Influence of chiral chemical potential to QCD properties

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In collaboration with:

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Outline:

- Introduction
- SU(2) QCD with nonzero chiral chemical potential
- SU(3) QCD with nonzero chiral chemical potential
- Theoretical explanation
- Conclusion

Introduction

Motivation:

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  - Neutron stars and supernovae
  - Early Universe
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How nonzero chiral chemical potential influences the properties of QCD

Studies of the phase diagram of chiral QCD

- "Chiral magnetic effect in the PNJL model"
  Kenji Fukushima, Marco Ruggieri, Raoul Gatto, Phys.Rev. D81 (2010) 114031
- "Phase diagram of chirally imbalanced QCD matter"
  M.N. Chernodub, A.S. Nedelin, Phys.Rev. D83 (2011) 105008
- "Hot Quark Matter with an Axial Chemical Potential"
  Raoul Gatto, Marco Ruggieri, Phys.Rev. D85 (2012) 054013
- "Inverse magnetic catalysis induced by sphalerons"
  Jingyi Chao, Pengcheng Chu, Mei Huang, Phys.Rev. D88 (2013) 054009
- "Spontaneous generation of local CP violation and inverse magnetic catalysis
  Lang Yu, Hao Liu, Mei Huang, Phys.Rev. D90 (2014) 7, 074009

Results:

- Decrease of the critical temperature with chiral chemical potential
- Decrease of the chiral condensate with chiral chemical potential
- Crossover$\rightarrow$first order transition at large chiral chemical potential

Studies of the phase diagram of chiral QCD

- "Chemical potentials and parity breaking: the Nambu-Jona-Lasinio model"Alexander A. Andrianov, Domenec Espriu, Xumeu Planells, Eur.Phys.J. C74 (2014) 2, 2776
- "Effect of the chiral chemical potential on the position of the critical endpoint"Bin Wang, Yong-Long Wang, Zhu-Fang Cui, Hong-Shi Zong, Phys.Rev. D91 (2015) 3, 034017
- "Chiral phase transition with a chiral chemical potential in the framework of Dyson-Schwinger equations"Shu-Sheng Xu, Zhu-Fang Cui, Bin Wang, Yuan-Mei Shi, You-Chang Yang, Hong-Shi Zong, Phys.Rev. D91 (2015) 5, 056003

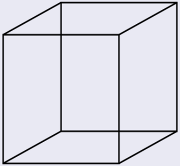
Results:

- Increase of the critical temperature with chiral chemical potential
- Increase of the chiral condensate with chiral chemical potential
- Crossover for all values of chiral chemical potential

SU(2) QCD
with
nonzero chiral chemical potential

Details of the calculation:

- Dynamical staggered fermions ($N_f = 4$) + Wilson action (SU(2))
- Link modification:
  $U \to U e^{\mu_5 \gamma_5}, \quad U^+ \to U^+ e^{-\mu_5 \gamma_5} \Rightarrow$ nonlocal action
- Chiral chemical potential:
  $\delta S_{\mu_5} = \frac{1}{2} \mu_5 a \sum_x (-1)^{x_2} (\bar{\psi}_{x+\delta} \bar{U}_{x+\delta,x} \psi_x - \bar{\psi}_x \bar{U}^{\dagger}_{x+\delta,x} \psi_{x+\delta})$



- Correct continuum limit: $\delta S_{\mu_5}|_{a \to 0} \to \mu_5 \int d^4x \; \bar{Q} (\gamma_4 \gamma_5 \times 1) Q$
- $6 \times 20^3 (m_\pi \sim 300\text{MeV}), \; 10 \times 28^3 (m_\pi \sim 500\text{MeV})$

Observables:

- The Polyakov loop (confinement/deconfinement transition)

  $$L = \frac{1}{N_\sigma^3} \sum_{n_1,n_2,n_3} \langle Tr \prod_{n_4=1}^{N_\tau} U_4(n_1,n_2,n_3,n_4) \rangle$$

- The chiral condensate (chiral symmetry breaking/restotation transition)

  $$a^3 \langle \bar\psi \psi \rangle = -\frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \frac{\partial}{\partial(ma)} \log Z = \frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \langle Tr \frac{1}{D+ma} \rangle$$

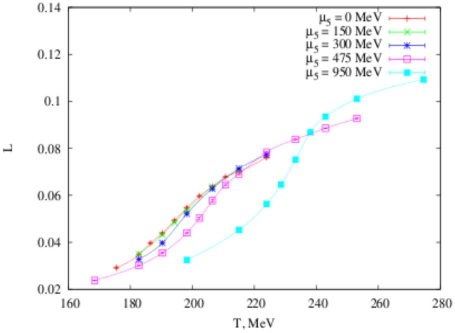
- The Polyakov loop susceptibility
  (position of the transition)

  $$\chi_L = N_\sigma^3 \left(\langle L^2 \rangle - \langle L \rangle^2 \right)$$

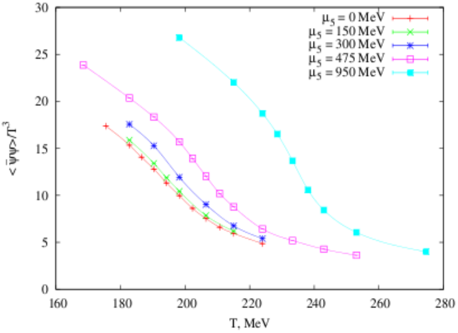
- The disconnected part of the chiral susceptibility
  (position of the transition)

  $$\chi_{disc} = \frac{1}{N_\tau N_\sigma^3} \frac{1}{16} \left(\langle (Tr \frac{1}{D+ma})^2 \rangle - \langle Tr \frac{1}{D+ma} \rangle^2 \right)$$

Polyakov loop and chiral condensate (6×20^3)



Polyakov loop



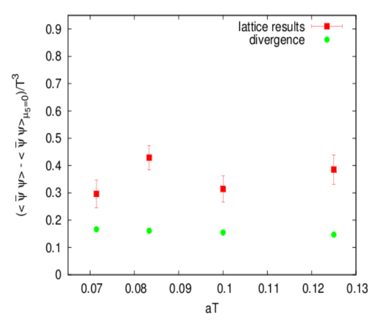
chiral condensate

Divergencies in the observables:

- Chiral condensate (free fermions):

$$\langle\bar\psi\psi\rangle_p = 0.309867\,\frac{m_p}{a^2} + m_p^3\left(\frac{\log(m_p a)}{\pi^2} - 0.172536\right) + m_p(\mu_5^p)^2\left(-2\,\frac{\log(m_p a)}{\pi^2} + 0.167759\right)$$

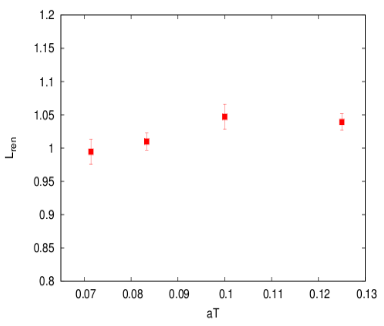
- Additional logarithmic divergence proportional to $\sim \mu_5^2$ (with small renormalization effects)
- In the deconfinement phase:



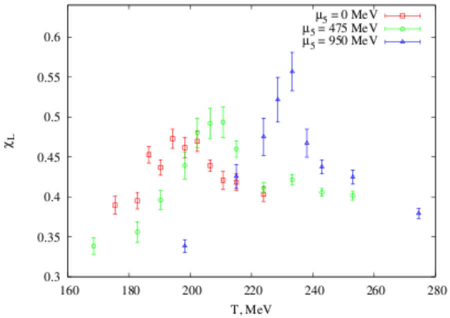
- In the confinement phase divergence is small

Divergencies in the observables:

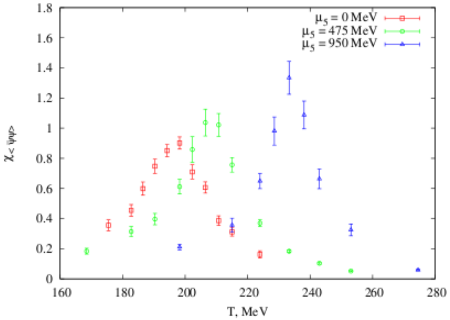
- Polyakov loop:
  $L = 1 - \beta \frac{g^2}{4} \int_0^\beta d\tau D_{00}^{aa}(\tau, \vec{0}) = 1 - \beta \frac{g^2}{4} \int \frac{d^3 q}{(2\pi)^3} \tilde{D}_{00}^{aa}(0, \vec{q}),$
- $\Pi_{00}(\vec{q}) - \Pi_{00}(\vec{q})|_{\mu_5=0} \sim \mu_5^2(c_1 \log \vec{q}^2 + c_2)$
- No divergency in the Polyakov loop



Susceptibilities (6×20^3)

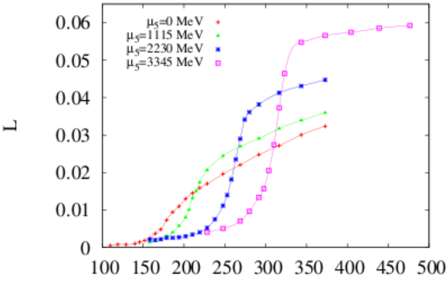


Polyakov loop susceptibility

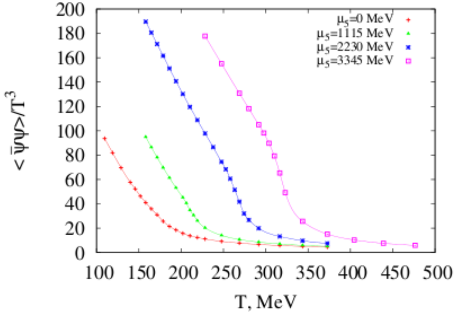


chiral susceptibility

Polyakov loop and chiral condensate (10×28^3)

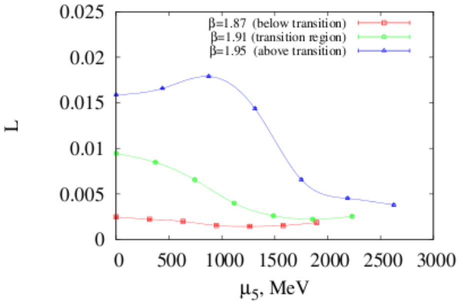


Polyakov loop

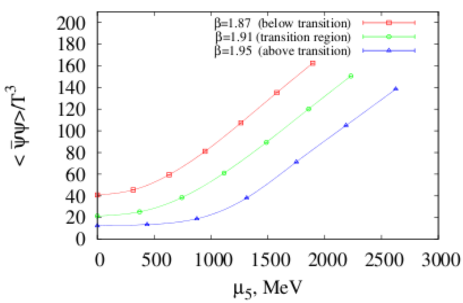


chiral condensate

Fixed temperature scan (10×28^3)

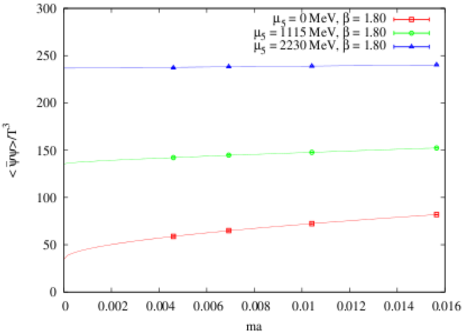


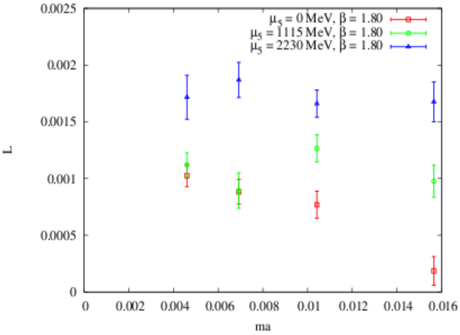
Polyakov loop



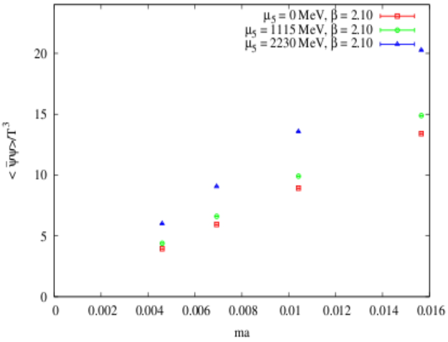
chiral condensate

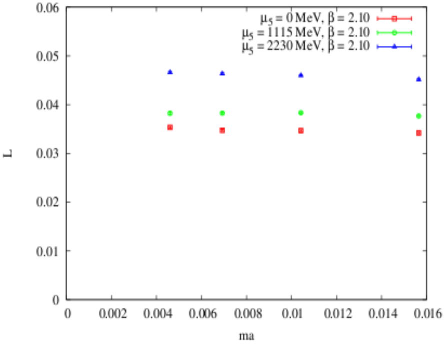
Chiral limit (confinement)





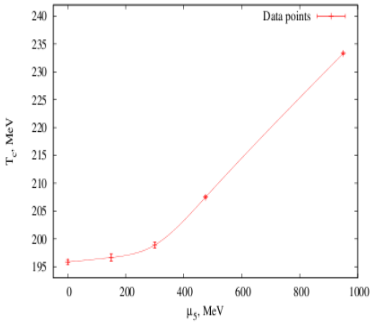
Chiral limit (deconfinement)





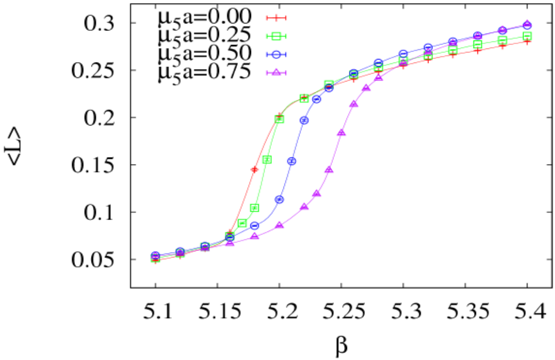
Results of the calculation:

- The critical temperatures increase
- The critical temperatures of the confinement/deconfinement phase transition and of the chiral symmetry breaking/restoration coincide
- The phase transitions confinement/deconfinement and chiral symmetry breaking/restoration are crossovers



SU(3) QCD with nonzero chiral chemical potential

Polyakov loop (SU(3), 4×16^3)



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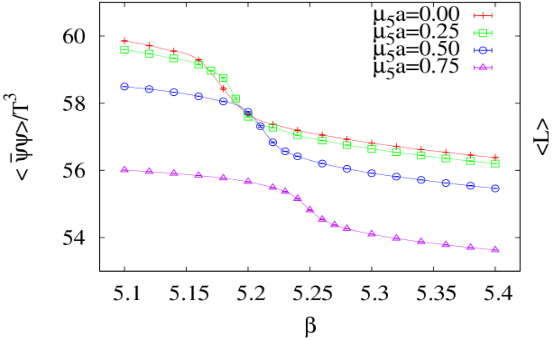
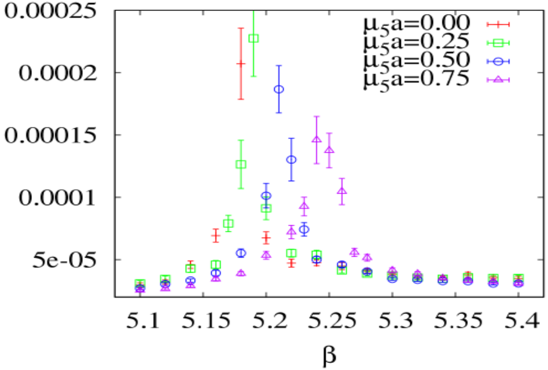


Polyakov loop susceptibility (SU(3), 4×16^3)



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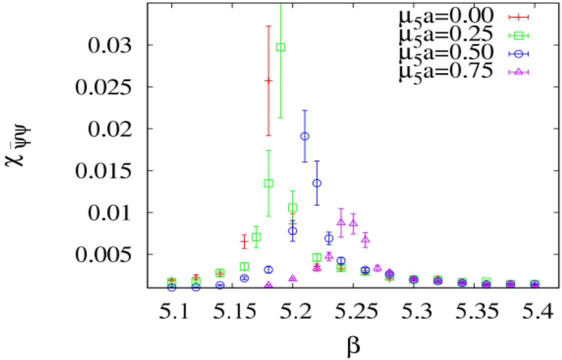

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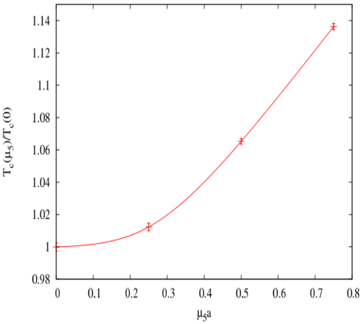

Theoretical explanation

NJL model ($U_L(1) \times U_R(1)$, N_c colors)

- $S_E = \int d^4x \left(\bar\psi (\hat\partial + m - \mu_5 \gamma_4 \gamma_5) \psi - G \left[(\bar\psi \psi)^2 + (\bar\psi i \gamma_5 \psi)^2 \right] \right)$

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- $S_E = \int d^4x \left(\bar{\psi}(\hat{\partial} + m - \mu_5 \gamma_4 \gamma_5 + \sigma + i\gamma_5 \pi)\psi + \frac{1}{4G}\left[\sigma^2 + \pi^2\right] \right)$

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- $S_{eff} = \int d^4x \left(\frac{1}{4G}(\sigma^2 + \pi^2) - \mathrm{Tr}\log(\hat\partial + m - \mu_5\gamma_4\gamma_5 + \sigma + i\gamma_5\pi) \right)$

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Gap equation ($N_c \to \infty$)

- $\frac{\delta S_{eff}}{\delta\sigma} = \frac{\sigma}{2G} - N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{i\hat k + m - \mu_5\gamma_4\gamma_5 + \sigma + i\gamma_5\pi} \right] = 0$

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- $\frac{\pi^2}{GN_c} = \int_0^\Lambda k^2 dk \left[\frac{1}{\sqrt{(|\vec k| - \mu_5)^2 + M^2}} + \frac{1}{\sqrt{(|\vec k| + \mu_5)^2 + M^2}} \right]$

Gap equation

$$\frac{1}{\alpha_{NJL}} - 1 = \left(y^2 - \frac{x^2}{2}\right) \log \frac{1}{x^2}$$

$$\alpha_{NJL} = \frac{G N_c \Lambda^2}{\pi^2}, \quad x = \frac{M}{\Lambda}, \quad y = \frac{\mu_5}{\Lambda}$$

Gap equation

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$$\alpha_{NJL} = \frac{G N_c \Lambda^2}{\pi^2}, \quad x = \frac{M}{\Lambda}, \quad y = \frac{\mu_5}{\Lambda}$$

Properties:

- $\alpha_{NJL} < 1$ no solutions
- $\alpha_{NJL} > 1$ there is solution $M \neq 0$

Weakly coupled chiral plasma ($\alpha_{NJL} \ll 1$)

- There is solution if $\mu_5 \neq 0$
  $M^2 = \Lambda^2 \exp\left[-\frac{\pi^2}{GN_c\mu_5^2}\right]$
- Very similar to superconductivity $\Delta = \omega_D \exp\left(-const/G_S\nu_F\right)$

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Chiral symmetry breaking as condensation of Cooper pairs

- Vacuum: $|vac\rangle = \hat{G}_1 \hat{G}_2 \hat{G}_3 |p_F\rangle$,
  $\hat{G}_1 = \prod_p \left(\cos(\theta_L) - \sin(\theta_L) \hat{a}^+_{L,p} \hat{b}^+_{L,-p}\right)$
  $\hat{G}_2 = \prod_{p>\mu_5} \left(\cos(\theta_R) + \sin(\theta_R) \hat{a}^+_{R,p} \hat{b}^+_{R,-p}\right)$
  $\hat{G}_3 = \prod_{p<\mu_5} \left(\cos(\tilde{\theta}_R) + \sin(\tilde{\theta}_R) \hat{b}_{R,-p} \hat{a}_{R,p}\right)$

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  $\hat{G}_3 = \prod_{p<\mu_5} \left(\cos(\tilde{\theta}_R) + \sin(\tilde{\theta}_R)\hat{b}_{R,-p}\hat{a}_{R,p}\right)$
- Energy:

$$E_{vac} = 2N_c\left[\int_{p<\mu_5}\frac{d^3p}{(2\pi)^3}(p-\mu)\cos^2\tilde{\theta}_R + \int_{p>\mu_5}\frac{d^3p}{(2\pi)^3}(p-\mu)\sin^2\theta_R + \int\frac{d^3p}{(2\pi)^3}(p+\mu)\sin^2\theta_L\right]$$

$$-GN_c^2\left(\int_{p<\mu_5}\frac{d^3p}{(2\pi)^3}\sin 2\tilde{\theta}_R + \int_{p>\mu_5}\frac{d^3p}{(2\pi)^3}\sin 2\theta_R + \int\frac{d^3p}{(2\pi)^3}\sin 2\theta_L\right)^2$$

Weakly coupled chiral plasma ($\alpha_{NJL} \ll 1$)

- Chiral plasma is unstable with respect to chiral symmetry breaking and condensation of Cooper pairs for $T < T_c$

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- Chiral plasma is unstable with respect to generation of magnetic field for $T > T_c$

Weakly coupled chiral plasma ($\alpha_{NJL} \ll 1$)

- Chiral plasma is unstable with respect to chiral symmetry breaking and condensation of Cooper pairs for $T < T_c$
- Chiral plasma is unstable with respect to generation of magnetic field for $T > T_c$
- CME cannot be realized for $T < T_c$ and it is not a vacuum state

Strongly coupled chiral plasma ($\alpha_{NJL} > 1$)

- $\mu_5 \ll M_0$: $\quad M^2 \simeq M_0^2 \left(1 + 2\frac{\mu_5^2}{M_0^2}\right)$.

Strongly coupled chiral plasma ($\alpha_{NJL} > 1$)

- $\mu_5 \ll M_0$: $\quad M^2 \simeq M_0^2 \left(1 + 2\frac{\mu_5^2}{M_0^2}\right).$
- $\mu_5 \sim M$: $\quad M^2 \simeq 2\mu_5^2 \left(1 - \frac{1-\alpha_{NJL}}{\alpha_{NJL}} \frac{1}{2y^2 \log\left(\frac{1}{2y^2}\right)}\right).$

Strongly coupled chiral plasma ($\alpha_{NJL} > 1$)

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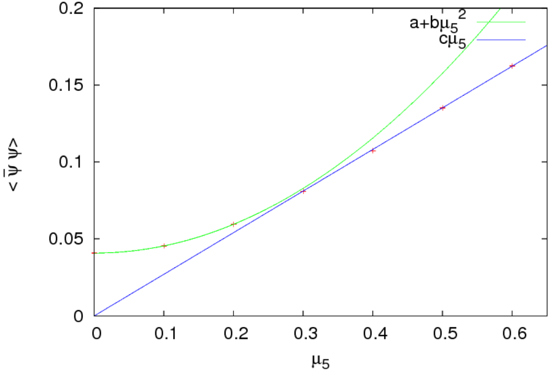
- $\mu_5 \sim M$: $\quad M^2 \simeq 2\mu_5^2 \left(1 - \frac{1-\alpha_{NJL}}{\alpha_{NJL}} \frac{1}{2y^2 \log\left(\frac{1}{2y^2}\right)}\right).$

Prediction:

In strong coupling region NJL model predicts that dynamical fermion mass is quadratically rising function at small μ_5 which then switch to linear rising behaviour at large μ_5



0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6
< ψ̄ ψ>
μ5
a+bμ5 2
cμ5

Conclusion

Conclusion:

- CHIRAL CATALYSIS: Chiral chemical potential enhances chiral symmetry breaking and rises critical temperature of chiral symmetry breaking/restoration transition.
- Chiral plasma is unstable with respect to chiral symmetry breaking and condensation of Cooper pairs for $T < T_c$