О международной конференции The 33rd International Symposium on Lattice Field Theory Кобе, Япония, 14-18 июля 2015

В.Г. Борняков

15 сентября 2015, ИФВЭ

- The electric dipole moment of the neutron from 2 + 1 flavor lattice QCD

- Ab initio calculation of the neutron-proton mass difference

- Lattice inputs to flavor physics (FLAG)

- Fluctuations at finite temperature and density

LATTICE 2015

The 33rd International Symposium
on Lattice Field Theory

Kobe International Conference Center, Kobe, Japan

Tuesday, July 14 - Saturday, July 18, 2015

http://www.aics.riken.jp/sympo/lattice2015

Kenneth Geddes Wilson

1936 - 2013

the 1982 Nobel Prize in Physics for his work on phase transitions

Method

- Imaginary time $t \to it$

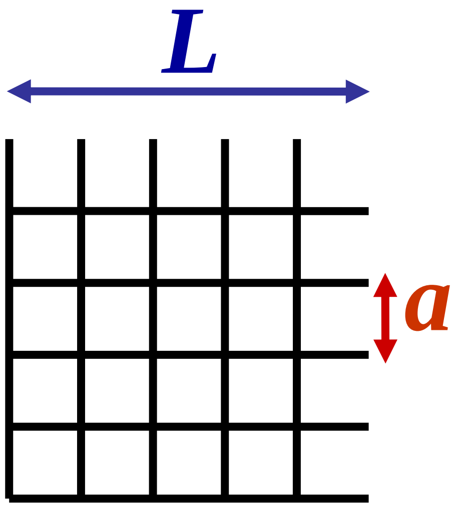
$$Z = \int D\varphi \, e^{iS}$$ $\longrightarrow$ $$Z = \int D\varphi \exp\{-S_E[\varphi]\}$$

- Space-time discretization

$$D\varphi = \prod_x d\varphi_x \qquad Z = \int \prod_x d\varphi_x \exp\{-S_E[\varphi]\}$$

- Thus we get from functional integral the partition function for statistical theory in four dimensions

Three limits



Lattice spacing $a \to 0$

Lattice size $L \to \infty$

Quark mass $m_q \to 0$

Typical values

$$a \approx 0.1\,fm$$

$$L \approx 2 \div 4\,fm$$

$$m_q \approx 100\,MeV$$

Typical multiplicity of integrals

For lattice with

L=48, L^4=5,308,416

- **The multiplicity of integrals over gluon fields is *32L^4* *(L=48, 32L^4=169,869,312)***

- **For quark fields we work with matrices**
  12L^4* x *12L^4 ***(L=48, 12L^4=63,700,992)***

Mike Creutz

first computations in 1979

$$< \mathcal{O} > = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \, \mathcal{O}(U) \, e^{-S_{\text{eff}}(U)} \, ,$$

$$\mathcal{Z} = \int \mathcal{D}U \, e^{-S_{\text{eff}}(U)} \, ,$$

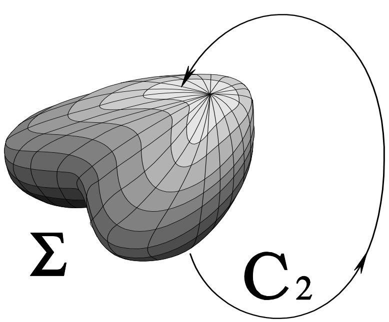
$$S_{\text{eff}}(U) = S_W^G(U) - N_f \ln \det M(U),$$

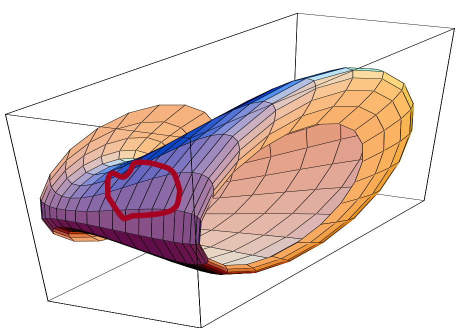
Three methods of investigations

Experimental

LHC **RHIC**

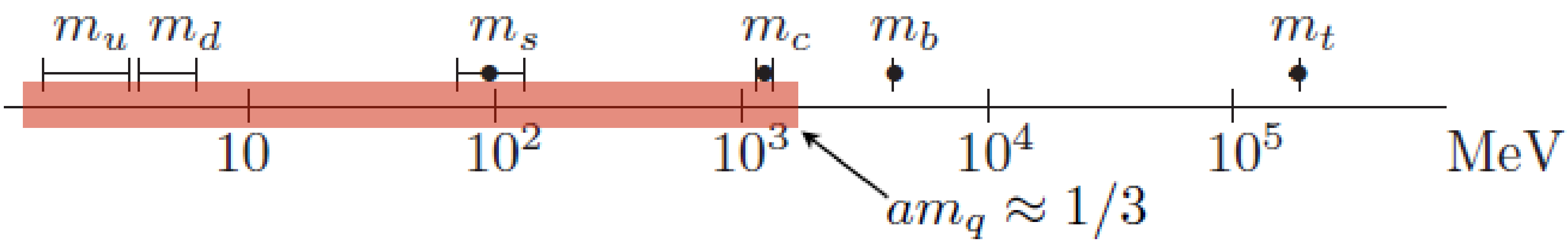
Theoretical

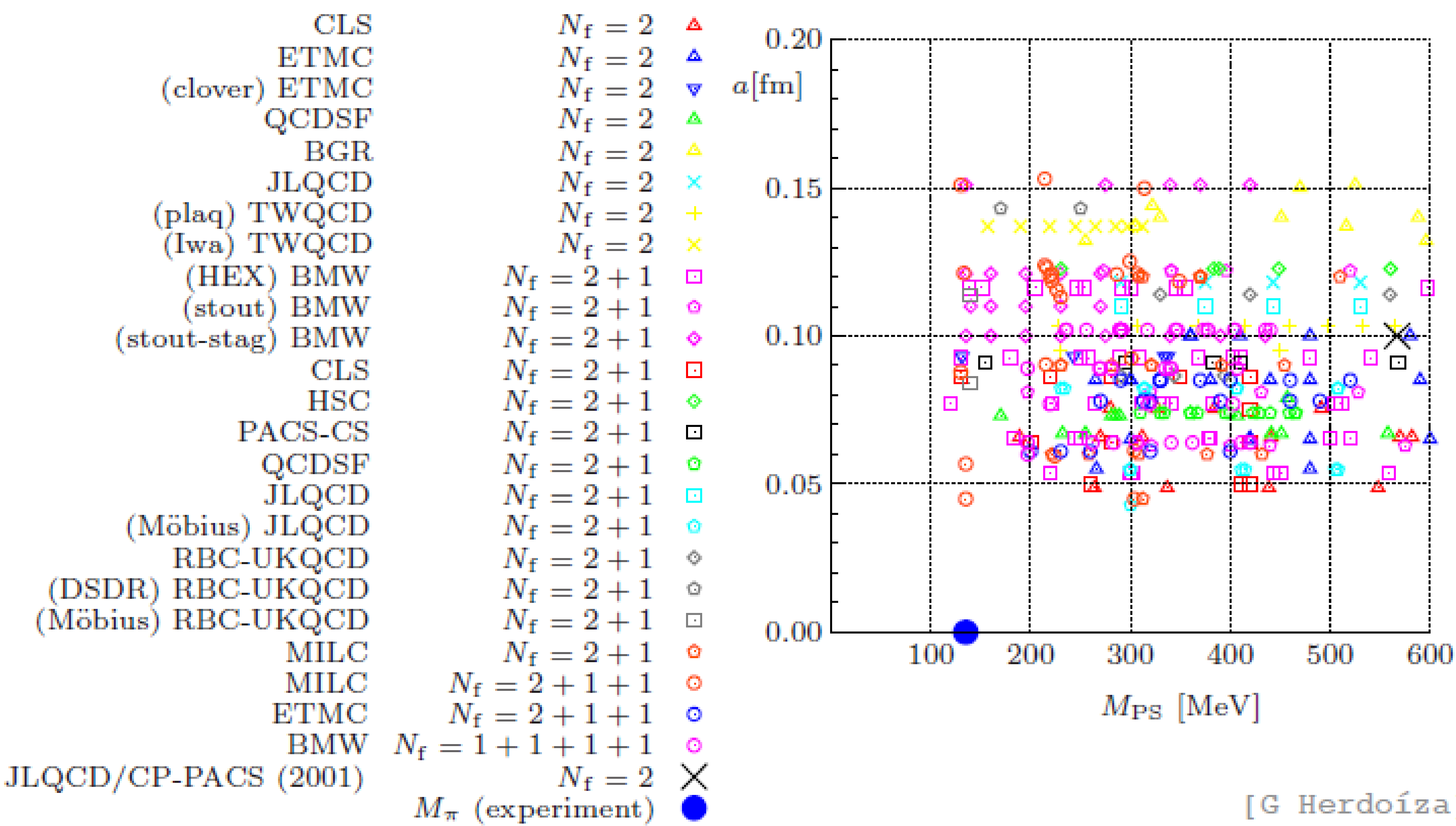




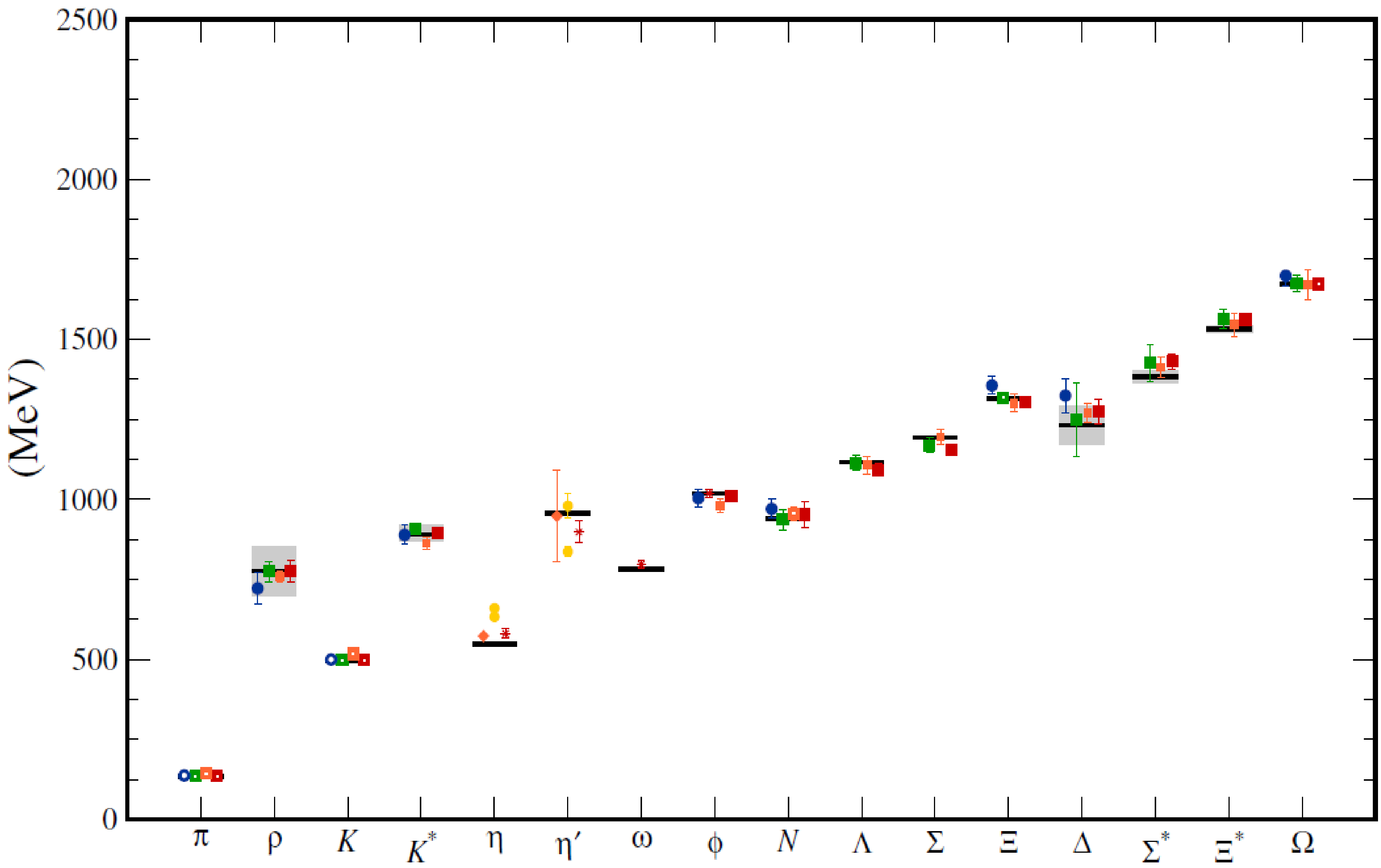
Numerical computations

ensembles/physics reach





[G Herdoíza]



Hadron Mass Spectrum

The electric dipole moment of the neutron from 2 + 1 flavor lattice QCD

QCDSF collaboration Phys.Rev.Lett. 115 (2015) 6, 062001

$$S = S_0 + S_\theta , \qquad S_\theta = i\,\theta\, Q$$

$$S_\theta = -\frac{i}{3}\theta\,\hat{m}\,a^4 \sum_x \left(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s\right)$$

$$\hat{m}^{-1} = \frac{1}{3}\left(m_u^{-1} + m_d^{-1} + m_s^{-1}\right)$$

V. Baluni, Phys. Rev. D **19** (1979) 2227.

$$\theta = i\,\bar{\theta}\,.$$

At nonvanishing vacuum angle θ the nucleon matrix element of the electromagnetic current

$$\langle p', s'|J_\mu|p, s\rangle = \bar{u}_\theta(\vec{p}\,', s')\,\mathcal{J}_\mu\, u_\theta(\vec{p}, s)$$

$$\mathcal{J}_\mu = \gamma_\mu F_1^\theta(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2^\theta(q^2)}{2m_N^\theta} + (\gamma q\, q_\mu - \gamma_\mu\, q^2)\,\gamma_5\, F_A^\theta(q^2) + \sigma_{\mu\nu} q_\nu\, \gamma_5 \frac{F_3^\theta(q^2)}{2m_N^\theta}$$

$$d_n = \frac{e\, F_3^\theta(0)}{2m_N^\theta}$$

calculations are done on $24^3 \times 48$ lattices

$$a = 0.074(2)\ \text{fm}$$

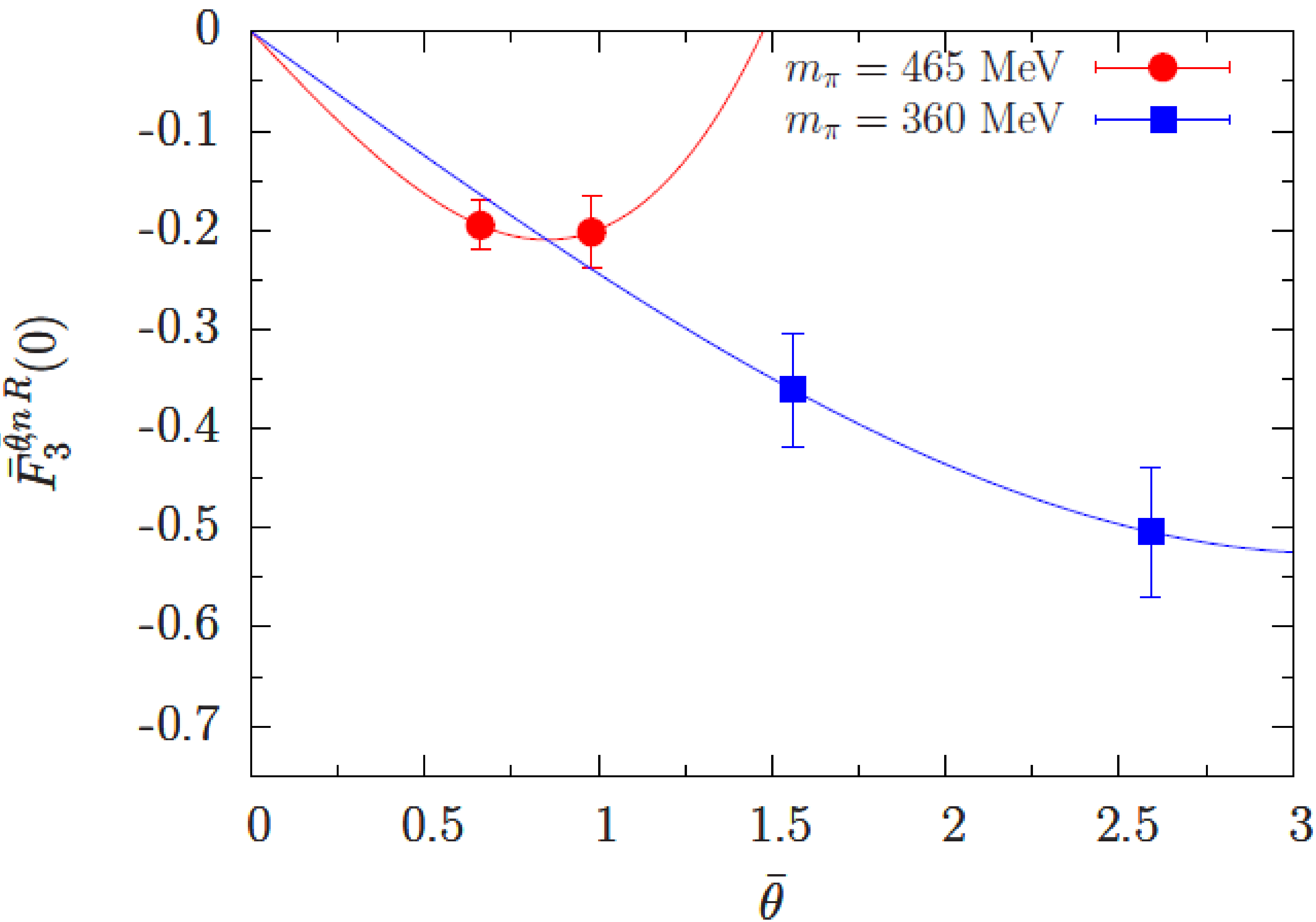
$$m_\pi = 465(13)\ \ \text{MeV}$$

$$m_\pi = 360(10)\ \ \text{MeV}$$

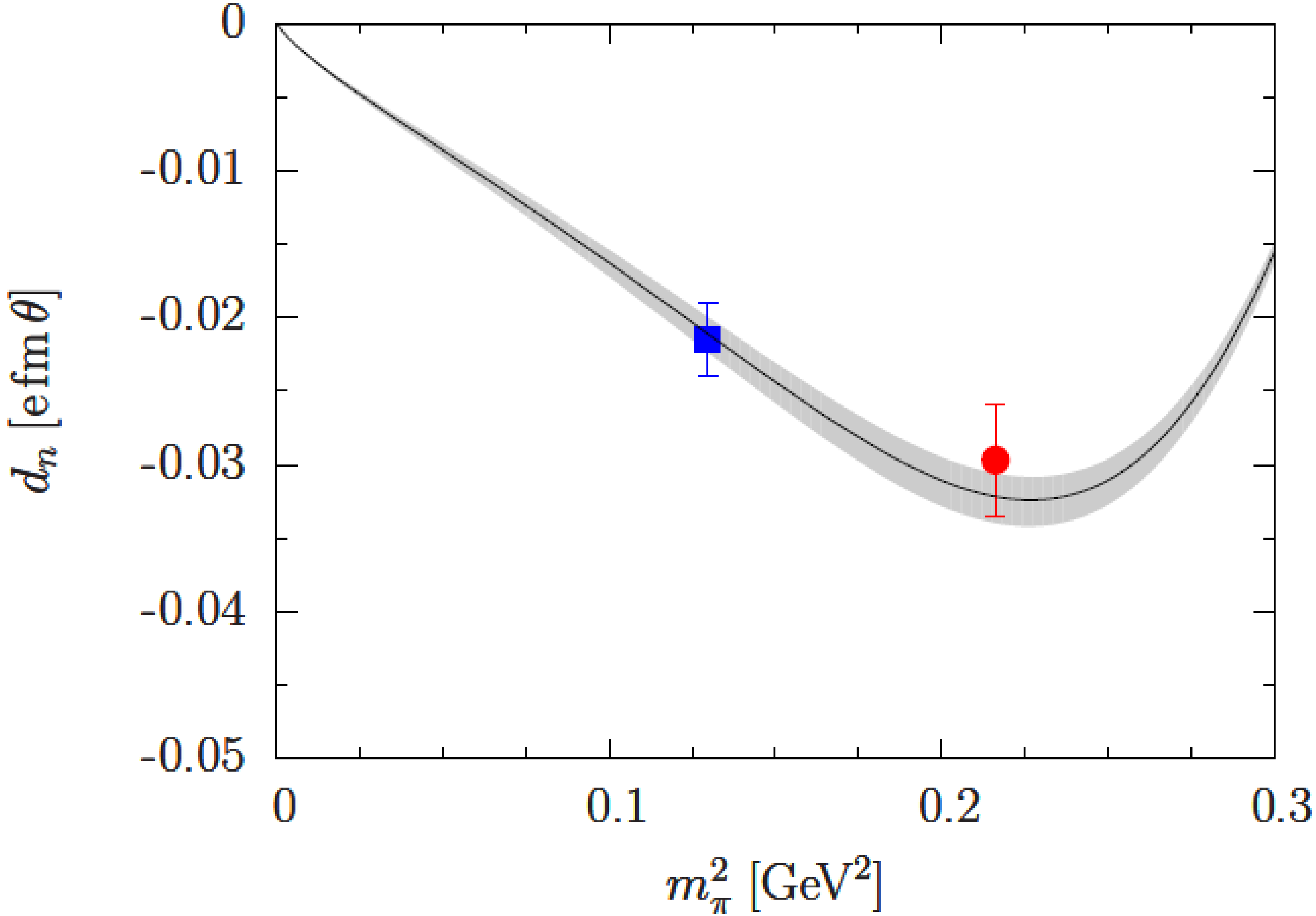
$$d_n = -0.0039(2)(9)\ [e\,\text{fm}\,\theta]$$

experimental bound on d_n : $|d_N^n| \leq 2.9 \times 10^{-13}\ [e\,\text{fm}]$.

$$|\theta| \lesssim 7.4 \times 10^{-11}$$



$m_\pi = 465$ MeV
$m_\pi = 360$ MeV
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
0.5
1
1.5
2
2.5
3
$\bar{\theta}$
$\bar{F}_3^{\bar{\theta}, nR}(0)$



0
-0.01
-0.02
-0.03
-0.04
-0.05
d_n [e fm θ]
0
0.1
0.2
0.3
m_π^2 [GeV^2]

Ab initio calculation of the neutron-proton mass difference

Wuppertal Science 347 (2015) 1452-1455

Lattice QCD + QED is simulated with 1+1+1+1 flavors:

$$S[U, A, \overline{\psi}, \psi] = S_g[U; g] + S_\gamma[A] + \sum_f \overline{\psi}_f D[U, A; e, q_f, m_f]\psi_f.$$

$$(D[U,A;e,q,m]\psi)_x =$$

$$(\frac{4}{a}+m)\psi_x -$$

$$-\frac{1}{2a}\sum_{\mu}\left[(1+\gamma_\mu)\exp(ieqa\tilde{A}_{\mu,x})\tilde{U}_{\mu,x}\psi_{x+\mu}+(1-\gamma_\mu)\exp(-ieqa\tilde{A}_{\mu,x-\mu})\tilde{U}^{\dagger}_{\mu,x-\mu}\psi_{x-\mu}\right]+$$

$$-\frac{ia}{4}\sum_{\nu>\mu}\left(F^{(\tilde{U})}_{\mu\nu,x}+eqF^{(\tilde{A})}_{\mu\nu,x}\right)[\gamma_\mu,\gamma_\nu]\psi_x.$$

$$S^{naive}_{\gamma}[A]=\frac{a^4}{4}\sum_{\mu,\nu,x}(\partial_\mu A_{\nu,x}-\partial_\nu A_{\mu,x})^2$$

$\alpha =$ 0, 1/6, 1/10, 1/137

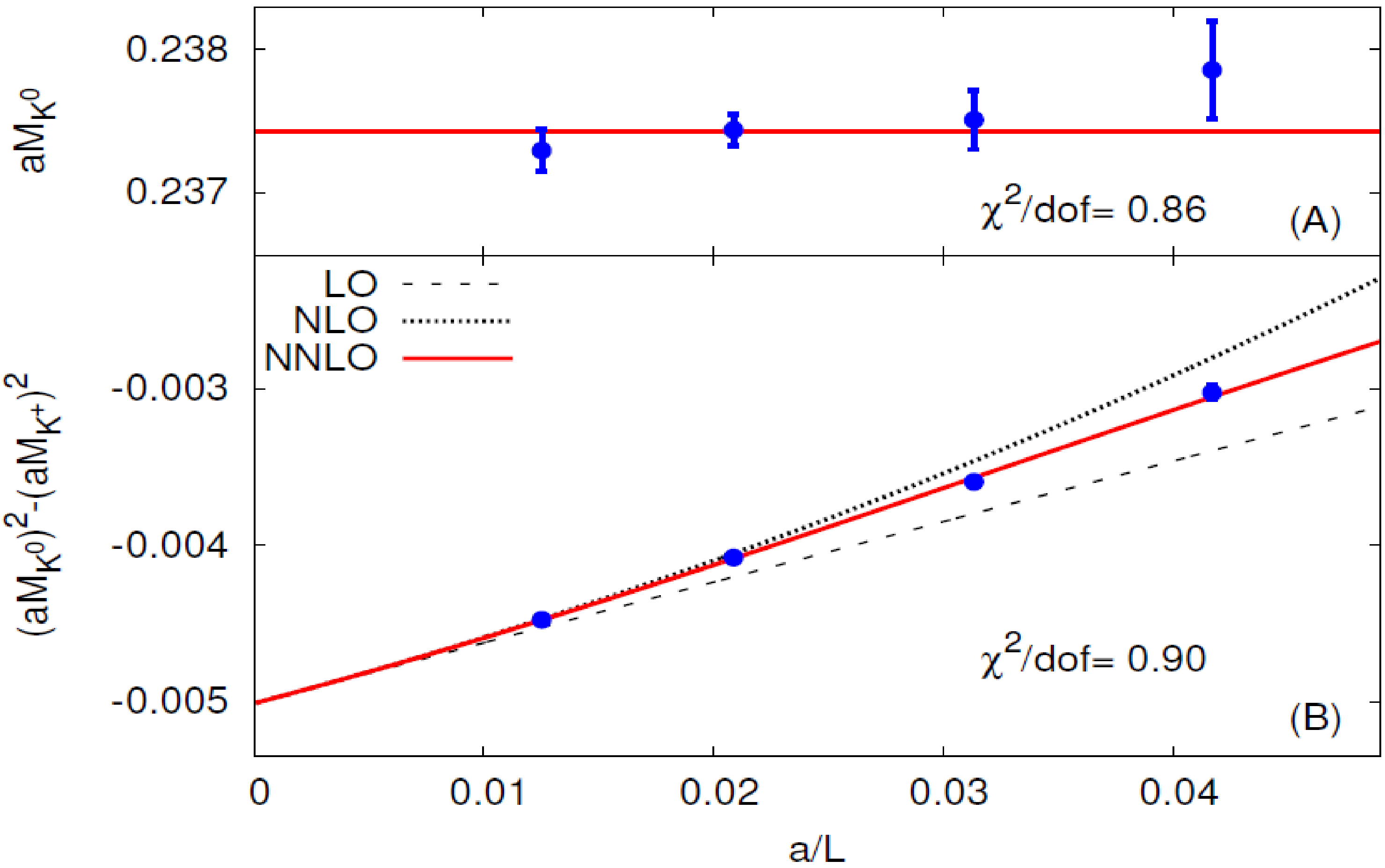


Figure 1: **Finite-volume behavior of kaon masses.** (**A**) The neutral kaon mass, M_{K^0}, shows no significant finite volume dependence; L denotes the linear size of the system. (**B**) The mass-squared difference of the charged kaon mass, M_{K^+}, and M_{K^0} indicates that M_{K^+} is strongly dependent on volume. This finite-volume dependence is well described by an asymptotic expansion in $1/L$ whose first two terms are fixed by QED Ward-Takahashi identities (*17*). The solid curve depicts a fit of the lattice results (points) to the expansion up to and including a fitted $O(1/L^3)$ term. The dashed and dotted curves show the contributions of the leading and leading plus next-to-leading order terms, respectively. The computation was performed by using the following parameters: bare $\alpha \sim 1/10$, $M_\pi = 290\,\text{MeV}$, and $M_{K^0} = 450\,\text{MeV}$. The mass difference is negative because a larger-than-physical value of α was used. The lattice spacing a is $\sim 0.10\,\text{fm}$.

	mass splitting [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta\Sigma = \Sigma^- - \Sigma^+$	8.09(16)(11)	8.09(16)(11)	0
$\Delta\Xi = \Xi^- - \Xi^0$	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D = D^\pm - D^0$	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta\Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^{+}$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{CG} = \Delta N - \Delta\Sigma + \Delta\Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)

Table 1: **Isospin mass splittings of light and charm hadrons.** Also shown are the individual contributions to these splittings from the mass difference $(m_d - m_u)$ (QCD) and from electromagnetism (QED). The separation requires fixing a convention, which is described in (*17*). The last line is the violation of the Coleman-Glashow relation (*30*), which is the most accurate of our predictions.

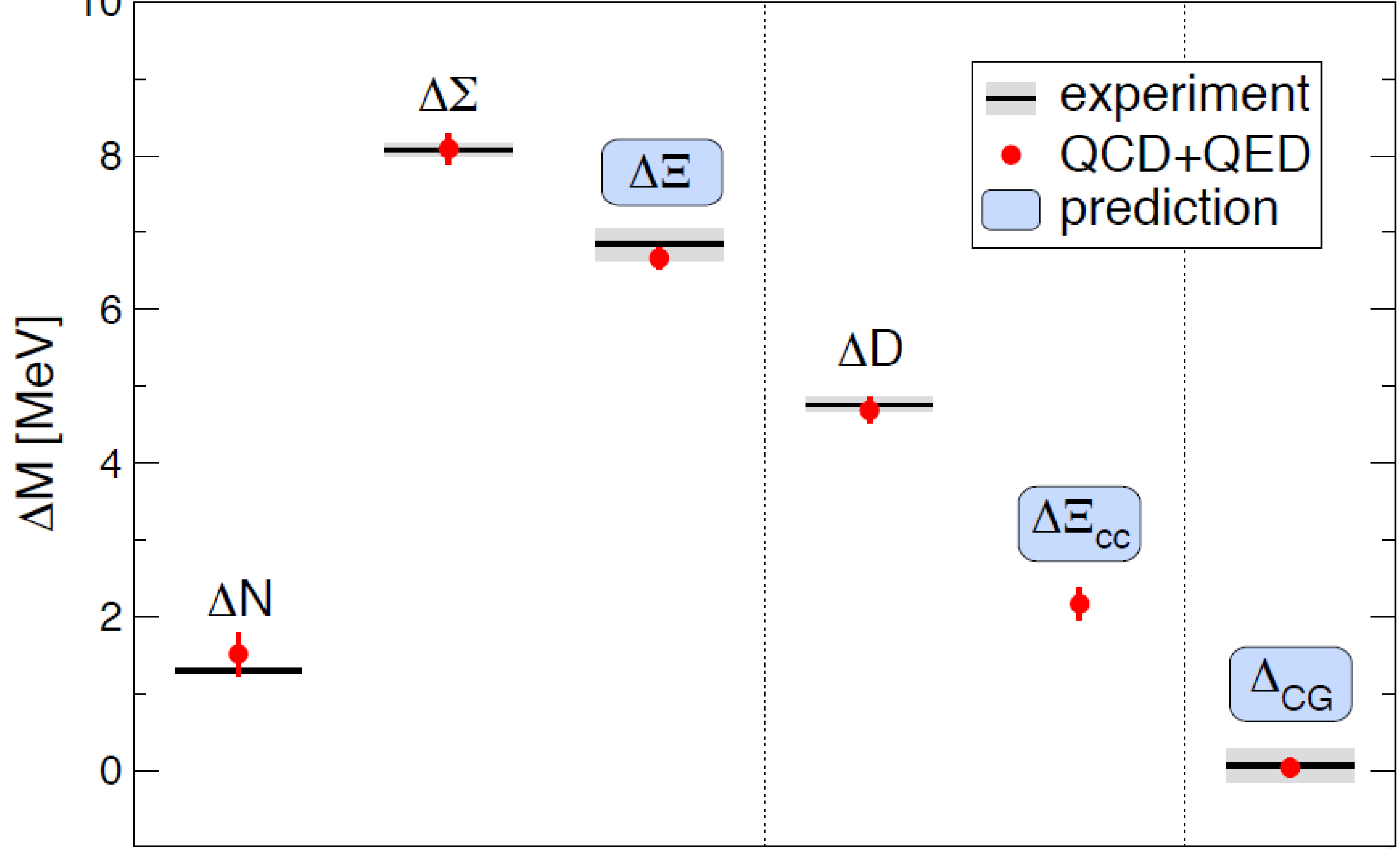


Figure 2: **Mass splittings in channels that are stable under the strong and electromagnetic interactions.** Both of these interactions are fully unquenched in our 1+1+1+1 flavor calculation. The horizontal lines are the experimental values and the grey shaded regions represent the experimental error (2). Our results are shown by red dots with their uncertainties. The error bars are the squared sums of the statistical and systematic errors. The results for the ΔM_N, ΔM_Σ, and ΔM_D mass splittings are post-dictions, in the sense that their values are known experimentally with higher precision than from our calculation. On the other hand, our calculations yield ΔM_Ξ, $\Delta M_{\Xi_{cc}}$ splittings, and the Coleman-Glashow difference Δ_{CG}, which have either not been measured in experiment or are measured with less precision than obtained here. This feature is represented by a blue shaded region around the label.

The neutron–proton mass difference, one of the most consequential parameters of physics, has now been calculated from fundamental theories. This landmark calculation portends revolutionary progress in nuclear physics.

Frank Wilczek

Nature, 520, 303–304

LATTICE INPUTS TO FLAVOR PHYSICS

FLAG (Flavor Lattice Averaging Group) project

e-Print: arXiv:1507.04051 [hep-ph]

To produce global averages/estimates and to review virtues and shortcomings of the different computations in a transparent way, which should be accessible also to the non-experts. This is the goal of the FLAG initiative.

quality criteria:

- Chiral extrapolation:

  ★ $M_{\pi,\mathrm{min}} < 200$ MeV

  ○ $200 \text{ MeV} \leq M_{\pi,\mathrm{min}} \leq 400 \text{ MeV}$

  ■ $400 \text{ MeV} < M_{\pi,\mathrm{min}}$

  in addition it is assumed that the chiral extrapolation is done using at least three points.

- Continuum extrapolation:

  ★ 3 or more lattice spacings, at least 2 points below 0.1 fm

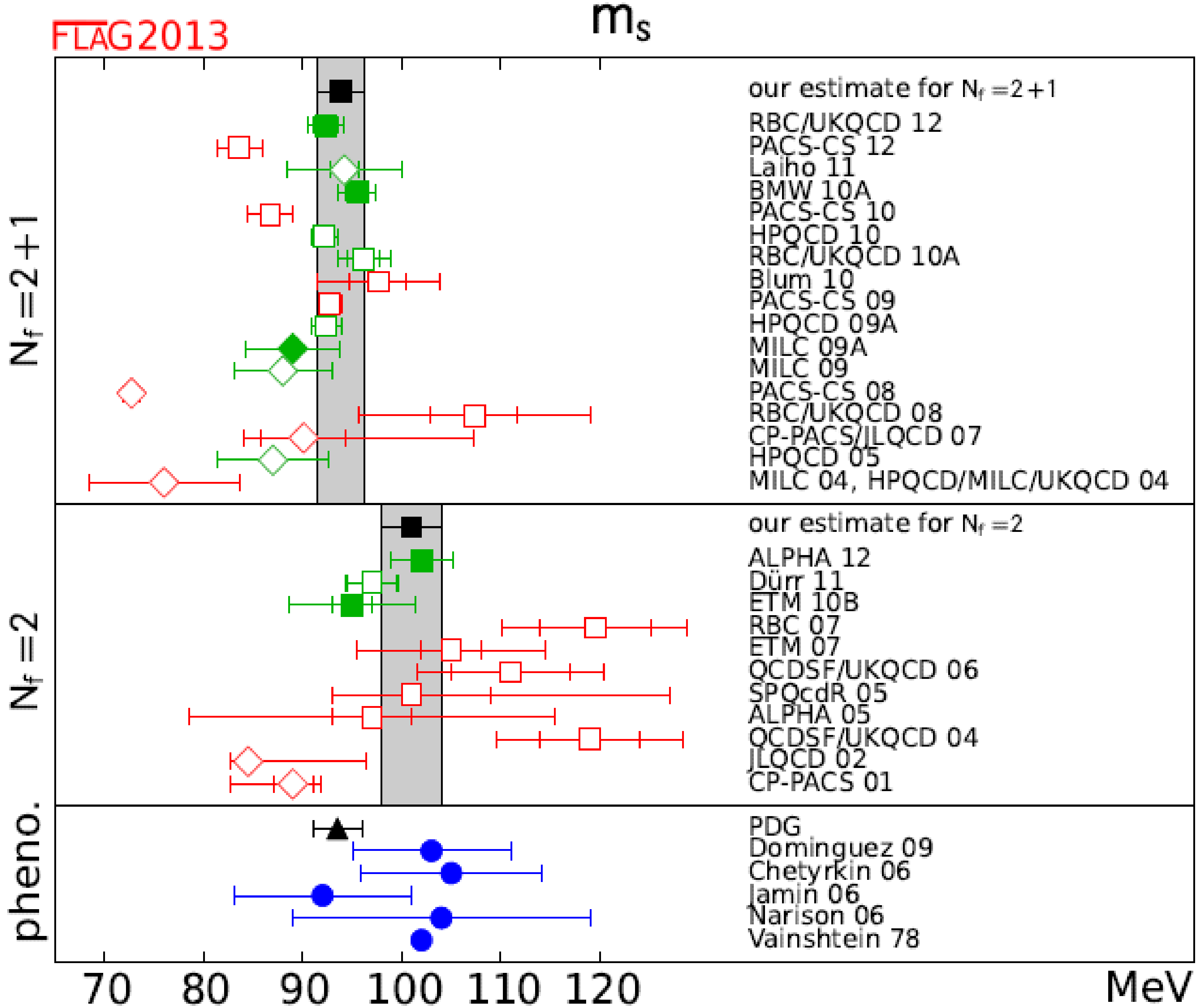
  ○ 2 or more lattice spacings, at least 1 point below 0.1 fm

  ■ otherwise

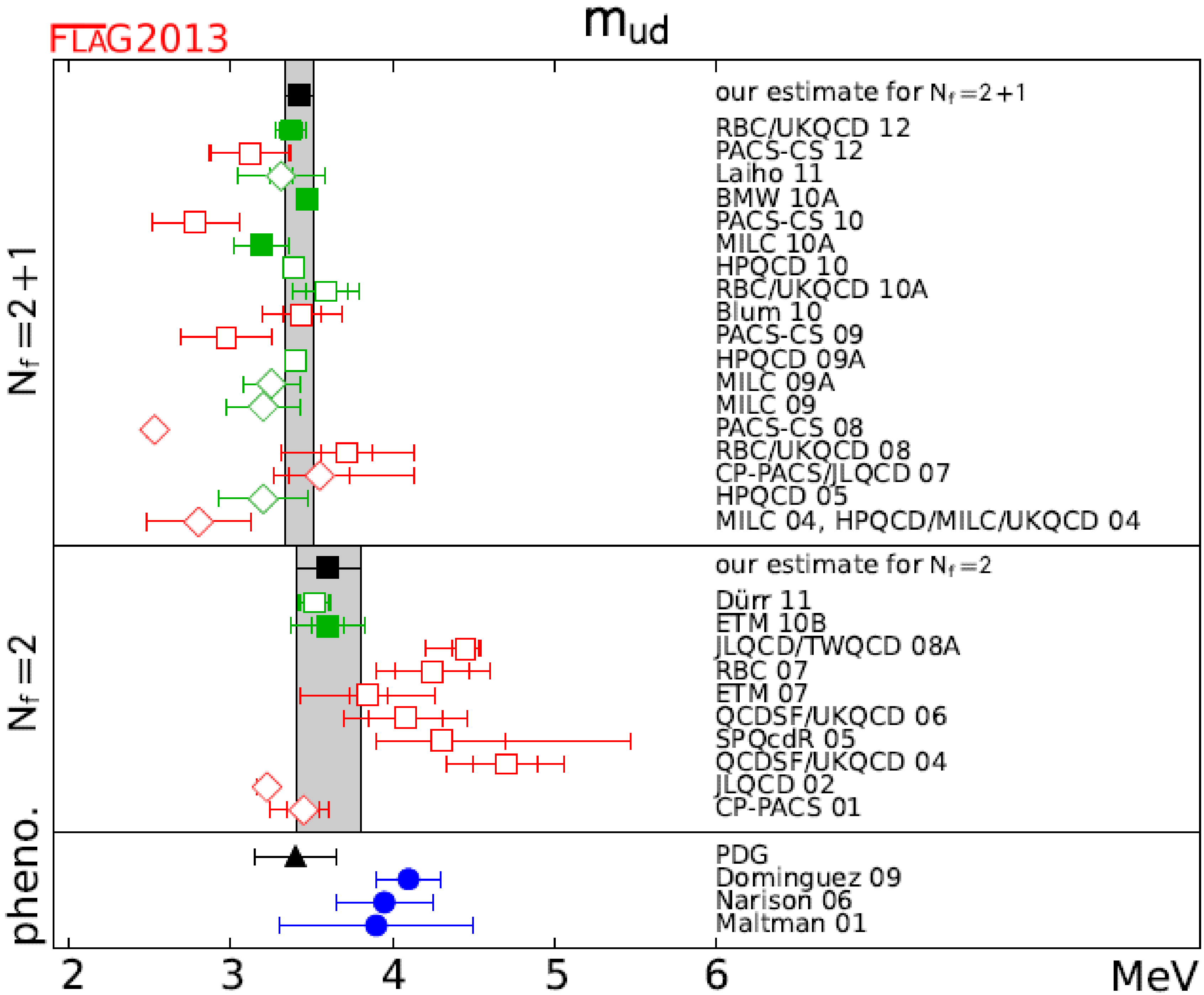
  in addition it is assumed that the action is $O(a)$-improved (i.e. the discretization errors vanish quadratically with the lattice spacing).

- Finite-volume effects:
  - ★ $M_{\pi,\mathrm{min}}L > 4$ or at least 3 volumes
  - ○ $M_{\pi,\mathrm{min}}L > 3$ and at least 2 volumes
  - ■ otherwise.

- Renormalization (where applicable):
  - ★ non-perturbative
  - ○ 1-loop perturbation theory or higher with a reasonable estimate of truncation errors
  - ■ otherwise.

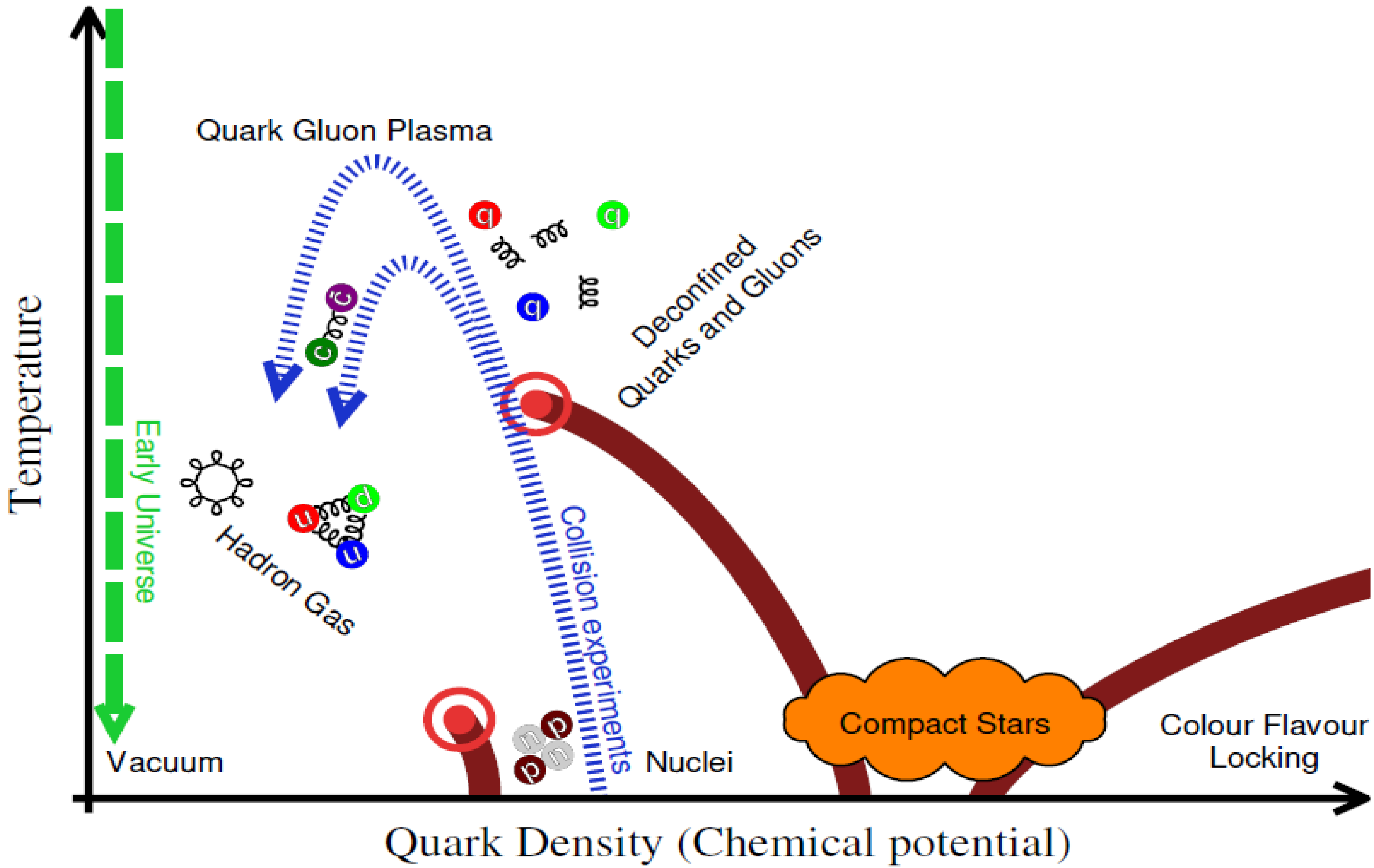


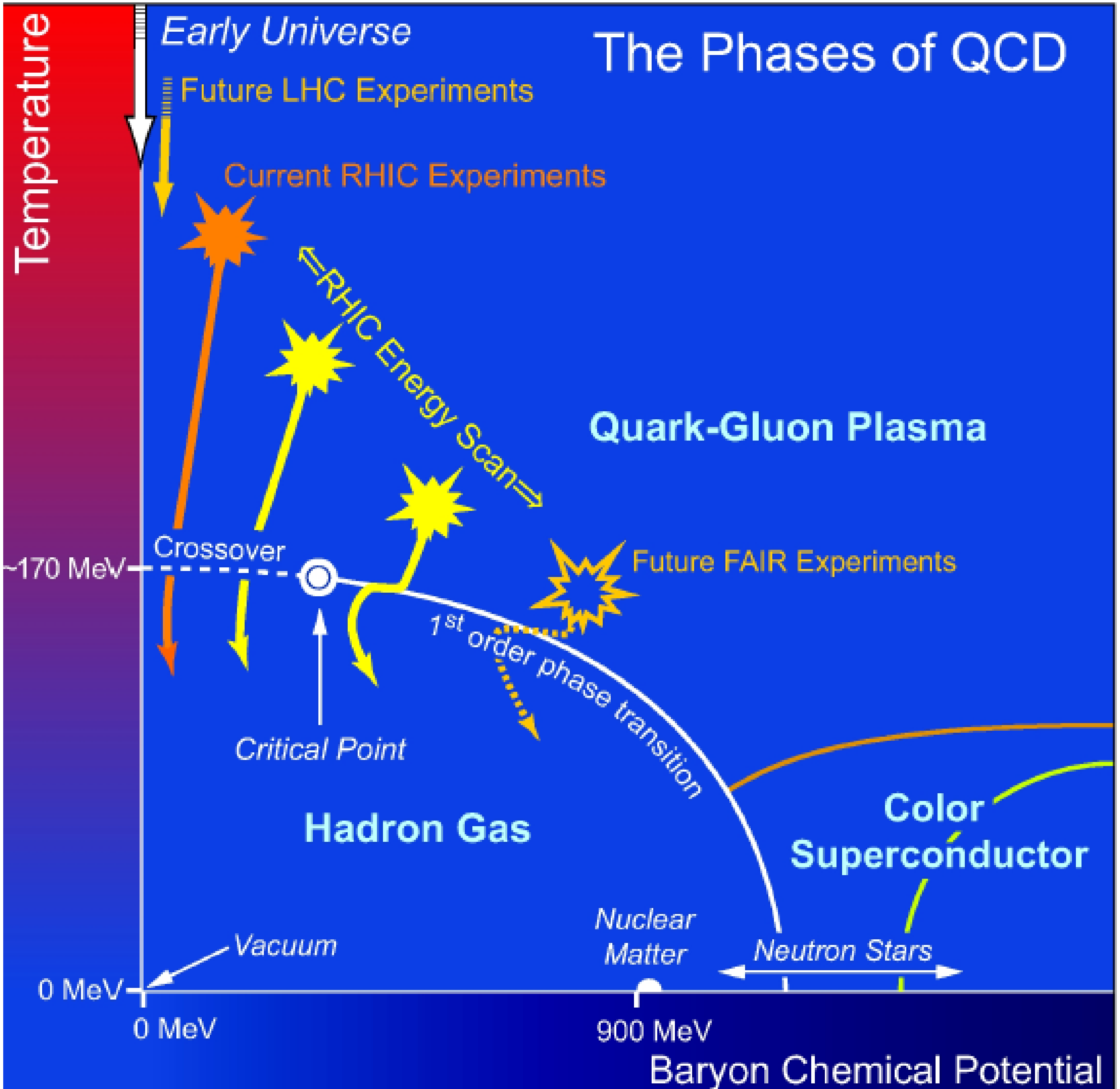
FLAG 2013
m_s
our estimate for $N_f = 2+1$
RBC/UKQCD 12
PACS-CS 12
Laiho 11
BMW 10A
PACS-CS 10
HPQCD 10
RBC/UKQCD 10A
Blum 10
PACS-CS 09
HPQCD 09A
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS/JLQCD 07
HPQCD 05
MILC 04, HPQCD/MILC/UKQCD 04
our estimate for $N_f = 2$
ALPHA 12
Dürr 11
ETM 10B
RBC 07
ETM 07
QCDSF/UKQCD 06
SPQcdR 05
ALPHA 05
QCDSF/UKQCD 04
JLQCD 02
CP-PACS 01
PDG
Dominguez 09
Chetyrkin 06
Jamin 06
Narison 06
Vainshtein 78
$N_f = 2+1$
$N_f = 2$
pheno.
70
80
90
100
110
120
MeV


FLAG 2013
m_{ud}
our estimate for $N_f = 2+1$
RBC/UKQCD 12
PACS-CS 12
Laiho 11
BMW 10A
PACS-CS 10
MILC 10A
HPQCD 10
RBC/UKQCD 10A
Blum 10
PACS-CS 09
HPQCD 09A
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS/JLQCD 07
HPQCD 05
MILC 04, HPQCD/MILC/UKQCD 04
our estimate for $N_f = 2$
Dürr 11
ETM 10B
JLQCD/TWQCD 08A
RBC 07
ETM 07
QCDSF/UKQCD 06
SPQcdR 05
QCDSF/UKQCD 04
JLQCD 02
CP-PACS 01
PDG
Dominguez 09
Narison 06
Maltman 01
$N_f = 2+1$
$N_f = 2$
pheno.
2
3
4
5
6
MeV

Fluctuations at finite temperature and density

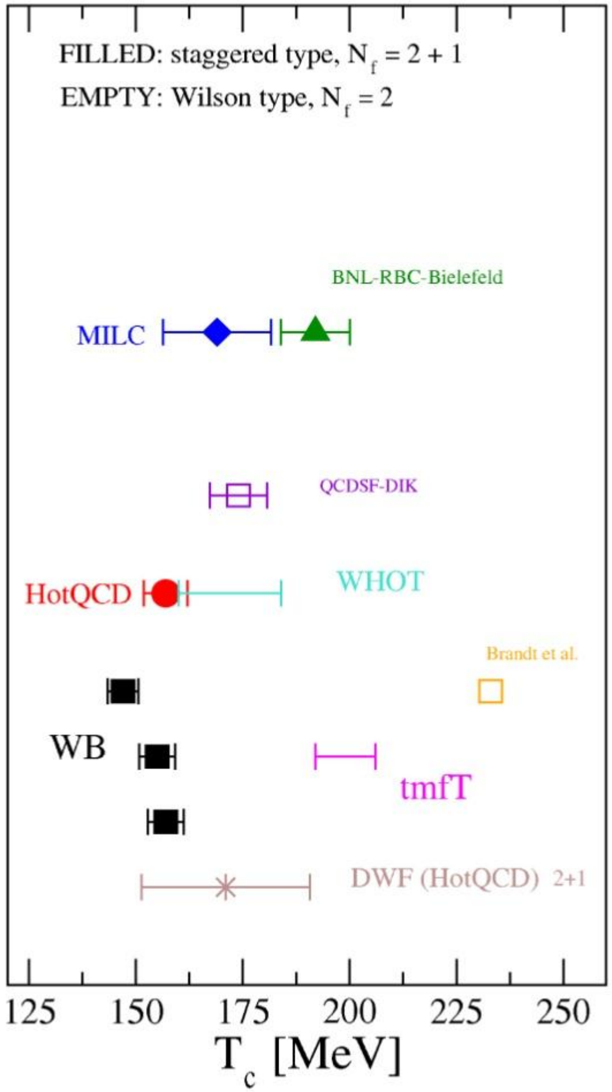
Phys Rev Lett 113(2014)052301





The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
Crossover
~170 MeV
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Nuclear Matter
Vacuum
Neutron Stars
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

Transition temperature from a variety of studies



- Staggered types, $N_f = 2 + 1$: p4, asqtad, HISQ, stout — already introduced. Data from only chiral type observables.
- Wilson types, $N_f = 2$:
  - QCDSF-DIK [arXiv:0910.2392], clover + plaquette, $N_t = 8 - 14$
  - WHOT-QCD [arXiv:0909.2121], clover + Iwasaki, $N_t = 4 - 6$
  - Brandt et al. [arXiv:1011.6172], clover, $N_t = 16$
  - tmfT (Florian Burger talk), mtmWilson + tree-level Symazik, $N_t = 8 - 12$
- DWF (HotQCD), $N_f = 2{+}1$:+Iwasaki, $N_t = 8$, $L_s = 32 - 96$

The expectation value of a conserved charge is a derivative with respect to the chemical potential.

$$\langle N_q \rangle = T \frac{\partial \log Z(T, V, \{\mu_q\})}{\partial \mu_q}$$

The response of the system to the thermodynamic force μ_q is proportional to the fluctuation of the conserved charge:

$$\frac{\partial \langle N_i \rangle}{\partial \mu_j} = T \frac{\partial^2 \log Z(T, V, \{\mu_q\})}{\partial \mu_j \partial \mu_i} = \frac{1}{T}(\langle N_i N_j \rangle - \langle N_i \rangle \langle N_j \rangle)$$

The higher derivatives are the generalized quark number susceptibilities:

$$\chi_{i,j,k,l}^{u,d,s,c} = \frac{\partial^{i+j+k+l}(p/T^4)}{(\partial \hat{\mu}_u)^i (\partial \hat{\mu}_d)^j (\partial \hat{\mu}_s)^k (\partial \hat{\mu}_c)^l}$$

with $\hat{\mu}_q = \mu_q / T$.

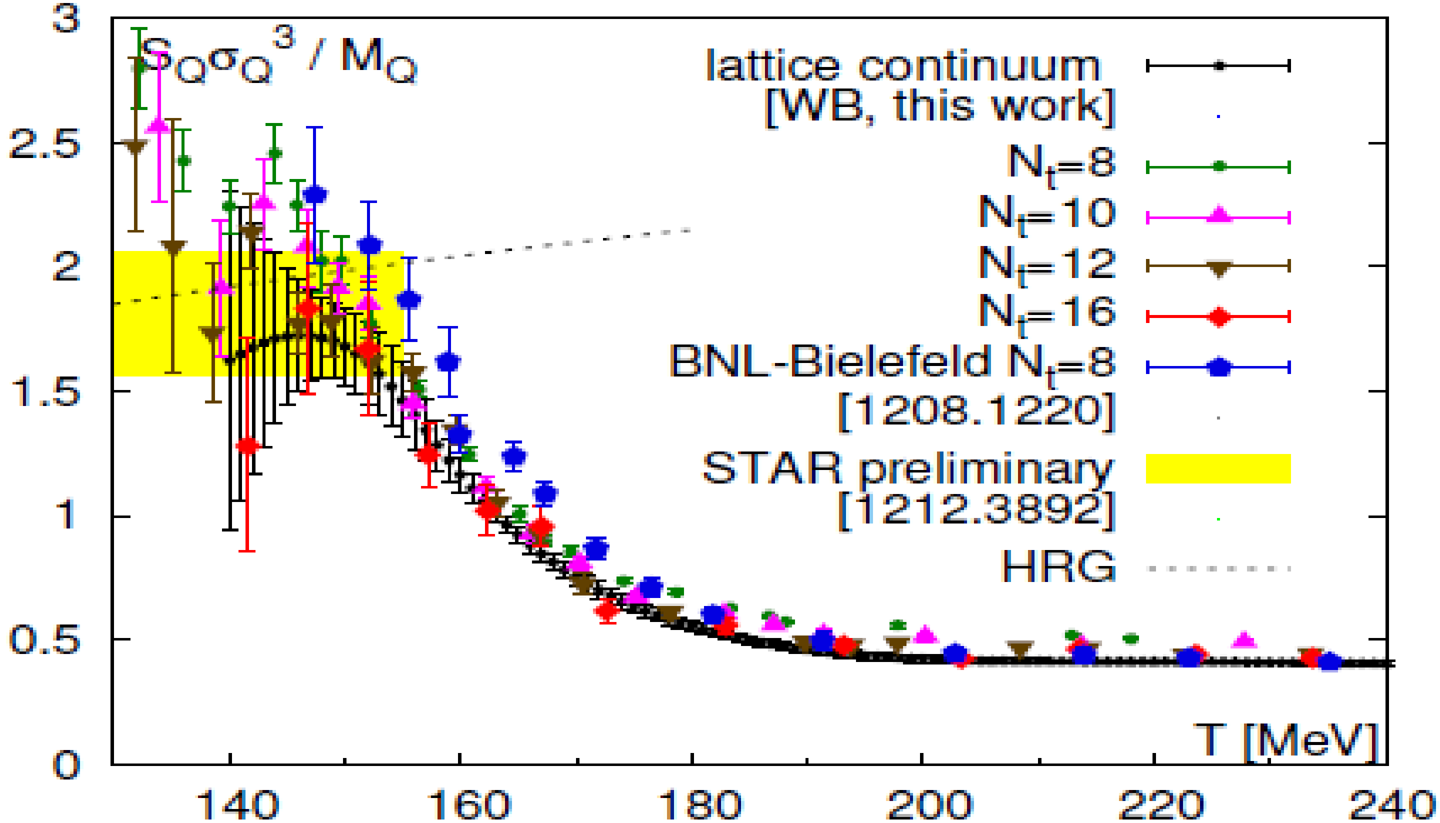
$$\text{mean}: \; M = \chi_1 \quad \text{variance}: \; \sigma^2 = \chi_2$$
$$\text{skewness}: \; S = \chi_3/\chi_2^{3/2} \quad \text{kurtosis}: \; \kappa = \chi_4/\chi_2^2 .$$

The comparison between the experimental and lattice QCD results for the electric charge and baryon number fluctuations allows a first-principles determination of the freeze-out temperature and chemical potential, under the assumption that the experimentally measured fluctuations can be described in terms of the equilibrium system simulated on the lattice.

A possible thermometer for T_{ch}

$$S\sigma^3/M\big|_{\text{experiment}}\,(\text{beam energy}) = S\sigma^3/M\big|_{\text{lattice}}\,(T_{ch})$$

Net electric charge:

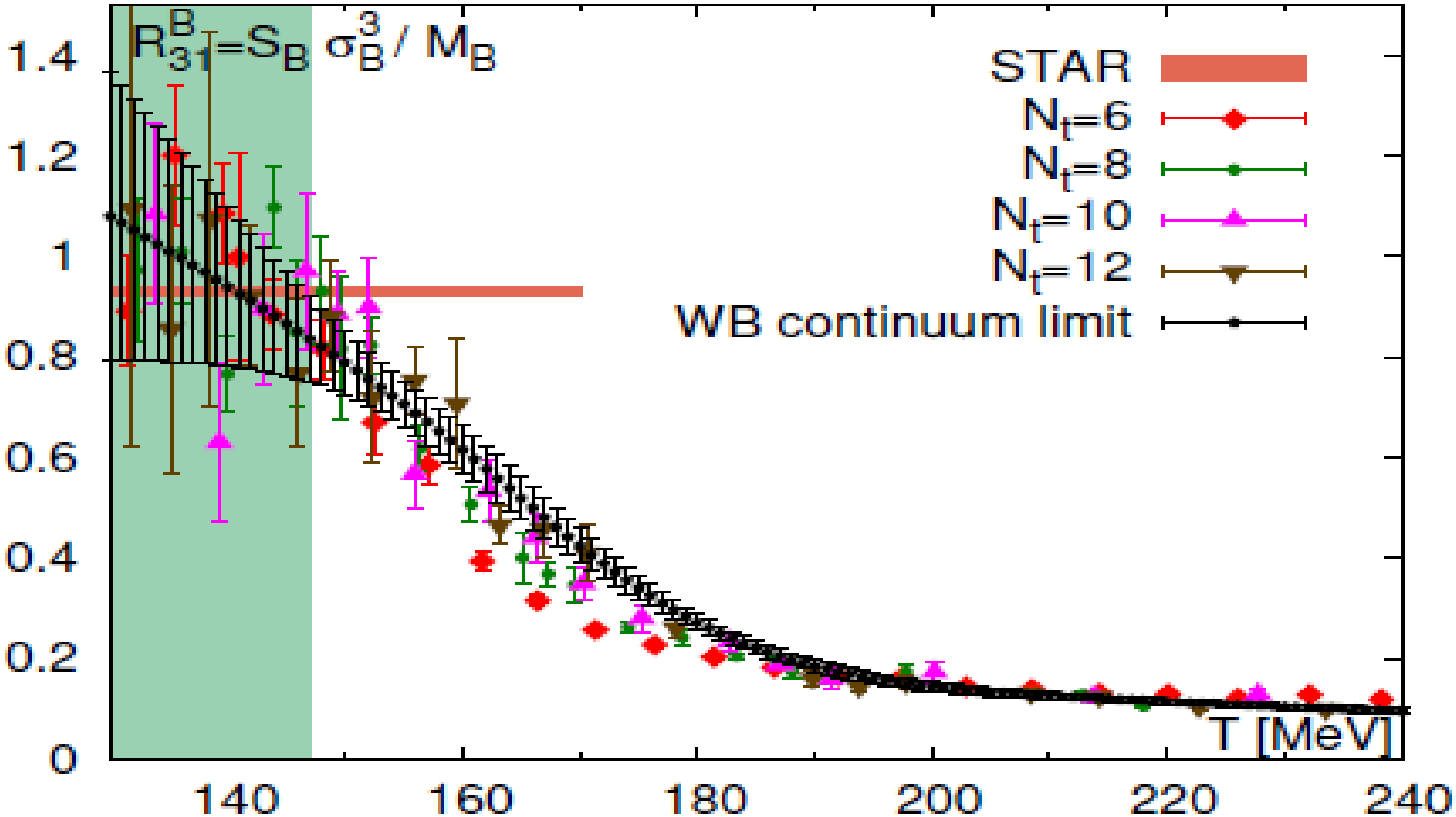


[Wuppertal-Budapest 1304.5161],

[STAR 1402.1558].

Conclusion $T_{ch} \leq 157\mathrm{MeV}$

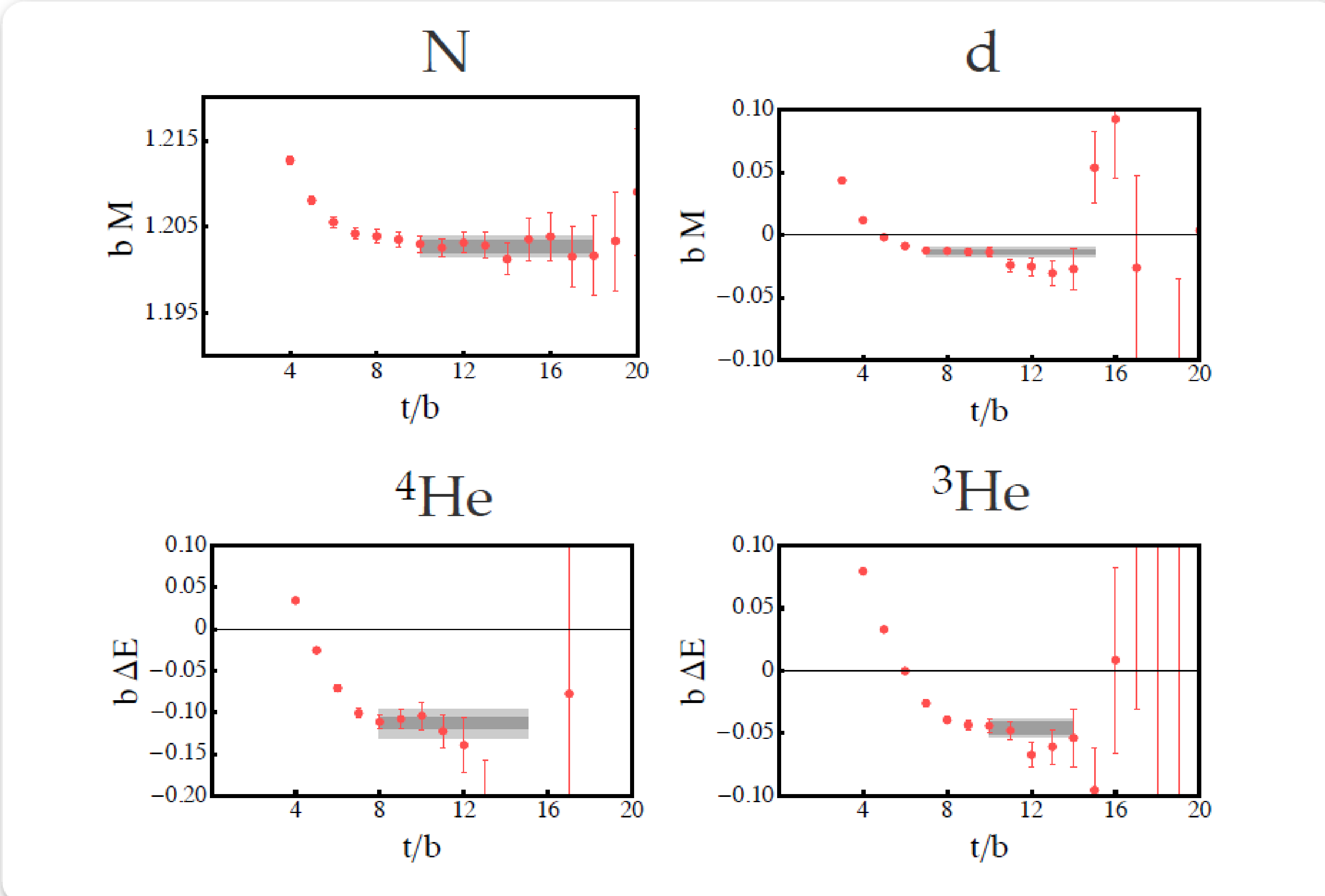
Net baryon number:



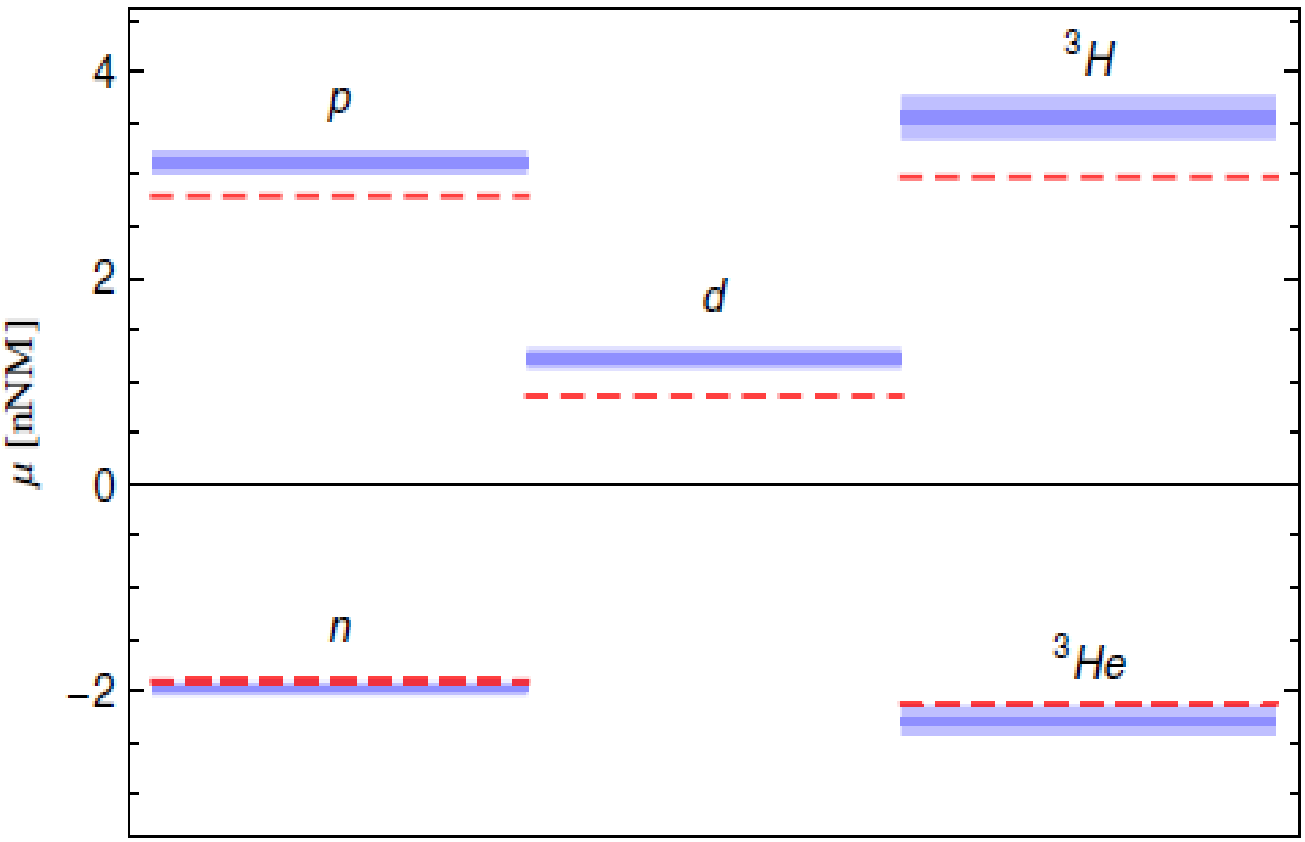
[Wuppertal-Budapest 1403.4576],

[STAR 1309.5681] (protons).

Conclusion $T_{ch} \leq 151\mathrm{MeV}$



Mass of neucleon, binding energy of dueteron, ^{4}He, ^{3}He (in units 1/b)



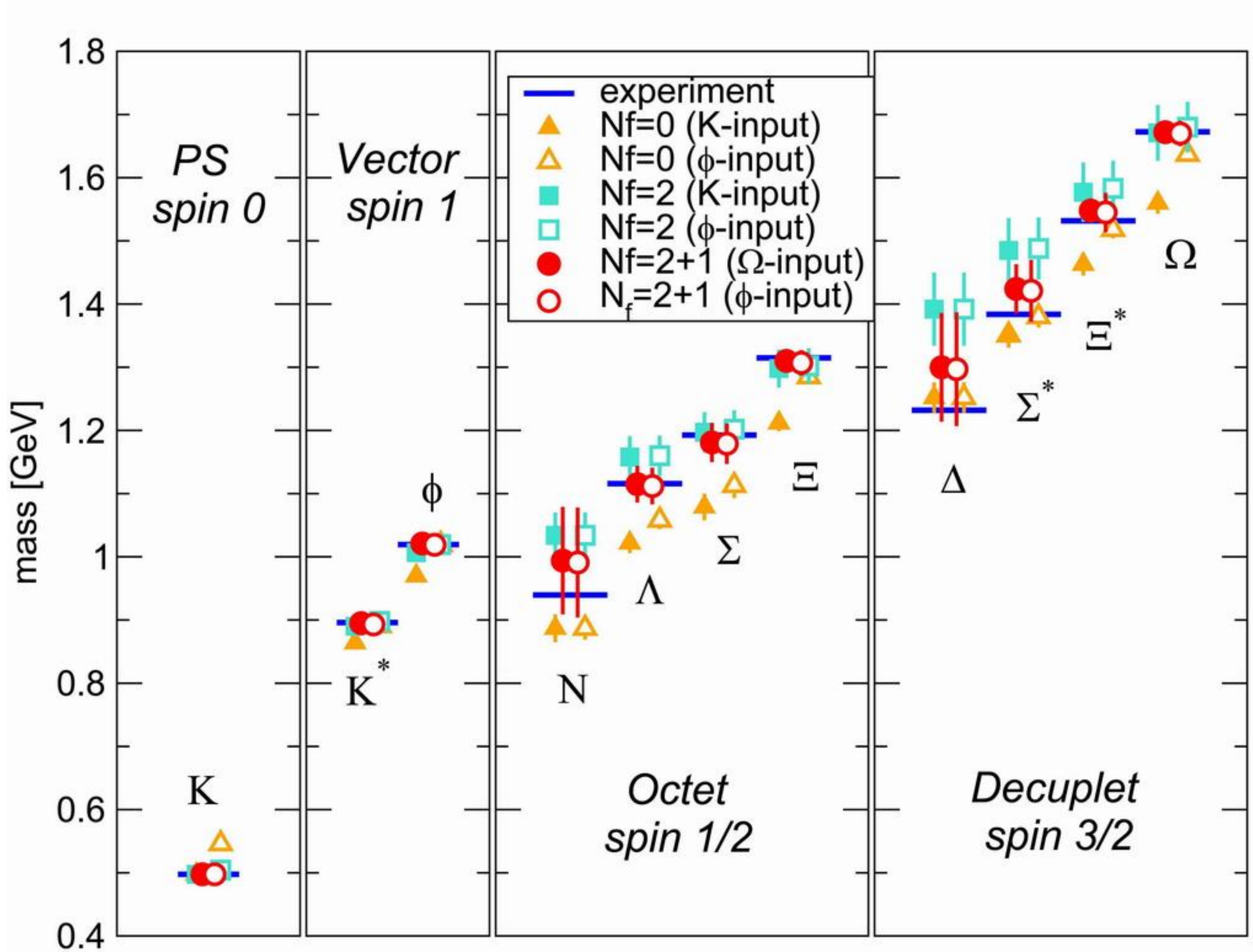
The magnetic moments of proton, nuetron, dueteron, triton and ^{3}He.
Dashed lines show experimental values

SU(2) glue SU(3) glue 2qQCD (2+1)QCD

Wilson non-perturbatively improved Fermions
“WORKING HORSE” of lattice QCD calculations

Y. Kuramashi Lattice 2007

Iwasaki gauge action +
clover quarks
a^(−1) = 2.2GeV,
lattice size: 32^3 × 64



APS 2005, Tampa meeting

created matter at a temperature of about 4 trillion degrees Celsius — the hottest temperature ever reached in a laboratory, about 250,000 times hotter than the center of the Sun

using a giant atom smasher said on they have created a new state of matter - a hot, dense liquid made out of basic atomic particles - and said it shows what the early universe looked like for a very, very brief time.

"We think we are looking at a phenomenon ... in the universe 13 billion years ago when free quarks and gluons ... cooled down to the particles that we know today," Aronson told a news conference carried by telephone from a meeting of the American Physical Society in Tampa, Fla.

Liquid, not a gas
The quark-gluon plasma was made in the Relativistic Heavy Ion Collider — a powerful atom smasher at Brookhaven National Laboratory in Upton, N.Y. Unexpectedly, the quark-gluon plasma behaved like a perfect liquid of quarks, instead of a gas, the physicists said.

Evidence of 5-th state of matter in heavy ions collisions

1. Thermalisation
2. Elliptic flow
3. Jet quenching
4. Spectrum of photons
5. Shear viscosity eta/s and hydrodynamic approach
6. Lattice calculations vs experiment
…………………………

After freezeout we have thermalized hadron gaz!

Chemical equilibrium,a hadro-chemical equilibrium model.

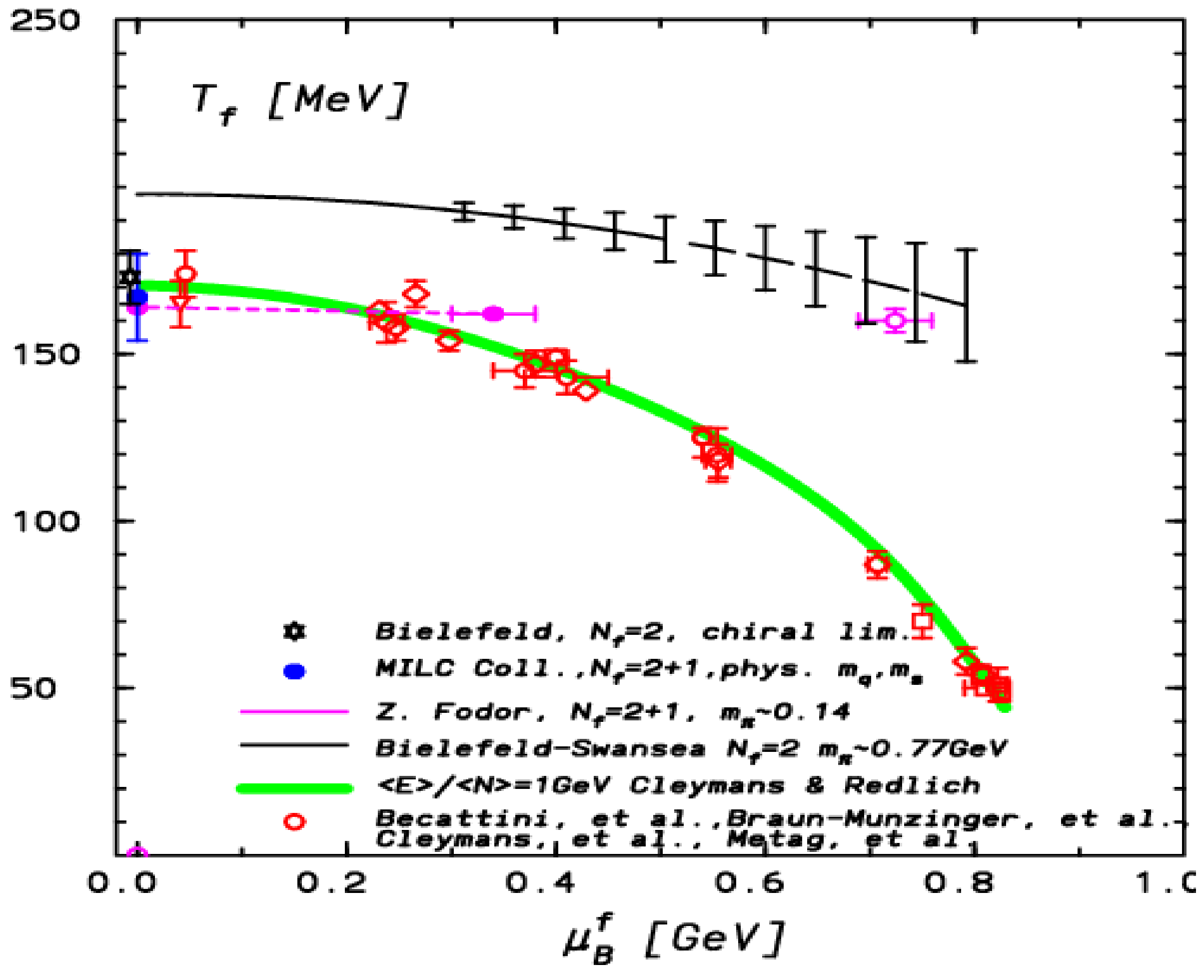
We know that from experimental distributions

Grand Canonical Ensemble for ideal gaz

$$Z = \sum_{N=0}^{\infty} \sum_{\{n_i\}} \prod_i e^{-\beta n_i(\varepsilon_i - \mu)}, \quad \beta = \frac{1}{kT}$$

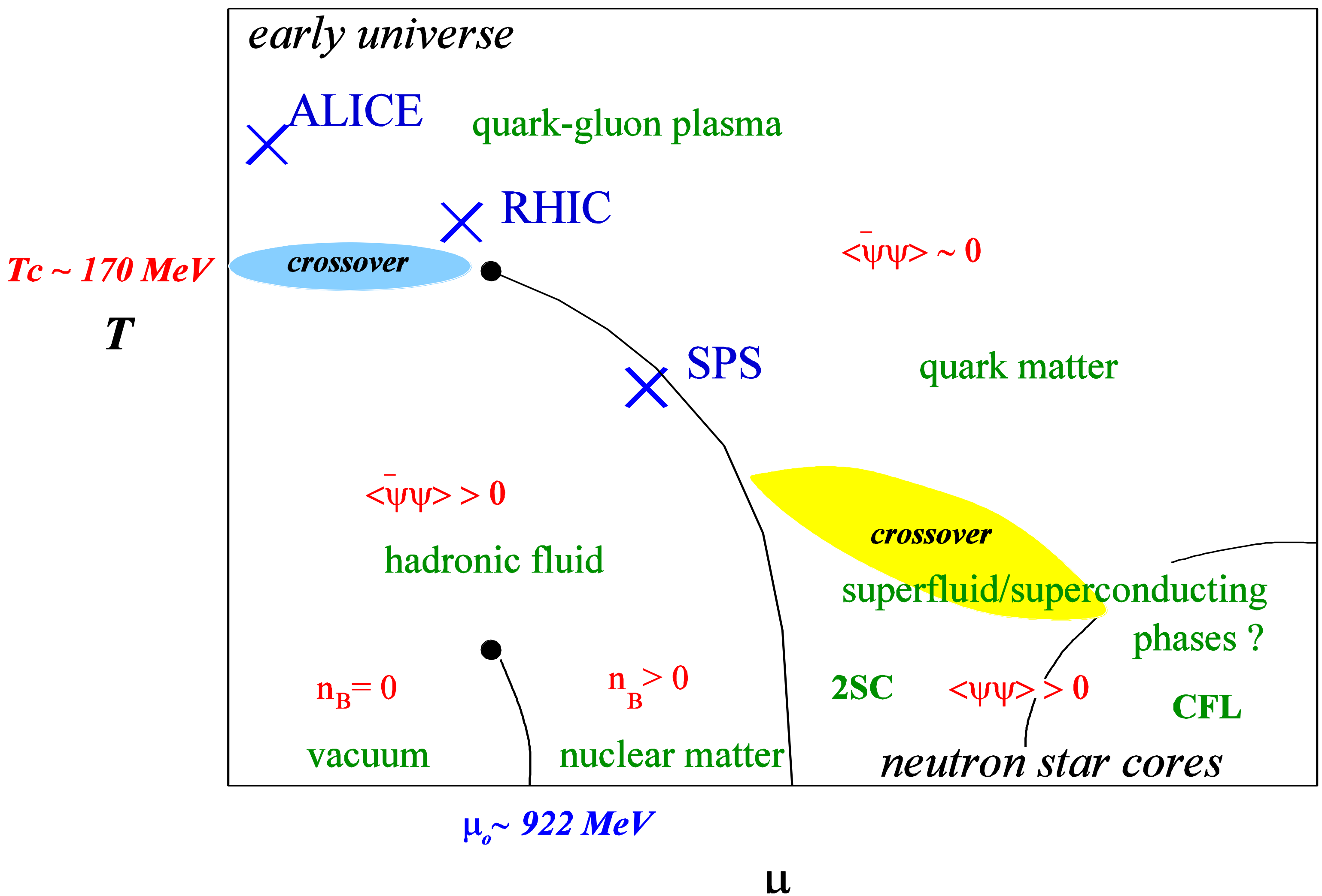
❍ is the chemical potential; n_i is the number of particles in the i-th state.

Lattice calculations
QCD on supercomputers



The main result of these investigations was that the extracted temperature values rise rather sharply from low energies on towards √sNN ≃10 GeV and reach afterwards constant values near T=160 MeV, while the baryochemical potential decreases smoothly as a function of energy. arXiv:1106.6321v1, A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel

Phase diagram of QCD



Critical temperature

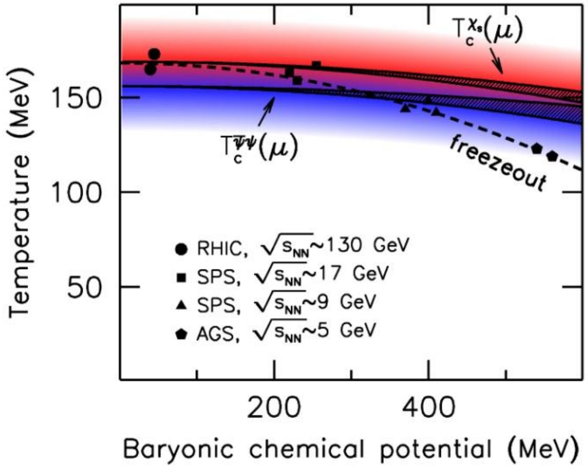
Critical line in the $\mu_B - T$ plane for $N_f = 2+1$

- Results from G. Endrödi et al. [arXiv:1102.1356] with Taylor expansion method, $N_t = 6, 8, 10$, physical quarks and stout action; continuum limit taken:

$$T_c(\mu_B^2) = T_c\left(1 - \kappa \cdot \mu_B^2/T_c^2\right), \qquad \kappa = -T_c \left.\frac{dT_c(\mu_B^2)}{d(\mu_B^2)}\right|_{\mu_B=0} = -T_c\left[-\left.\left(\frac{\partial\phi}{\partial\mu_B^2}\right)\right|_{T_c,\mu_B=0} \Big/ \left.\left(\frac{\partial\phi}{\partial T}\right)\right|_{T_c,\mu_B=0}\right]$$

$$\kappa^{(\chi_s/T^2)} = 0.0089(14), \qquad \kappa^{(\bar\psi\psi_r)} = 0.0066(20).$$

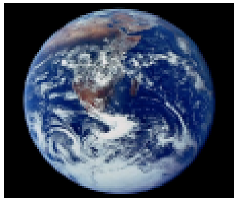
- Kaczmarek et al. [PRD83 2011] for 2+1 flavors: $\kappa = 0.0066(5)$.
- Compare with $N_f = 2$ (Leonardo Cosmai talk) $\kappa = 0.0059(1)$, WHOT-QCD (2010) $\kappa \approx 0.0078$. Freezeout curve: $\kappa \approx 0.02$.

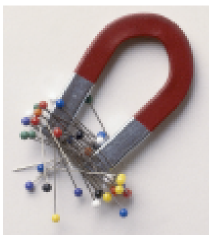


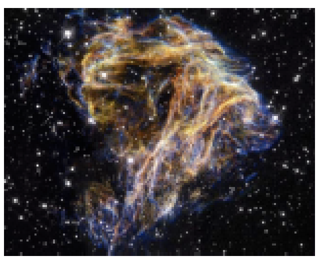
L. Levkova; Talk at Lattice 2011

Comparison of magnetic fields

D.Kharzeev

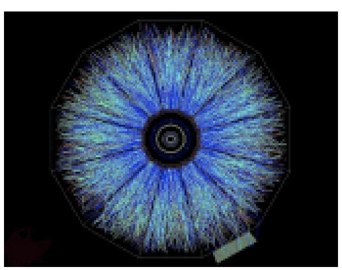






The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau = 0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

Chiral Magnetic Effect

[Fukushima, Kharzeev, Warringa, McLerran '07-'08]

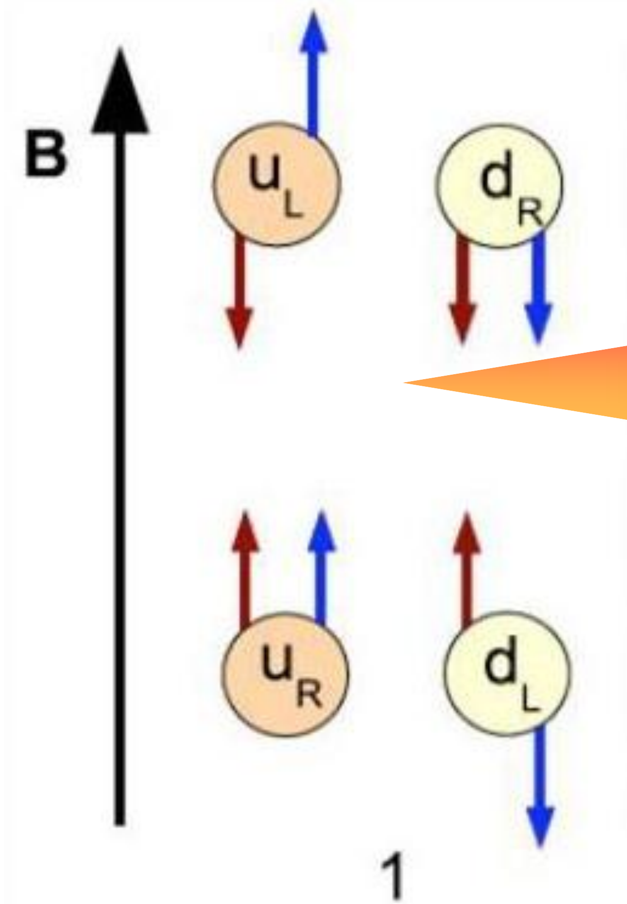
Electric current appears at regions
1. with non-zero topological charge density
2. exposed to external magnetic field

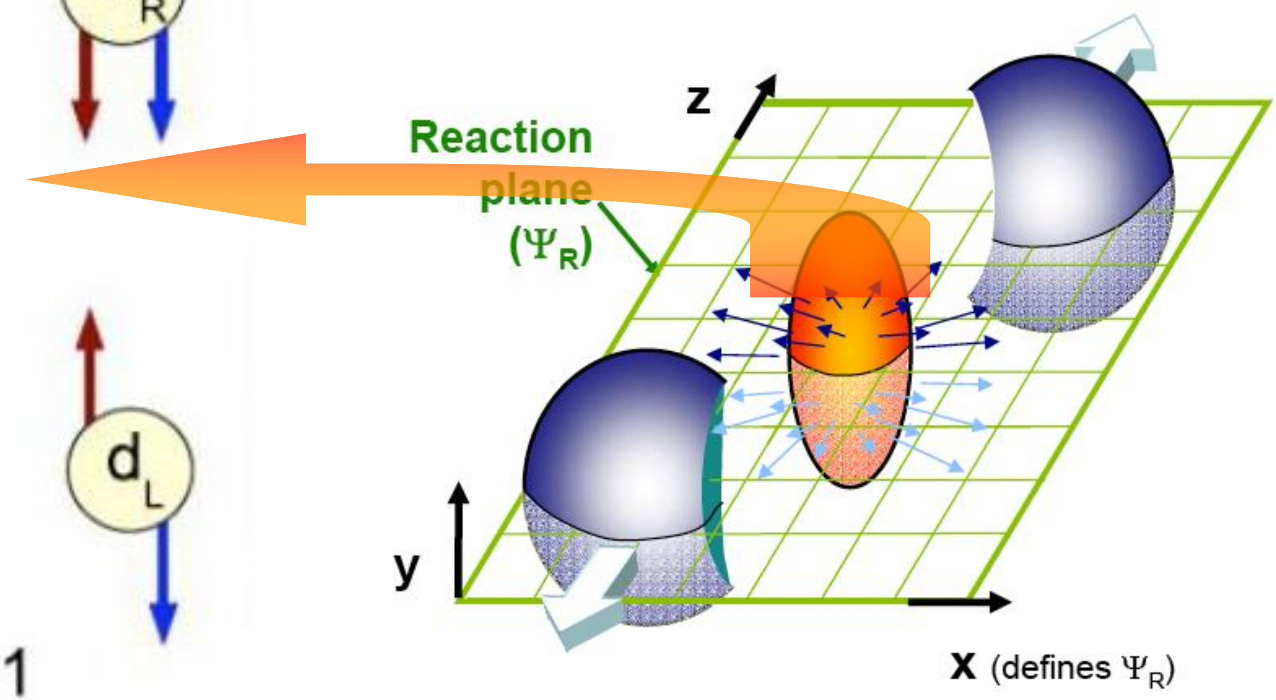
Experimentally observed at RHIC **:**
charge asymmetry of produced particles at heavy ion collisions

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

1. Massless quarks in external magnetic field.

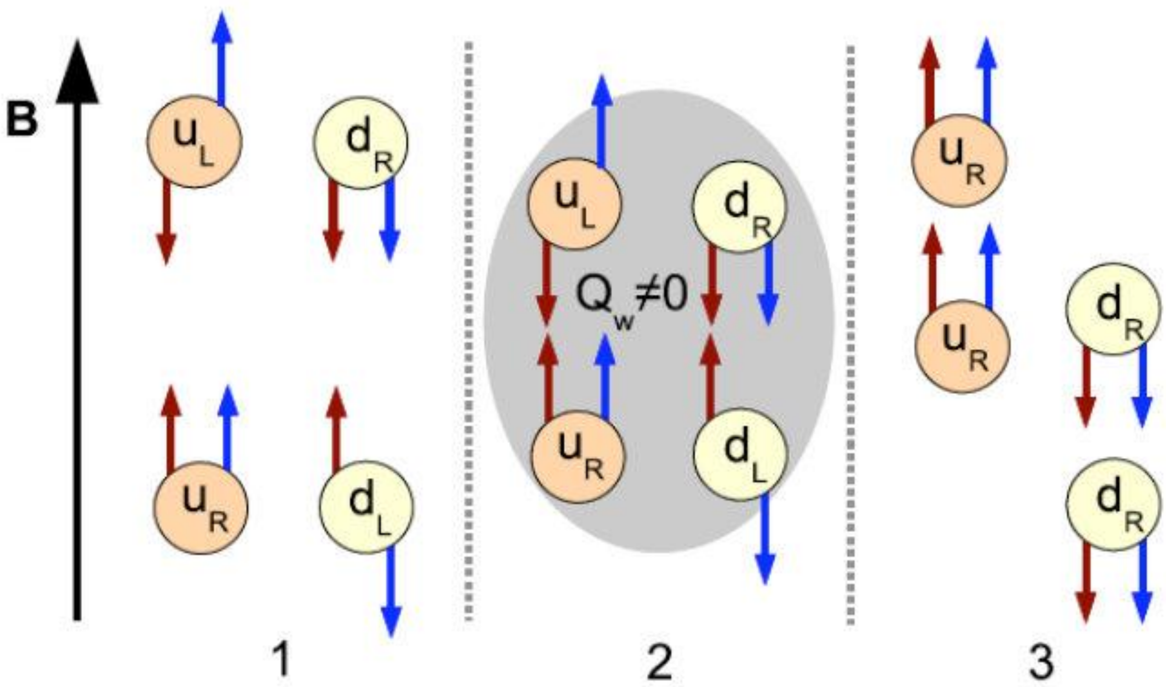
Red: momentum   Blue: spin





Chiral Magnetic Effect by **Fukushima, Kharzeev, Warringa, McLerran**

3. Electric current is along magnetic field
In the *instanton* field

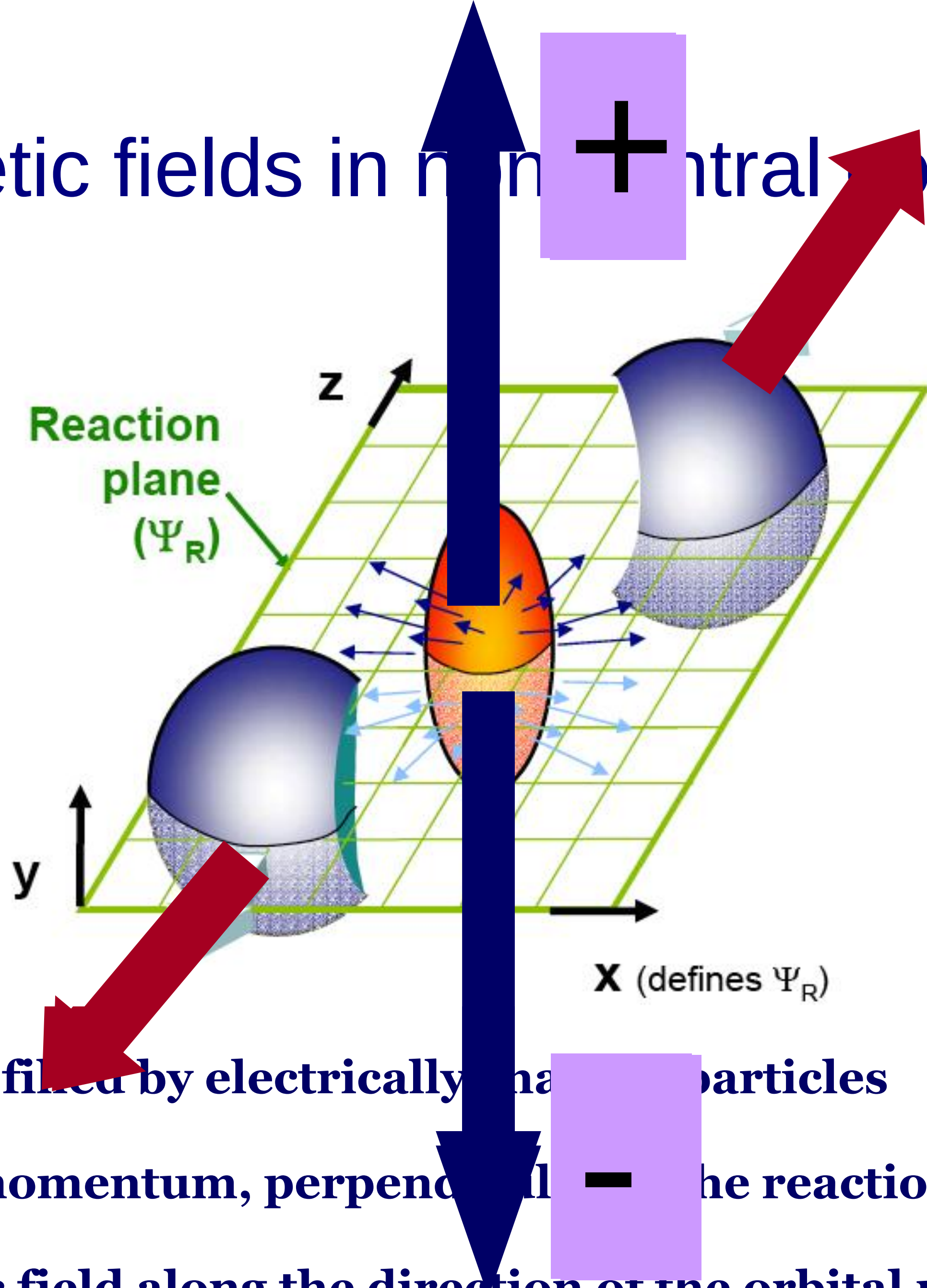


Red: momentum
Blue: spin

Effect of topology:
$u_L \rightarrow u_R$
$d_L \rightarrow d_R$

u-quark: q=+2/3
d-quark: q= - 1/3

Magnetic fields in non-central collisions



Two very big currents produce a very

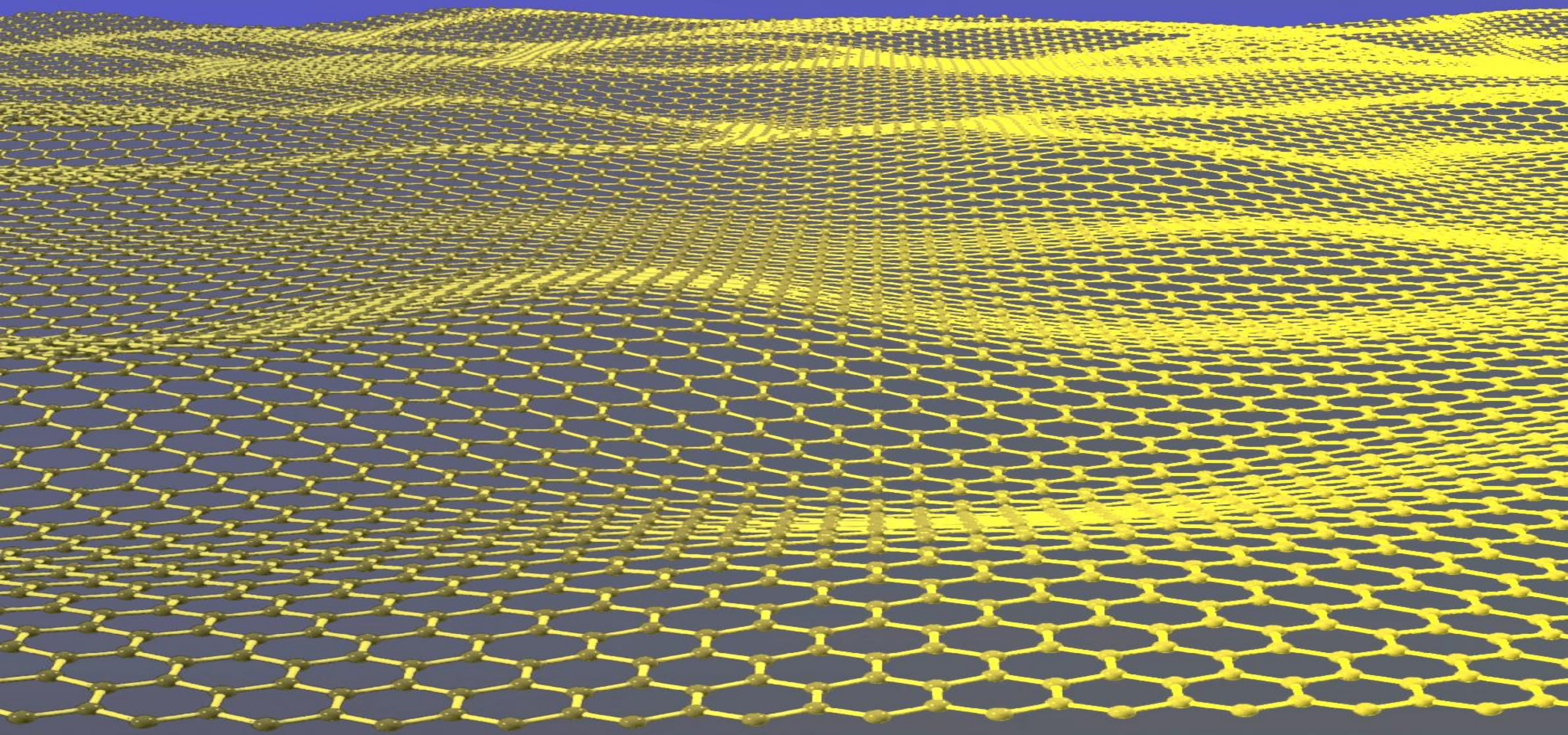
big magnetic field

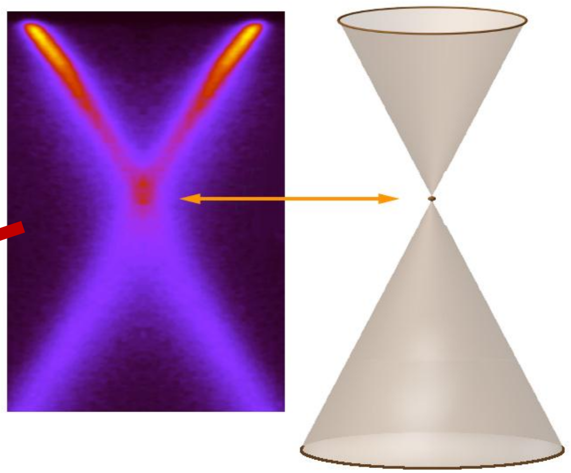
The medium is filled by electrically charged particles

Large orbital momentum, perpendicular to the reaction plane

Large magnetic field along the direction of the orbital momentum

Graphene





Relativistic particle $E=\sqrt{m^2c^4+p^2c^2}$

Massless particle $E=cp$

Graphene $E=v_F p;\quad v_F=\frac{c}{300};$

$\alpha_g=300\alpha=2.16>1$

$\alpha_g>\alpha_g^{crit}=1.11\pm0.06$ Pure graphene is the insulator!

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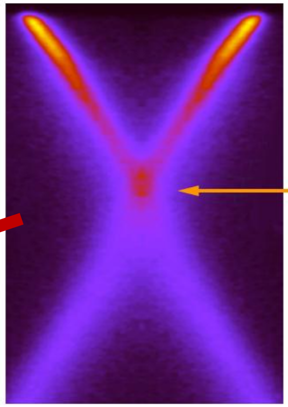
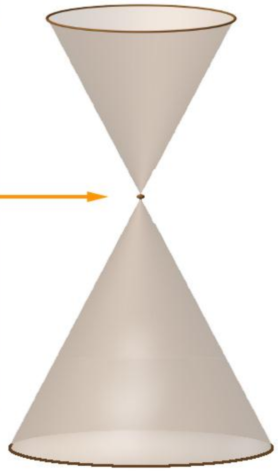




Graphene $E=v_F p$; $v_F=\dfrac{c}{300}$;

$\alpha_g=300\alpha=2.16>1$

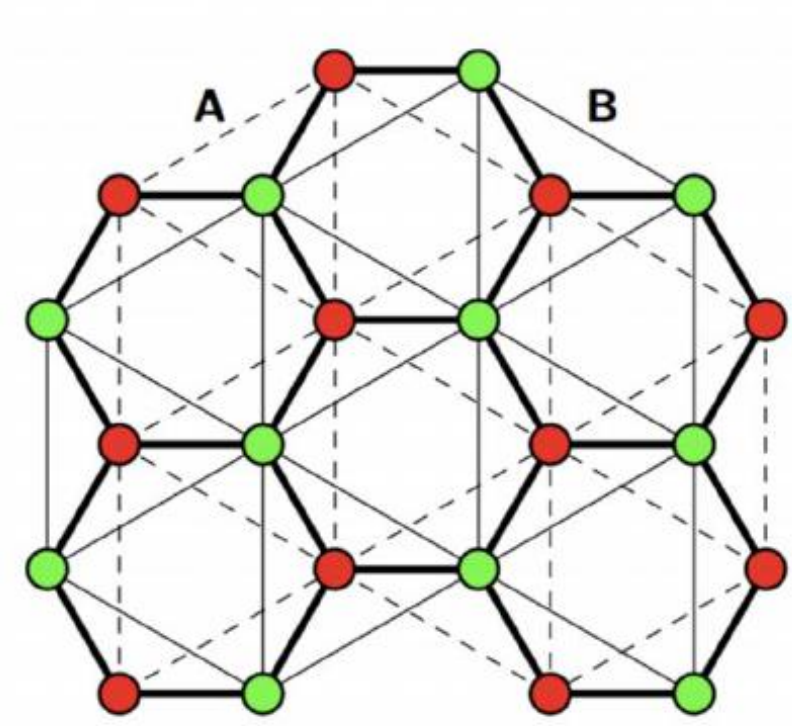
$\alpha_g>\alpha_g^{crit}=1.11\pm0.06$ Pure graphene is the insulator!

If we put graphene on a substrate we can get conductor: $\alpha_g \rightarrow \dfrac{2}{1+\varepsilon}\alpha_g$

Hexagonal lattice = Triangular lattice + Triangular lattice

On such lattice nonrelativistic electrons are "equivalent" to *massless four component Dirac fermions* moving with

$$v_F = \frac{c}{300};$$



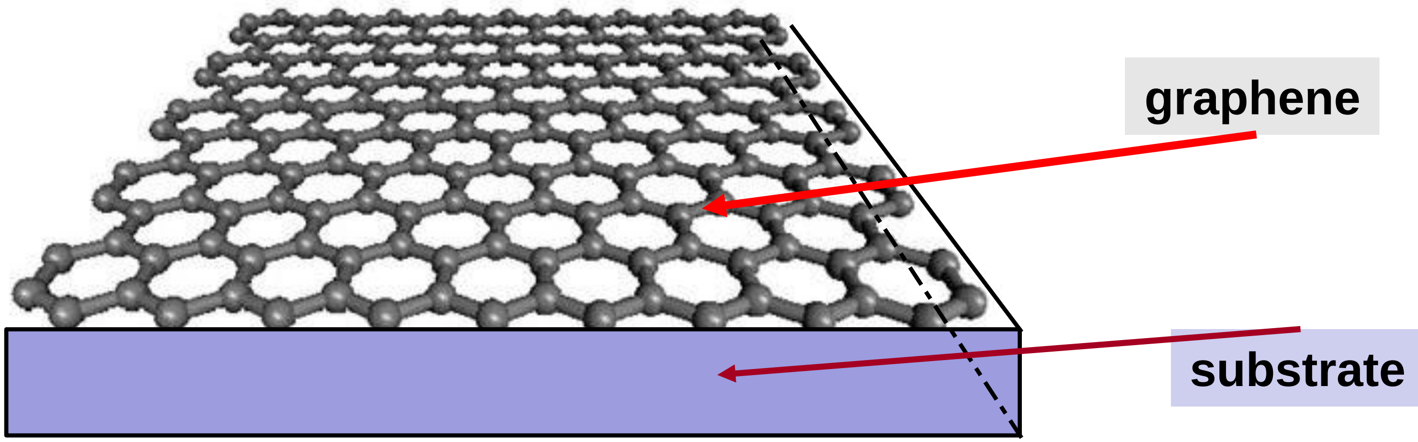
the effective charge is:

$$\alpha_g = 300\alpha = 2.16 > 1$$

We can vary the effective coupling in graphene!

Graphene in the dielectric media $\alpha_g \to \frac{\alpha_g}{\varepsilon}$

Graphene on substrate $\alpha_g \to \frac{2}{1+\varepsilon}\alpha_g$



Effective theory of charge carriers in graphene

1. “Massless” four component Dirac fermions

2. Fermi velocity is $$v_F = c/300$$

3. The effective charge is $$\alpha_g \approx 300\alpha \approx 2.16 > 1$$

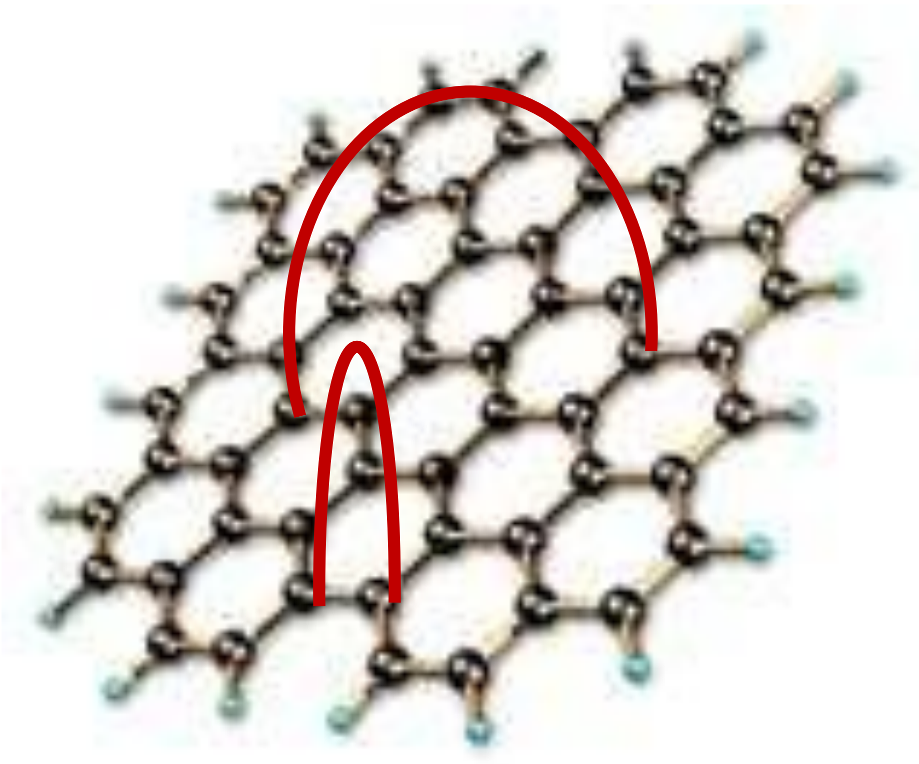
4. We can vary the effective charge if we vary the dielectric permittivity of the substrate

$$\alpha_g \longrightarrow \frac{2}{1+\varepsilon}\alpha_g$$

$$S_E \equiv \frac{1}{2g^2}\int d^3x\,dt\,(\partial_i A_0)^2 - \sum_{a=1}^{N_f}\int d^2x\,dt\,\bar{\psi}_a D[A_0]\psi_a,$$

On substrate

$$\alpha_g \longrightarrow \frac{2}{1+\varepsilon}\alpha_g$$



(2+1)D fermions

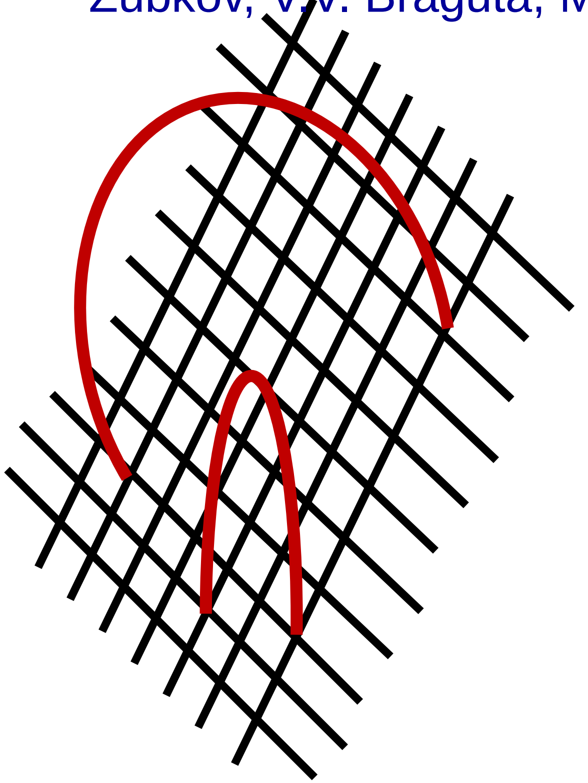
(3+1)D Coulomb

Simulation of the effective graphene theory
***Approach 1*, hypercubic lattice**

J. E. Drut, T. A. Lahde, and E. Tolo (2009-2011)

W. Armour, S. Hands, and C. Strouthos (2008-2011)

P.V. Buividovich, O.V. Pavlovsky, M.V. Ulybyshev, E.V. Luschevskaya, M.A. Zubkov, V.V. Braguta, M.I. Polikarpov (2012)



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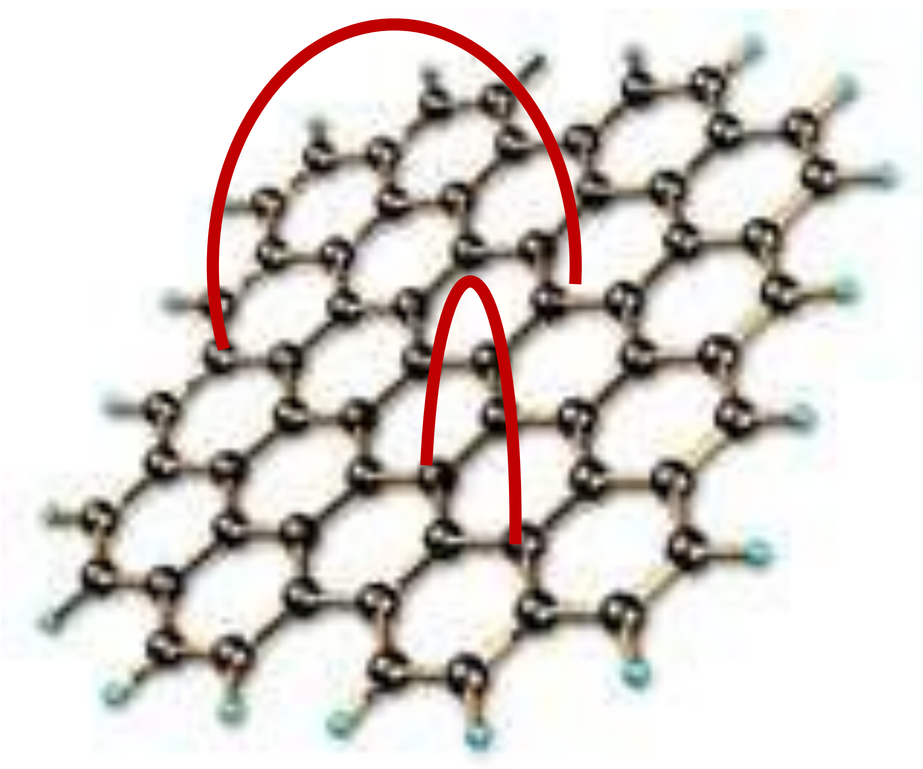
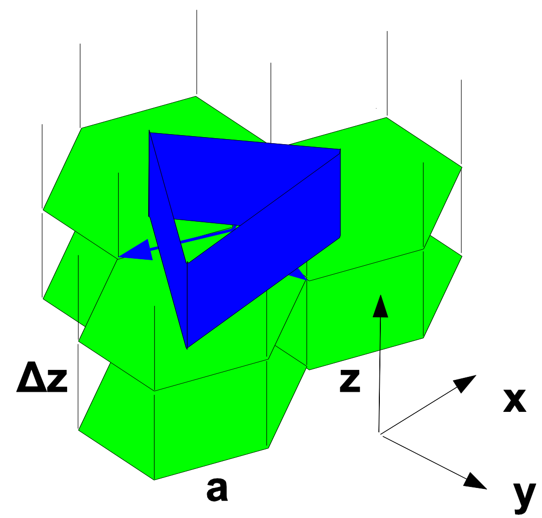
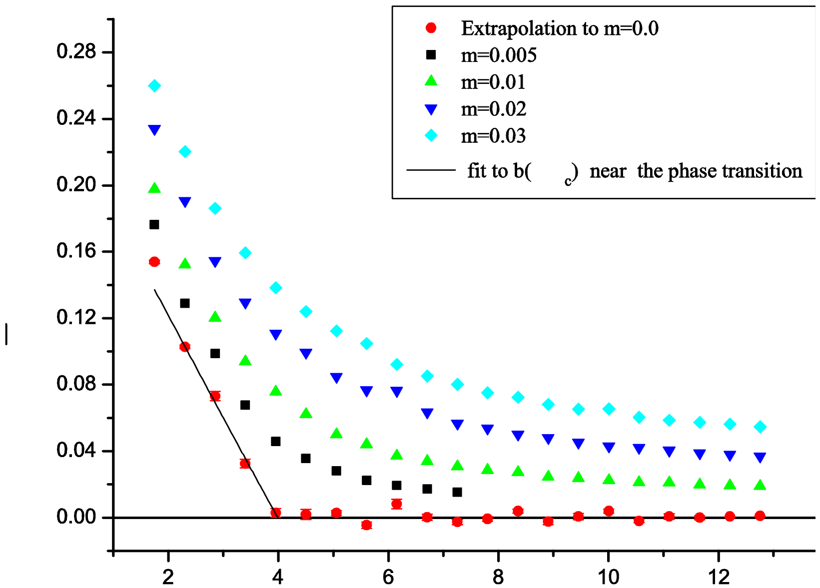
Simulation of the effective graphene theory
<u>Approach 2,</u> 2D hexagonal lattice and rectangular lattice in z and time dimensions

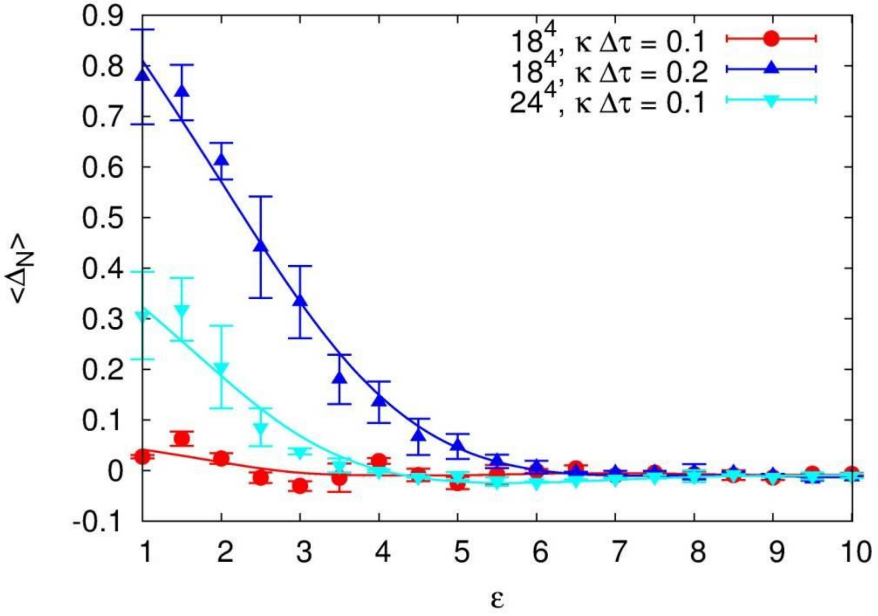
R. Brower, C. Rebbi, and D. Schaich (2011-2012)
P.V. Buividovich, M.I.P. (2012)





Fermion condensate as a function of substrate dielectric permittivity





Approach 1
Hypercubic lattice

Approach 2
Hexagonal lattice