda \mapsto 10^{-12}$ GeV $\Rightarrow$ **30 orders of magnitude!**

The very beginning of inflationary Universe

Consider a time-dependent scalar field $\phi(t)$, which is spatially global and free in a finite physical volume V_R

$$S = V_R \int dt\,\frac{1}{2}\left(\dot{\phi}^2 - m^2\phi^2\right), \tag{18}$$

The minimal energy shift from the minimum of potential is referred to the energy level of *zero-point mode*,

$$\delta_{\min} E = \frac{1}{2}\,m. \tag{19}$$

So, it defines the bare cosmological constant,

$$\rho_\Lambda^{\rm bare} = \frac{m}{2V_R} = \langle \mathrm{vac}|\rho|\mathrm{vac}\rangle = m^2\langle \mathrm{vac}|\phi^2|\mathrm{vac}\rangle = \langle \mathrm{vac}|\dot{\phi}^2|\mathrm{vac}\rangle. \tag{20}$$

CC problem

We expect (now due to the introduction of fundamental scale Λ_c, cut off)

$$m^2 \sim \langle \text{vac}|\phi^2|\text{vac}\rangle \sim \Lambda_c^2, \qquad \langle \text{vac}|\dot{\phi}^2|\text{vac}\rangle \sim \Lambda_c^4, \qquad V_R \sim \Lambda_c^{-3},$$

hence,

$$\rho_\Lambda^{\text{bare}} \sim \Lambda_c^4 \sim (10^{16}\ \text{GeV})^4 \gg \rho_\Lambda \sim (10^{-12}\ \text{GeV})^4,$$

Statistically occasional excitation of field: the coherent state $|\alpha\rangle$.

Average number of quanta $\langle n\rangle$ determines the Poisson distribution:

$$\langle\alpha|\rho|\alpha\rangle = |\mathcal{A}_{\text{vac}}|^2 \langle \text{vac}|\rho|\text{vac}\rangle + |\mathcal{A}_q|^2 \langle \text{quanta}|\rho|\text{quanta}\rangle, \tag{21}$$

Therefore, the average density of energy observed in the gravity is

$$\langle\rho\rangle = |\mathcal{A}_{\text{vac}}|^2\, \rho_\Lambda^{\text{bare}} + \rho_q, \qquad \text{at}\ \ |\mathcal{A}_{\text{vac}}|^2 = \mathrm{e}^{-\langle n\rangle}, \tag{22}$$

Suppression of vacuum fluctuations

Zero-point fluctuations are suppressed exponentially

$$\rho_\Lambda = |\mathcal{A}_{\rm vac}|^2\, \rho_\Lambda^{\rm bare} \mapsto {\rm e}^{-\langle n\rangle}\, \rho_\Lambda^{\rm bare}. \tag{23}$$

Estimate of average number of "quanta"

$$\langle n\rangle \approx \frac{\langle E\rangle}{m} \sim \frac{m_{\rm Pl}}{\Lambda_c} \sim \xi \sim \frac{4\pi}{\alpha} \sim 250. \qquad \text{O.K. !!!}$$

Contributions of other sources are also suppressed exponentially!

Since the vacuum is **common** for all particles and condensates:

$$|{\rm vac}\rangle = \mathcal{A}_{\rm vac}|0_{\phi(t)}\rangle \otimes |{\rm vac}_{\rm QCD}\rangle \otimes \ldots |{\rm vac}_{\rm H}\rangle \;\Rightarrow$$

$$\rho^{\rm QCD}_{\rm vac} \mapsto {\rm e}^{-\langle n\rangle} \rho^{\rm QCD}_{\rm vac}, \ldots$$

Inferences

- The pseudo-Goldstone nature of global shift to the vacuum energy in cooperation with the pseudo-Galilean symmetry is realistic for the description of inflation and cosmological constant
- Higher kinetic structures of Galileon are uniquely fixed and can influence the additional gravitational force in the region of dark matter, say, in addition to the implementation of non-minimal interaction (?!)
- The interaction of pG^2 with the Einstein tensor modify the inflation aspects
- New features could be revealed ...