

МОДИФИКАЦИЯ СВОЙСТВ АДРОНОВ В ЯДЕРНОЙ МАТЕРИИ

М. И. Криворученко

ИТЭФ, МФТИ, БЛТФ ОИЯИ

**Семинар ИФВЭ
Протвино, 20 октября 2015**

Содержание:

- 1. Сверхпроводимость в кварковой материи и ядрах**
- 2. Бозе конденсация дибарионов в ядерной материи**
- 3. Рождение резонансов на ядрах**
- 4. Рождение e^+e^- пар в столкновениях тяжелых ионов**
- 5. Квантовый транспорт в формализме деформационного квантования**
- 6. Свойства K мезонов в пионной материи**
- 7. Майорановское нейтрино в ядерной материи**

PRC	9
PRL	2
J. Phys. G	2
PLB	2
NPA	2
Ann. Phys. (N.Y.)	2
Ann. Phys. (Leipzig)	1
Z. Phys. A	1
PRD	1

SU(2) цветовая сверхпроводимость

- ◆ L. A. Kondratyuk, M. M. Giannini, M. I. K.,
Phys. Lett. B 269, 139 (1991);
- ◆ L. A. Kondratyuk and M. I. K., Z. Phys. A 344, 99 (1992).

Эфф. лагранжиан в среде (версия модели НИЛ):

$$L_{eff} = \bar{\psi}(i\hat{\nabla} + \hat{\mu} - m)\psi - \frac{1}{2}g^2(\bar{\psi}\gamma_\sigma t^a\psi)(\bar{\psi}\gamma^\sigma t^a\psi),$$

Система уравнений Горькова-Дайсона:

Функция Грина:



Аномальная функция Грина:



Аномальная функция Грина:

$$iF(1,2) = \langle N | T(\psi(1)\psi(2)) | N+2 \rangle,$$

$$i\bar{F}(1,2) = \langle N+2 | T(\bar{\psi}(1)\bar{\psi}(2)) | N \rangle,$$

$$N_f = 1 \Rightarrow J = 1$$

$\forall \alpha, \beta, \gamma \langle 11 | D_{1m}^1(\alpha, \beta, \gamma) \neq \delta_{m0} \langle 10 | \rightarrow 2$ решения:

1. Спаривание в канале $J_3 = 0$
2. Спаривание в канале $J_3 = \pm 1$

Найдены:

1. функции Грина (нормальная и аномальная)
2. щель в спектре возбуждений, закон дисперсии
3. распределение夸арков по импульсам
4. спиновая плотность
5. температура фазового перехода

Щель в спектре возбуждений в канале $J_3 = 0$

$$\Delta^2 = g^4 \left((\mathbf{a}^* \mathbf{a}) - (\mathbf{a}^* \mathbf{n})(\mathbf{a} \mathbf{n}) \frac{-m^2 + \mu^2 + g^4 (\mathbf{a}^* \mathbf{n})(\mathbf{a} \mathbf{n})}{\mu^2 + g^4 (\mathbf{a}^* \mathbf{n})(\mathbf{a} \mathbf{n})} \right) \geq 0,$$

$$\Delta = \Delta(\mathbf{p}) \geq 0$$

$\Delta = 0$ при $m = 0$, когда $\mathbf{n} \parallel \mathbf{a}$

$$\mathbf{n} = \mathbf{p}/|\mathbf{p}|$$

Уравнение щели:

$$\bar{F}(0) = \int \frac{d^4 p}{(2\pi)^4} \bar{F}(p), \quad \rightarrow \quad g^4 (\mathbf{a}^* \mathbf{a}) \propto \exp \left(-\frac{8\pi^2}{(6g^2)\mu^2 v_F (1 - v_F^2/3)} \right),$$

SU(2) цветовая сверхпроводимость на решетках:

S. Hands, S. Kim, J.-I. Skallerud, Eur. Phys. J. C 48, 193 (2006).

origin. The behaviour of thermodynamic observables and the superfluid order parameter are consistent with a Fermi surface disrupted by a BCS diquark condensate. The energy per baryon as a function of μ exhibits

SU(2) цветовая кварковая материя в модели ПНЛ:

PHYSICAL REVIEW D 80, 074035 (2009)

Two-color quark matter: U(1)_A restoration, superfluidity, and quarkyonic phase

Tomáš Brauner,^{1,*†} Kenji Fukushima,² and Yoshimasa Hidaka³

¹*Institut für Theoretische Physik, Goethe-Universität, Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany*

²*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

³*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

(Received 4 August 2009; published 29 October 2009)

TWO-COLOR QUARK MATTER: U(1)_A ...

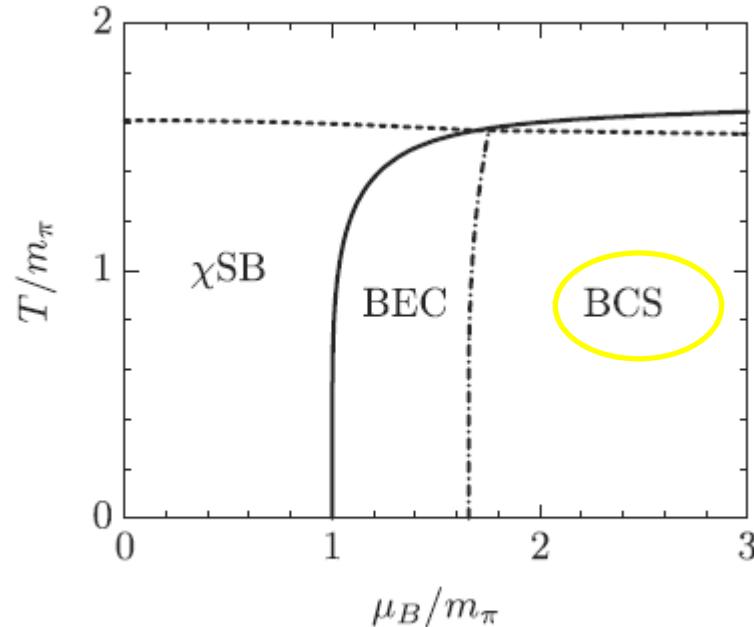
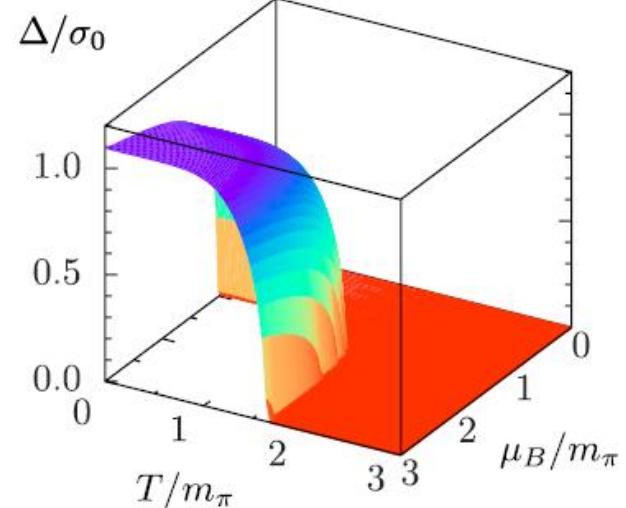


FIG. 4. Conventional presentation of the phase diagram of two-color QCD from the PNJL model in the $\mu_B - T$ plane.

PHYSICAL REVIEW D 80, 074035 (2009)



Polyakov-quark-meson-diquark (PQMD) model

Physics Letters B 731 (2014) 350–357



Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb



Polyakov-quark-meson-diquark model for two-color QCD

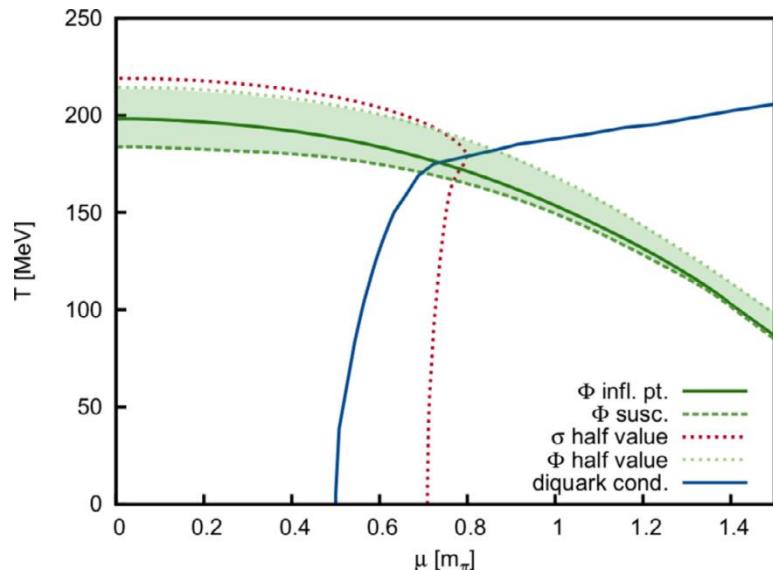


Nils Strodtthoff^{a,*}, Lorenz von Smekal^{b,c}

^a Institut für Theoretische Physik, Universität Heidelberg, 69120 Heidelberg, Germany

^b Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

^c Institut für Theoretische Physik, Jusus-Liebig-Universität Giessen, 35392 Giessen, Germany



СВОЙСТВА $SU(2)_c$ КВАРКОВОЙ МАТЕРИИ ПРИ $T = 0$ - ФАЗА БКШ

Ферми-сфера кварков определяет EoS:
энергию и давление vs. плотность
барионного числа

Спаривание, щель в спектре,
сверхпроводимость

Бесцветные элементарные возбуждения
над ферми-сферой - фаза
конфайнмента

Цветные элементарные возбуждения
над ферми-сферой - фаза
деконфайнмента

НИЛ
(1991)

Решетки
(2006)

ПНИЛ
(2009)

PQMD
(2014)

+

+

+

+

+

+

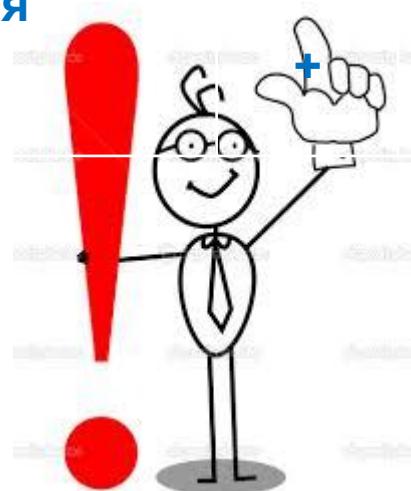
+

+

+

+

+



Проекционная теория БКШ

- Projected BCS for **U(1)**: B. F. Bayman, Nucl. Phys. 15, 33 (1960).
- Projected BCS for **SU(2)**: A. A. Raduta and E. Moya de Guerra,
Ann. Phys. (NY) 284, 134 (2000).
- Projected BCS for **SU(3)_c**: P. Amore et al.,
Phys. Rev. D 65, 074005 (2002).

EXACT RESULTS IN THEORY OF SUPERCONDUCTIVITY OF FINITE FERMI-SYSTEMS

- ◆ A. A. Raduta, M.I.K., Amand Faessler, PRC85, 054314 (2012)

PARTICLE NUMBER PROJECTION: BCS \rightarrow PBCS/FBCS

Particle number projection operator $P_N^2 = P_N$

$$P_N = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{i(\hat{N} - N)\varphi}$$

Projected BCS wave function:

$$|BCS, N\rangle \equiv C_N P_N |BCS\rangle$$

We define

$$Q(N) = C_N^{-2}$$

and get recursion:

$$Q(N) = \sum_{\beta} Q^{\beta}(N),$$

$$Q^{\beta}(N) = \frac{\Omega_{\beta}}{N} \sum_{n=1}^{N/2} (-)^{n+1} \tan^{2n}(\rho_{\beta}/2) Q(N - 2n).$$

with the initial value

$$Q(0) = \prod_{\alpha} [\cos^2(\rho_{\alpha}/2)]^{2j_{\alpha}+1}.$$

- Matrix elements in PBCS/FBCS can be calculated analytically in terms of single function $Q(N)$
- The one-dimensional recursion for $Q(N)$ was presented

Direct applications: Nuclear structure calculations

Possible applications:

Extending to non-abelian-symmetry projected BCS

Discovery potential of cold quark matter:

neutrino bursts from formation of quark stars

Длинная мягкая нейтринная вспышка

*Astronomy Letters, Vol. 20, No. 4, 1994, pp. 499 - 502. Translated from Pis'ma v Astronomicheskii Zhurnal, Vol. 20, No. 8, 1994, pp. 588 - 592.
Original Russian Text Copyright © 1994 by Martem'yanov.*

Neutrino Radiation during the Conversion of a Neutron Star into a Strange Star

B. V. Martem'yanov

Institute for Theoretical and Experimental Physics, Moscow, Russia

Received November 18, 1993; in final form, February 2, 1994

Abstract – Conversion of a neutron star into a strange star as a result of phase transition of neutron matter into quark matter is considered. The conversion rate and neutrino luminosity are calculated under certain assumptions about the heat-transfer process. A possible spectrum of neutrino radiation is obtained.

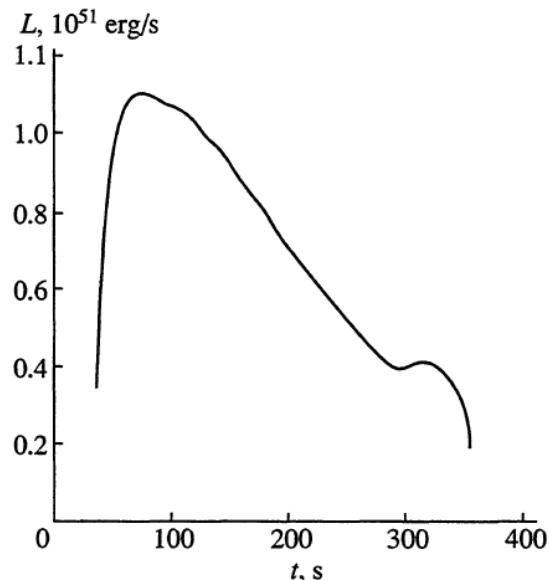


Fig. 7. Neutrino luminosity as a function of time (by the clock of a remote observer).

Длительность вспышки - минуты
vs. 10 секунд у сверхновой
Средняя энергия нейтрино 3-5 МэВ
vs. 20 МэВ у сверхновой
Полная энергия –
сравнима со сверхновой

Signals of the QCD Phase Transition in Core-Collapse Supernovae

I. Sagert,¹ T. Fischer,³ M. Hempel,¹ G. Pagliara,² J. Schaffner-Bielich,² A. Mezzacappa,⁴
F.-K. Thielemann,³ and M. Liebendörfer³

¹*Institut für Theoretische Physik, Goethe-Universität, Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany*

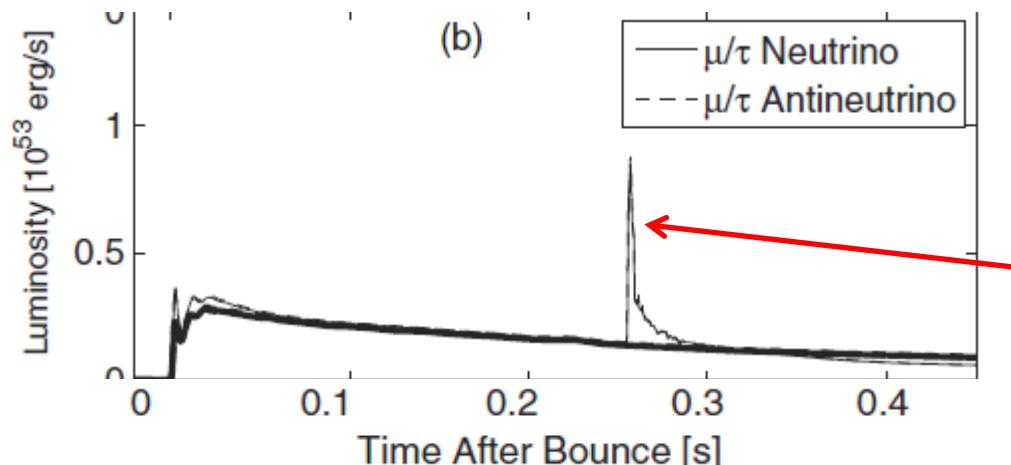
²*Institut für Theoretische Physik, Ruprecht-Karls-Universität, Philosophenweg 16, 69120 Heidelberg, Germany*

³*Department of Physics, University of Basel, Klingelbergstr. 82, 4056 Basel, Switzerland*

⁴*Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*

(Received 12 August 2008; published 26 February 2009)

We explore the implications of the QCD phase transition during the postbounce evolution of core-collapse supernovae. Using the MIT bag model for the description of quark matter, we model phase transitions that occur during the early postbounce evolution. This stage of the evolution can be simulated with general relativistic three-flavor Boltzmann neutrino transport. The phase transition produces a second shock wave that triggers a delayed supernova explosion. If such a phase transition happens in a future galactic supernova, its existence and properties should become observable as a second peak in the neutrino signal that is accompanied by significant changes in the energy of the emitted neutrinos. This second neutrino burst is dominated by the emission of antineutrinos because the electron degeneracy is reduced when the second shock passes through the previously neutronized matter.



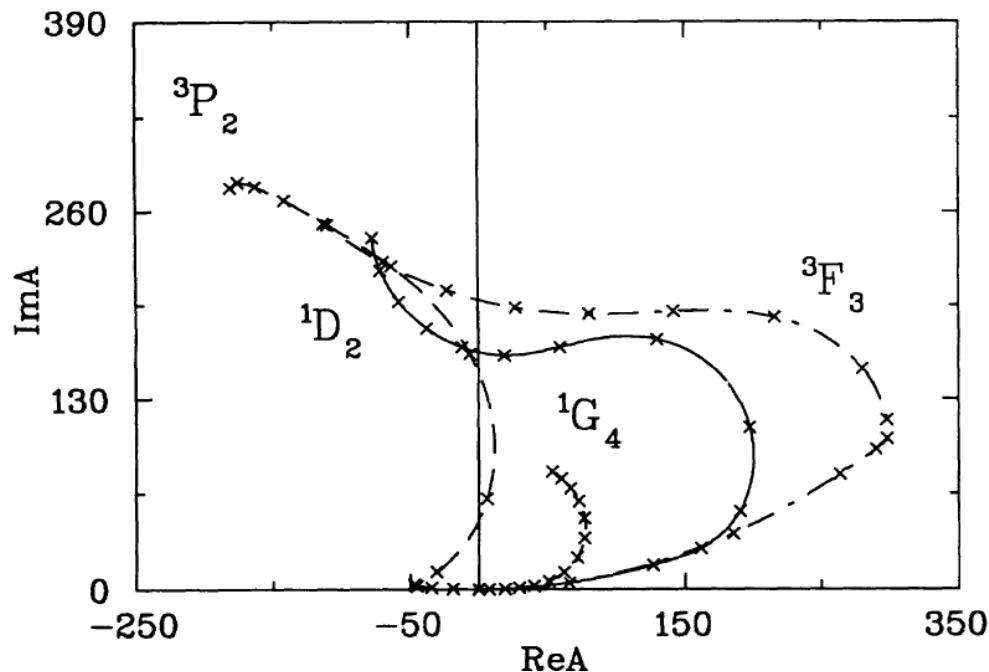
$\Delta t \sim 10$ милисекунд

Бозе конденсация дибарионов в ядерной материи

- ◆ M.I.K., Phys. Rev. C 82, 018201 (2010);
- ◆ A. Faessler, A. J. Buchmann, M.I.K., B. V. Martemyanov, J. Phys. G 24, 791 (1998);
- ◆ A. Faessler, A. J. Buchmann and M.I.K., Phys. Rev. C 57, 1458 (1998);
- ◆ A. Faessler, A. J. Buchmann and M.I.K., Phys. Rev. C 56, 1576 (1997);
- ◆ A. J. Buchmann, A. Faessler and M.I.K., Annals Phys. 254, 109 (1997);
- ◆ A. Faessler, A. J. Buchmann, M.I.K., B. V. Martemyanov Phys. Lett. B 391, 255 (1997).

Экспериментальный статус дибарионов

- 1 . R. L. Jaffe, PRL 38, 195 (1977). H-dibaryon
(экспериментально не подтвержден)
2. С конца 1980-х PDG перестает публиковать обзоры по дибарионам
3. R. A. Arndt et al., PRD 45, 3995 (1992);
R. A. Arndt et al., PRD 50, 2731 (1994).



PWA:
упругое NN рассеяние

TABLE II. Pole positions and residues for partial waves exhibiting resonancelike behavior. W_p is the pole position. G gives the function $(W_p - W)T_p$ evaluated at the pole. $G_r = \text{Re}G$ and $G_i = \text{Im}G$. Values from SM86 are given in square brackets.

State	W_p	$ G $ (MeV)	$\arctan(G_i/G_r)$ (deg)
1D_2	2148- <i>i</i> 59	8.8	-11
	[2148- <i>i</i> 63]	[10]	[-15]
3F_3	2170- <i>i</i> 72	9.4	74
	[2183- <i>i</i> 79]	[14]	[-78]
3P_2	2167- <i>i</i> 86	11	59
	[2163- <i>i</i> 75]	[7.7]	[52]
3F_2	2167- <i>i</i> 86	0.3	85
	[2163- <i>i</i> 75]	[0.3]	[86]

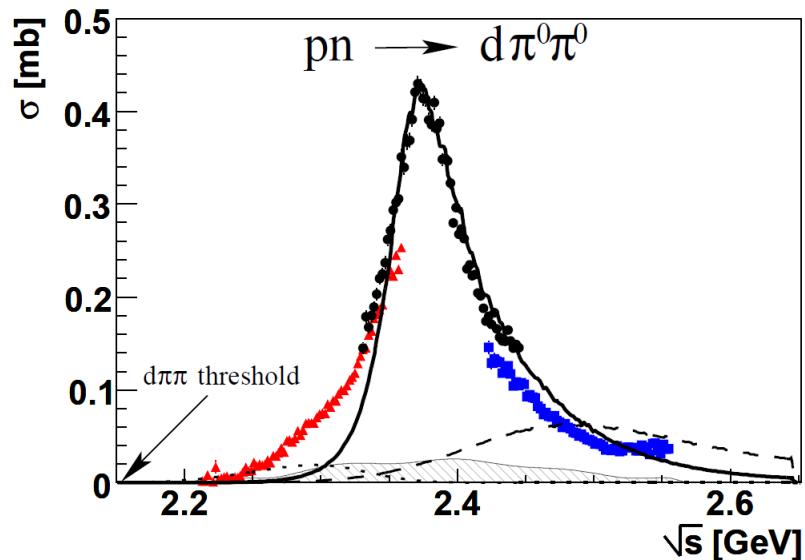
(NΔ)

4. Б. В. Мартемьянов и М. Г. Щепкин, Письма в ЖЭТФ 53, 132 (1991);
 R. Bilger, H. A. Clement, M. G. Schepkin, PRL 71, 42 (1993);

d'(2063) $IJ^P = 00^-$ (экспериментально не подтвержден)

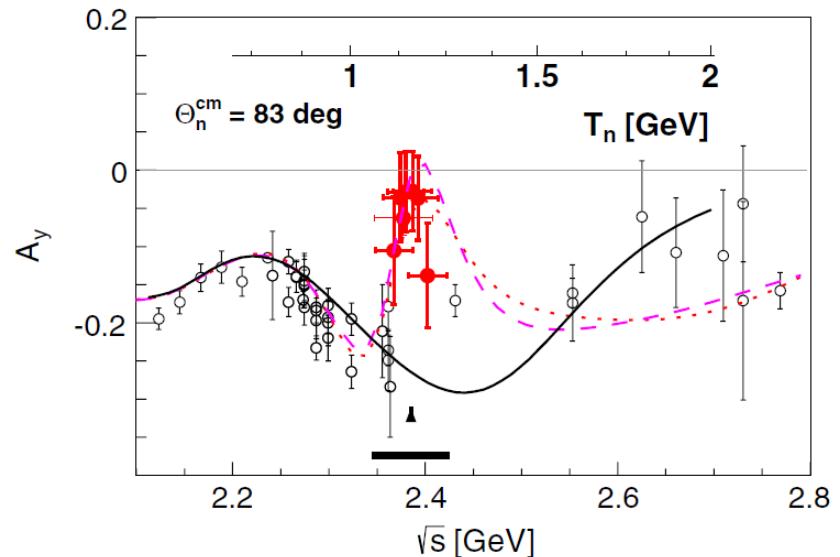
1D_2 also is seen in
 $pp(^1D_2) \leftrightarrow \pi^+ d(^3P_2)$

5. P. Adlarson et al. (WASA@COSY), PRL 106, 242302 (2011).
 $\text{pd} \rightarrow \text{d}\pi^0\pi^0 + \text{p}_s \quad \rightarrow \quad \text{d}^*(2370) \Gamma = 70 \text{ MeV}, IJ^P = 03^+ (\Delta\Delta)$



Also seen in: $\text{pd} \rightarrow \text{d}\pi^+\pi^- + \text{p}_s$

Also seen in: $\text{dp} \xrightarrow{\rightarrow} \text{pn} + \text{p}_s$
P. Adlarson et al.,
PRL 112, 202301 (2014)



ДИБАРИОНЫ КАК РЕЗОНАНСЫ И КАК ПРИМИТИВЫ

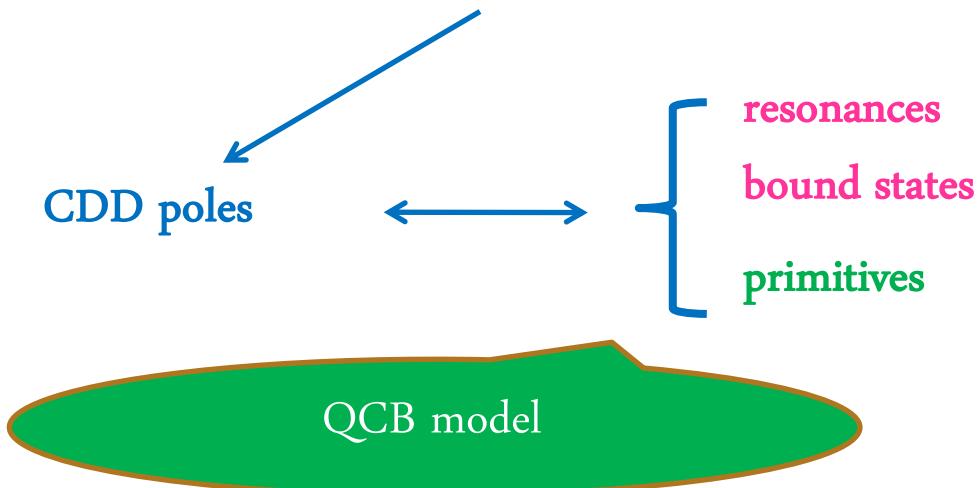
P matrix method for identifying exotic multiquark states with primitives
that appear as poles of the P matrix rather than the S matrix:

Jaffe and Low (1979)

A dynamical QCB model of the P matrix was developed and applied to the
description of nucleon-nucleon scattering

$$\text{Im } D(s) > 0$$

Simonov (1981) <- Dyson model <- Lee model

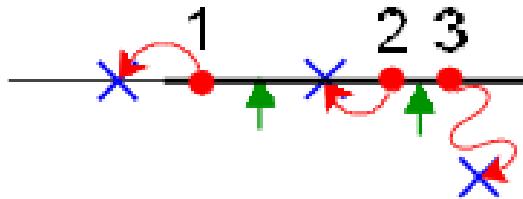


$$\text{Im } D(s) \geq 0$$

CDD poles correspond to resonances, bound states, and primitives

Связь полюсов Р матрицы с полюсами Кастильехо-Далица-Дайсона

D-FUNCTION ZEROS IN THE COMPLEX s -PLANE

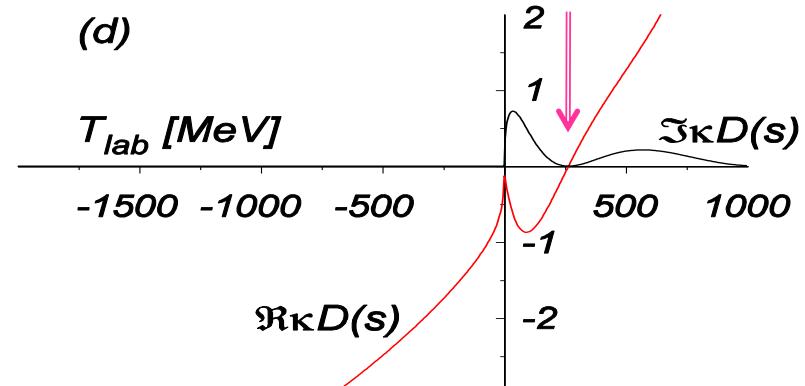
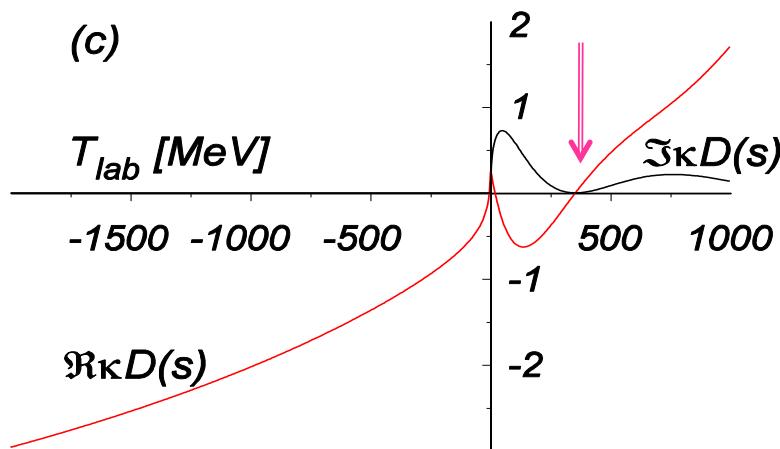
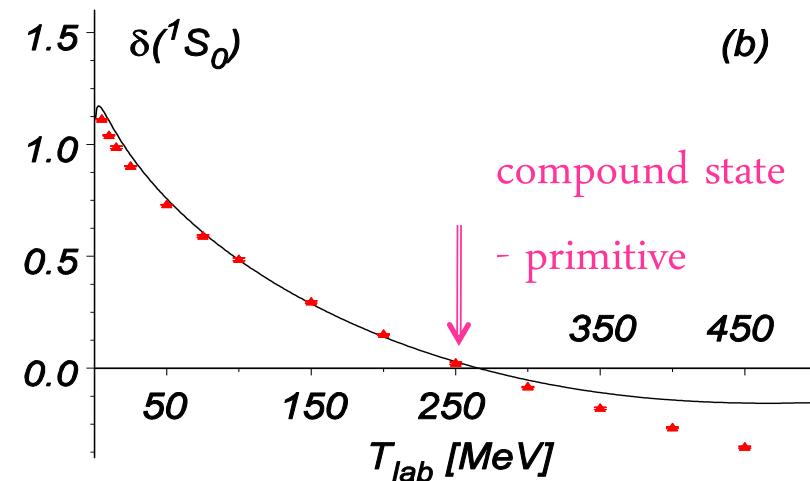
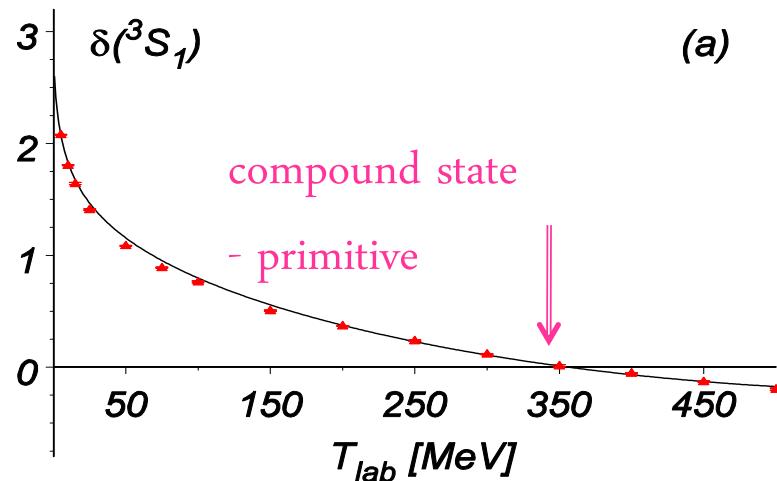


Compound states 1, 2, and 3 move to new positions:

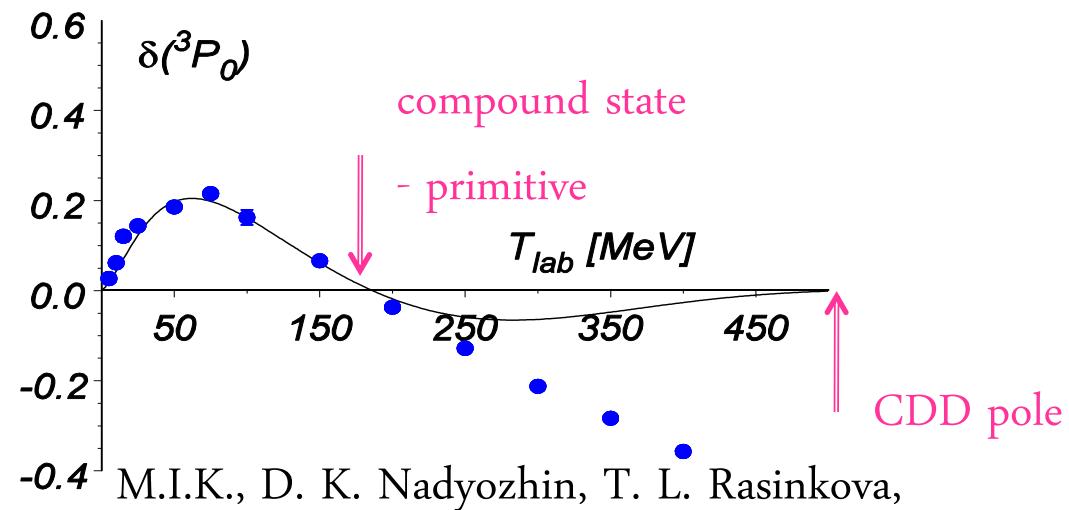
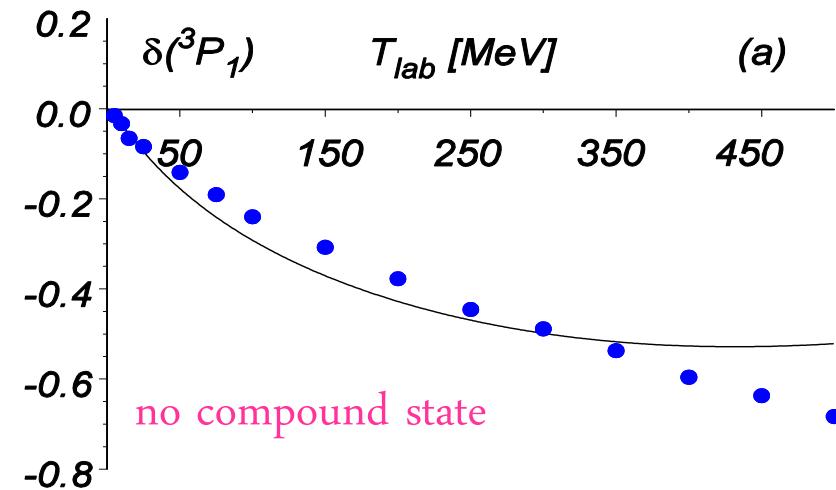
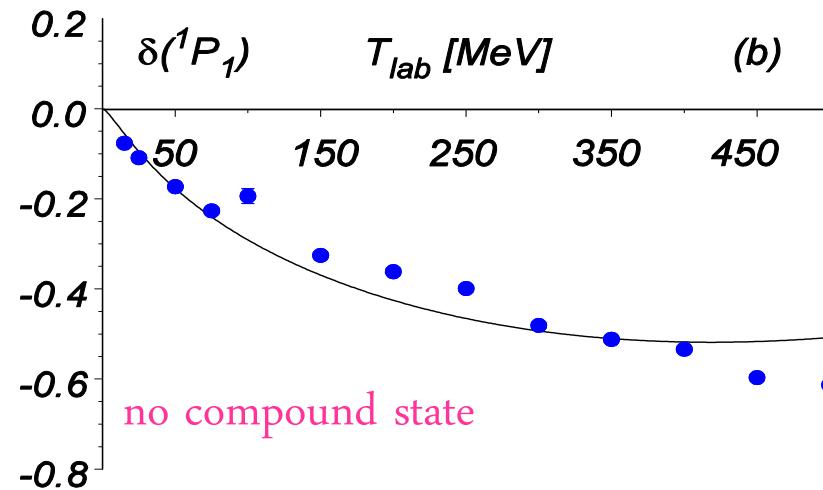
1-bound state, 2-primitive, and 3- resonance.

A pair of the CDD poles that squeezes compound state #2 of the primitive type is shown by arrows.

NN scattering S-wave phase shifts and the D functions
vs. the proton kinetic energy



NN scattering P-wave phase shifts
vs. the proton kinetic energy



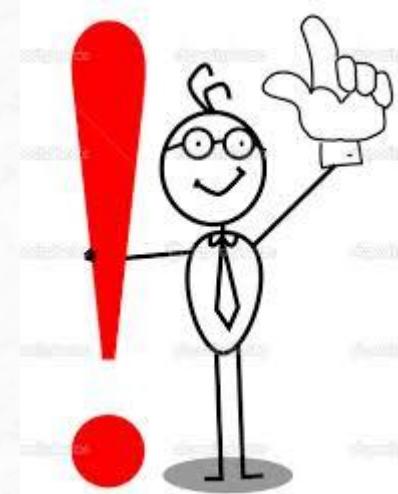
CDD poles are related to:

DYSON →

bound states, resonances

+ primitives.

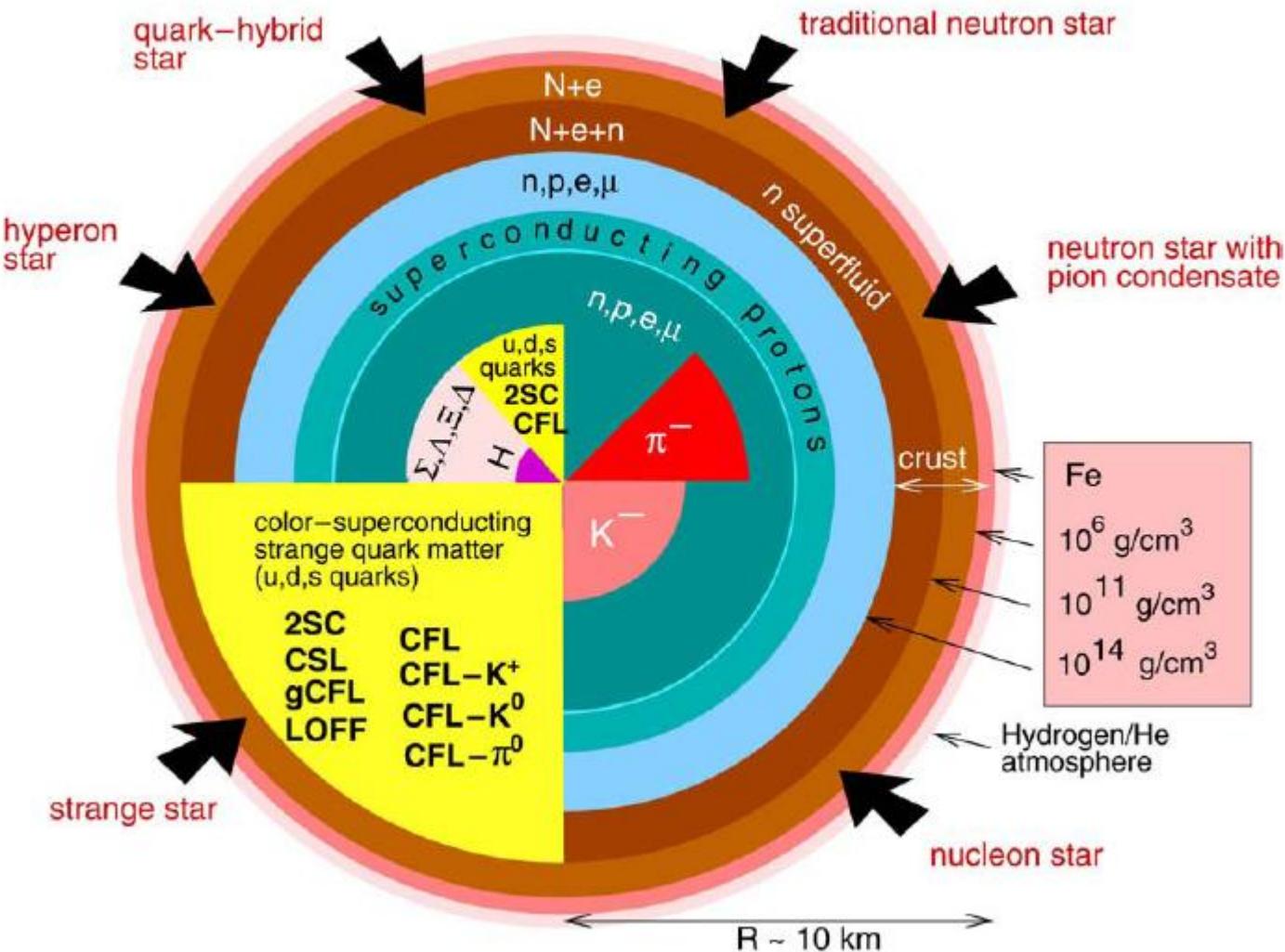
← M.I.K. (2010)



Possible global structures of neutron stars:

196

F. Weber / Progress in Particle and Nuclear Physics 54 (2005) 193–288

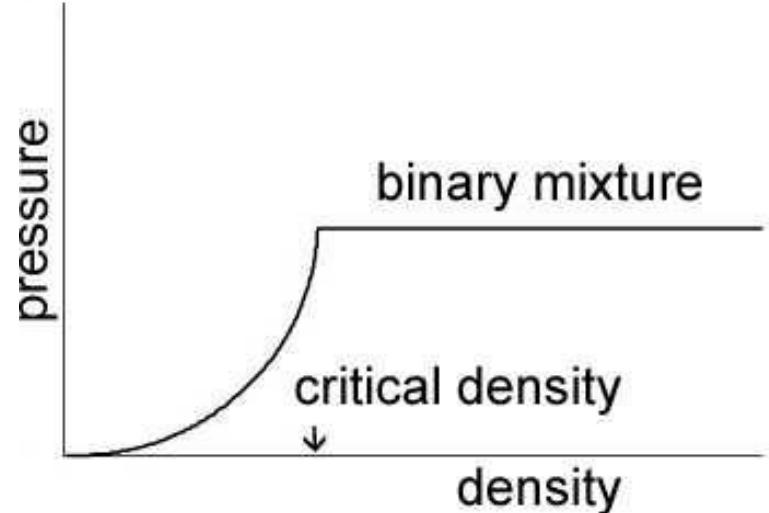
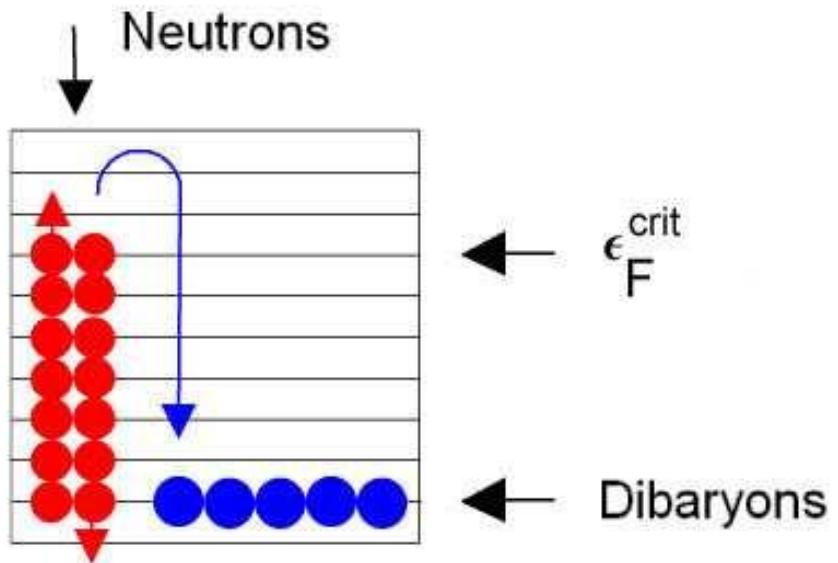


Current models for EoS & neutron stars:

like in the particle physics in 1960's:

a lot of data & THERE IS NO “standard model”

Бозе конденсация дибарионов



**Бозе конденсация дибарионов
в ядерной материи**

А. М. Балдин и др., Докл. Акад.
Наук СССР 279, 602 (1984).

**Схематический вид уравнения
состояния ядерной материи
с примесью дибарионов в
приближении идеального
газа** М.И.К., Письма в ЖЭТФ
46, 5 (1987)

Точно решаемая 1D модель Бозе конденсации двухфермионных резонансов в ферми-жидкости

$$\left(\sum_i \frac{\hat{p}_i^2}{2m} + \sum_{i < j} V(x_i - x_j) \right) \Psi(x_1, \dots, x_N) = E \Psi(x_1, \dots, x_N)$$

$V(x) \rightarrow \infty$ при $|x| < a$ & $a \rightarrow 0$, периодические граничные условия

фаза рассеяния $\delta_+(k)$: $\exp(2i\delta_+(k)) = \frac{(k + k_0)(k - k_0^*)}{(k + k_0^*)(k - k_0)}$,

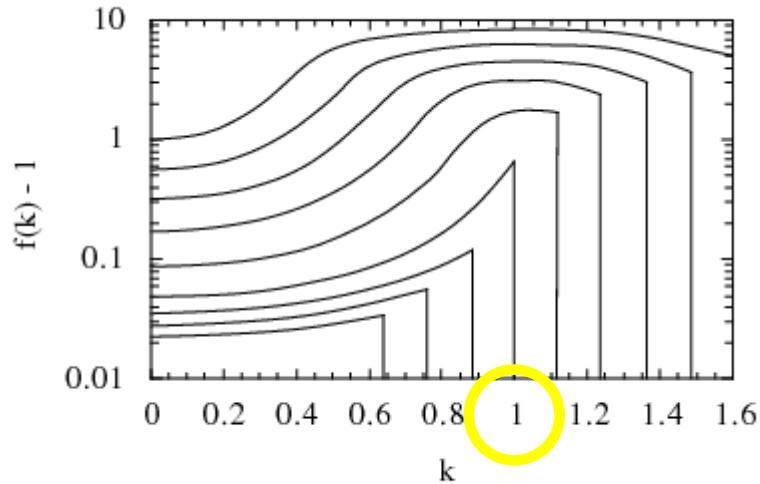
БЕТЕ АНЗАТЦ \Rightarrow правило квантования Бора-Зоммерфельда:

$$k_j L + \sum_{l=1}^N 2\delta_- \left(\frac{k_j - k_l}{2} \right) = 2\pi n_j.$$

Термодинамический предел: $\sum \rightarrow \int \frac{L dk}{2\pi} f(k), \quad f(k) = 2\pi L^{-1} dn/dk$

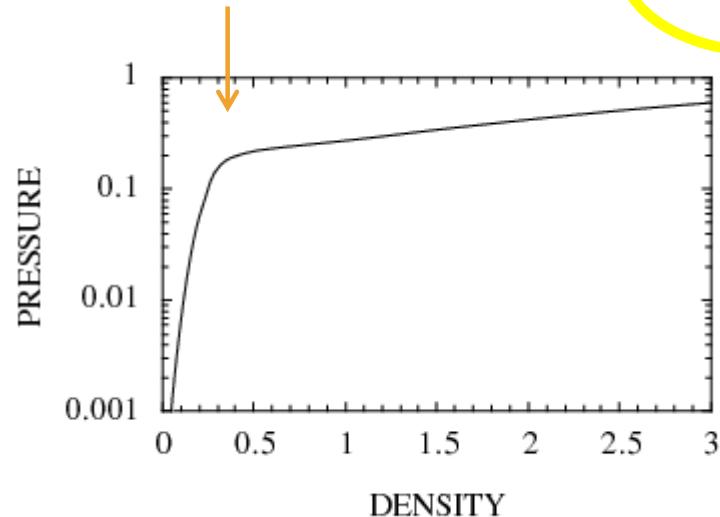
$$f(k) = 1 + \int_{-p_F}^{p_F} \frac{dk'}{2\pi} f(k') \delta'_+ \left(\frac{k - k'}{2} \right)$$

критическая плотность $n = 0.36$



Функция распределения
фермионов по импульсам
 $f(k)$ для отдельных значений
ферми-импульса $k (= k_F)$.

резонанс при импульсе $k_1 = 1$, ширина $k_2 = 0.05$.



Давление как функция
плотности.

В КАЧЕСТВЕННОМ СОГЛАСИИ С
МОДЕЛЬЮ ИДЕАЛЬНОГО ГАЗА

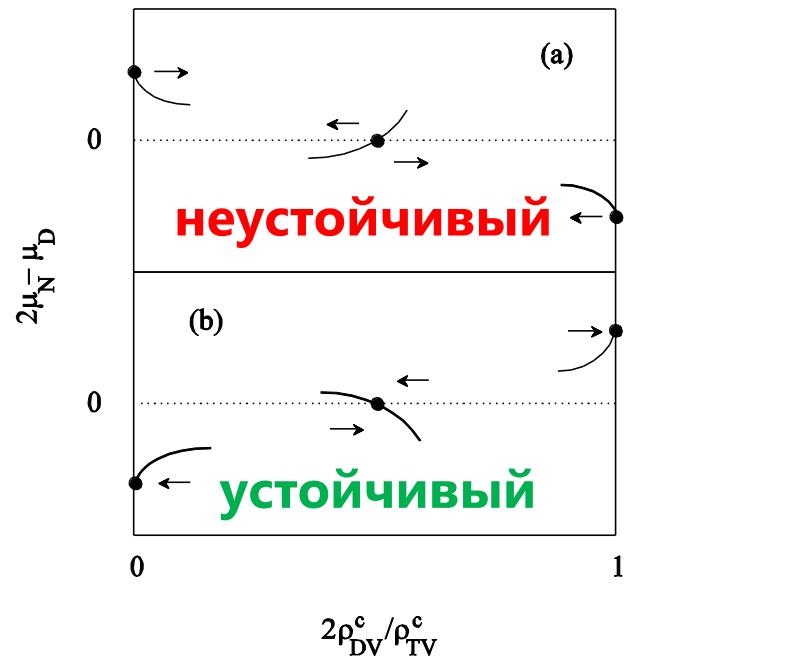
ДИБАРИОНЫ В ЯДЕРНОЙ МАТЕРИИ В МОДЕЛИ СРЕДНЕГО ПОЛЯ

Модель Валечки со скалярными дибарионами:

$$\begin{aligned} L = & \bar{\Psi}(i\partial_\mu\gamma_\mu - m_N - g_\sigma\sigma - g_\omega\omega_\mu\gamma_\mu)\Psi \\ & + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m_\omega^2\omega_\mu^2 - \frac{1}{2}\lambda(\partial_\mu\omega_\mu)^2 \\ & + (\partial_\mu - ih_\omega\omega_\mu)\varphi^*(\partial_\mu + ih_\omega\omega_\mu)\varphi - (m_D + h_\sigma\sigma)^2\varphi^*\varphi. \end{aligned}$$

Конденсат:

$$\varphi_c(t) = e^{-i\mu_D t} \sqrt{\rho_{DS}^c}.$$



Вершины дибарионов:

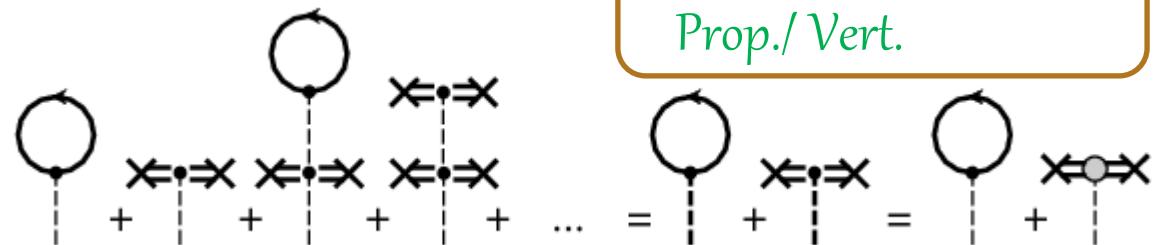
$$\begin{array}{c} \text{---} \\ | \quad \omega, \sigma \\ \text{---} \end{array} = \begin{cases} -ih\omega(2\mu_D + k' + k)_\mu \\ -ih_\sigma 2m_D \end{cases}$$

$$\begin{array}{c} \text{---} \\ | \quad \omega, \sigma \\ \text{---} \end{array} = \begin{cases} ih_\omega^2 g_{\mu\nu} \\ -ih_\sigma^2 \end{cases}$$

$$\Rightarrow = \sqrt{\rho_{DS}^c}$$

Renorm.:

Prop./ Vert.



Среднее поле:

Полные вершины в MFT :

$$\begin{array}{c} \text{---} \\ | \quad \omega, \sigma \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \quad \omega, \sigma \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \quad \omega, \sigma \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \quad \omega, \sigma \\ \text{---} \end{array} = \begin{cases} -ih_\omega^*(2\mu_D^* + k' + k)_\mu \\ -ih_\sigma^* 2m_N^* \end{cases}$$

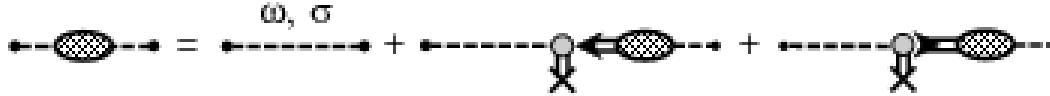
N

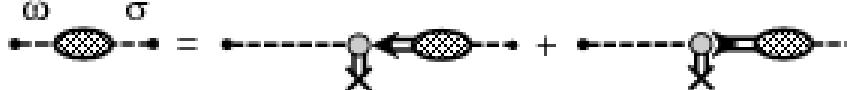
Уравнения Дайсона в MFT :

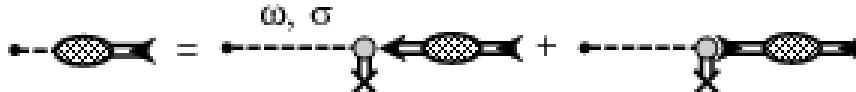
$$\begin{aligned} \text{---} &= \text{---} + \boxed{\text{---}} \\ \leftrightarrow &= \leftrightarrow + \boxed{\text{---}} \\ \leftrightarrow &= \leftrightarrow + \boxed{\text{---}} \\ + \boxed{\text{---}} &+ \boxed{\text{---}} + \end{aligned}$$

=0 ниже критической плотности

Уравнения Беляева-Дайсона в МФТ:

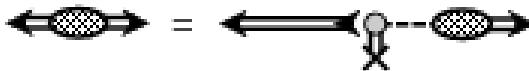
(a)  ω, σ

(b)  ω, σ

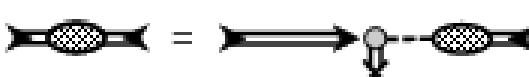
(c)  ω, σ

(d)  ω, σ

(e) 

(f) 

$\sigma-\omega-\varphi-\varphi^*$ mixing

(g) 

древесное приближение

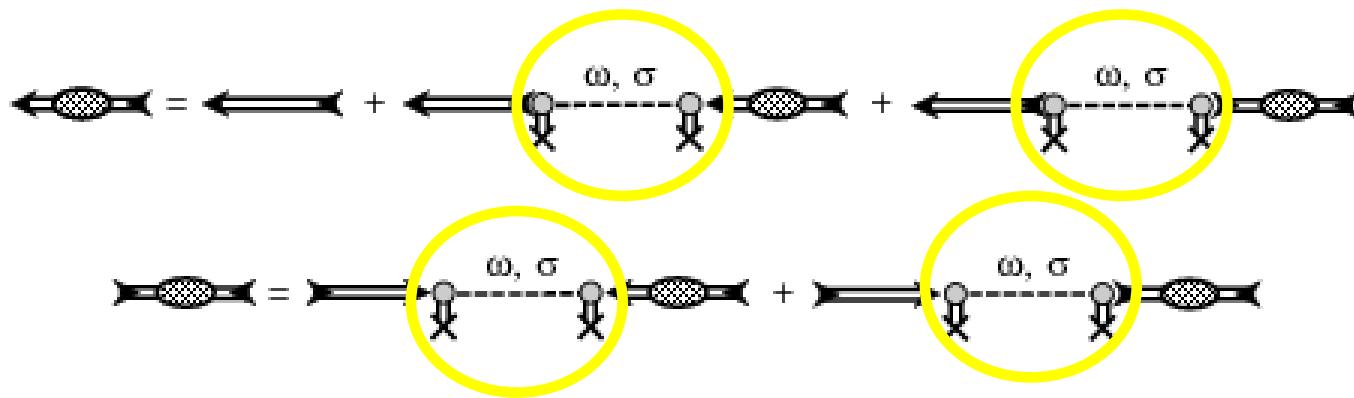
b,c,d,f,g - АНОМАЛЬНЫЕ ФУНКЦИИ ГРИНА

**Замкнутая система 2-х уравнений для функций Грина
дибариона :**

$$\begin{aligned} \text{Diagram 1: } & \text{Left side: } \text{Diagram of a closed loop with two internal vertices. The top vertex is shaded gray, and the bottom vertex is crossed out with an 'X'.} \\ & \text{Right side: } \text{Diagram of a closed loop with two internal vertices. The top vertex is crossed out with an 'X', and the bottom vertex is shaded gray.} \\ & = \text{Diagram of a closed loop with two internal vertices. The top vertex is crossed out with an 'X', and the bottom vertex is shaded gray.} + \text{Diagram of a closed loop with two internal vertices. The top vertex is shaded gray, and the bottom vertex is crossed out with an 'X'.} \\ & \quad \text{Diagram of a closed loop with two internal vertices. The top vertex is crossed out with an 'X', and the bottom vertex is shaded gray.} + \text{Diagram of a closed loop with two internal vertices. The top vertex is shaded gray, and the bottom vertex is crossed out with an 'X'.} \end{aligned}$$

$$\begin{aligned} \text{Diagram 2: } & \text{Left side: } \text{Diagram of a closed loop with two internal vertices. The top vertex is crossed out with an 'X', and the bottom vertex is shaded gray.} \\ & \text{Right side: } \text{Diagram of a closed loop with two internal vertices. The top vertex is shaded gray, and the bottom vertex is crossed out with an 'X'.} \\ & = \text{Diagram of a closed loop with two internal vertices. The top vertex is crossed out with an 'X', and the bottom vertex is shaded gray.} + \text{Diagram of a closed loop with two internal vertices. The top vertex is shaded gray, and the bottom vertex is crossed out with an 'X'.} \end{aligned}$$

Замкнутая система 2-х уравнений для функций Грина дибариона :



Аналог уравнений Горькова в теории сверхпроводимости

--> like

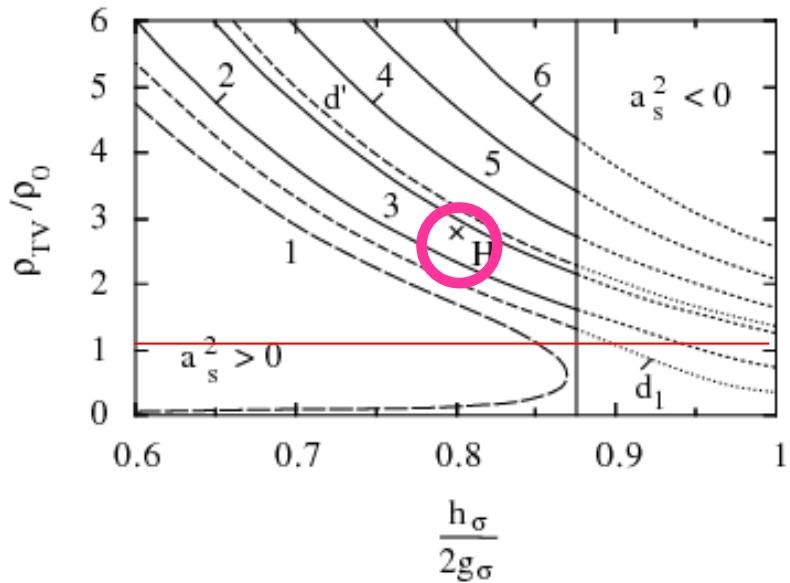
$$\mu_D^{*2} - m_D^{*2} = \Sigma^{\varphi\varphi^*}(0) - \Sigma^{\varphi\varphi}(0) \quad \rightarrow \quad \mu_D^* = m_D^* \quad \text{in the ideal gas}$$

& MF

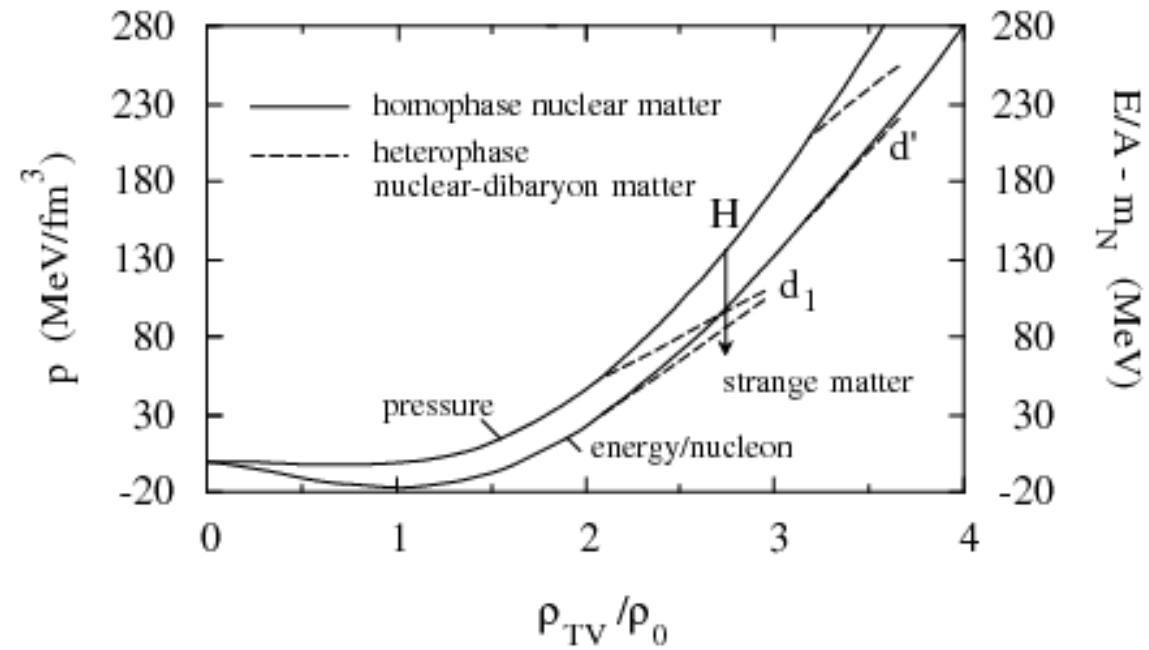
звук

$$a_s^2 = \left(\frac{\partial \omega_s(k)}{\partial k} \right) \Big|_{k=0}^2 = \frac{\alpha}{1 + \alpha} \quad \alpha = 2\rho_{DS}^c \frac{m_\sigma^2}{\tilde{m}_\sigma^2} \left(\frac{h_\omega^2}{m_\omega^2} - \frac{h_\sigma^2}{m_\sigma^2} \right) > 0$$

Уравнение состояния ядерной материи с дибарионами (MFT)



Критическая плотность образования $\text{H}(2220)$,
 $d'(2060)$, $d_1(1920)$

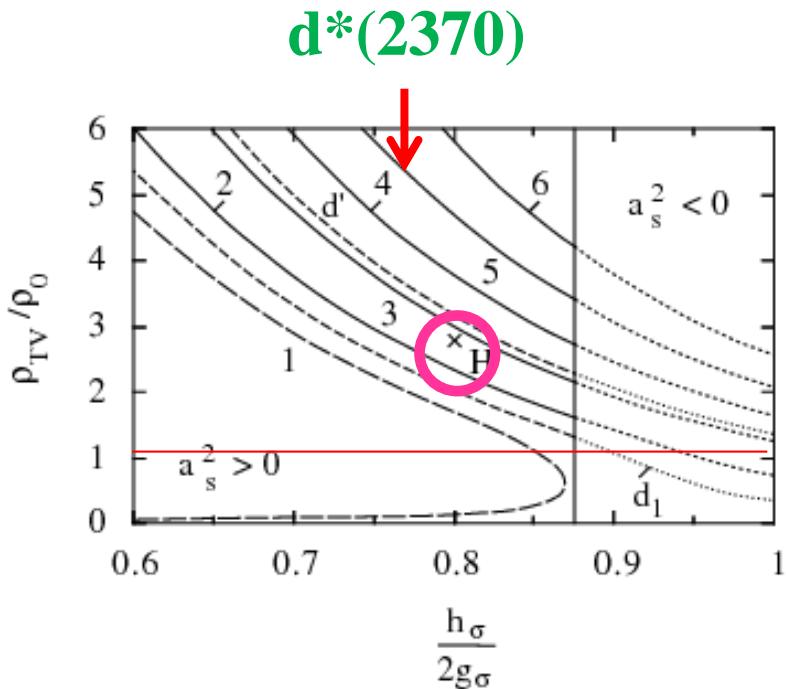


$$h_\omega = 2g_\omega$$

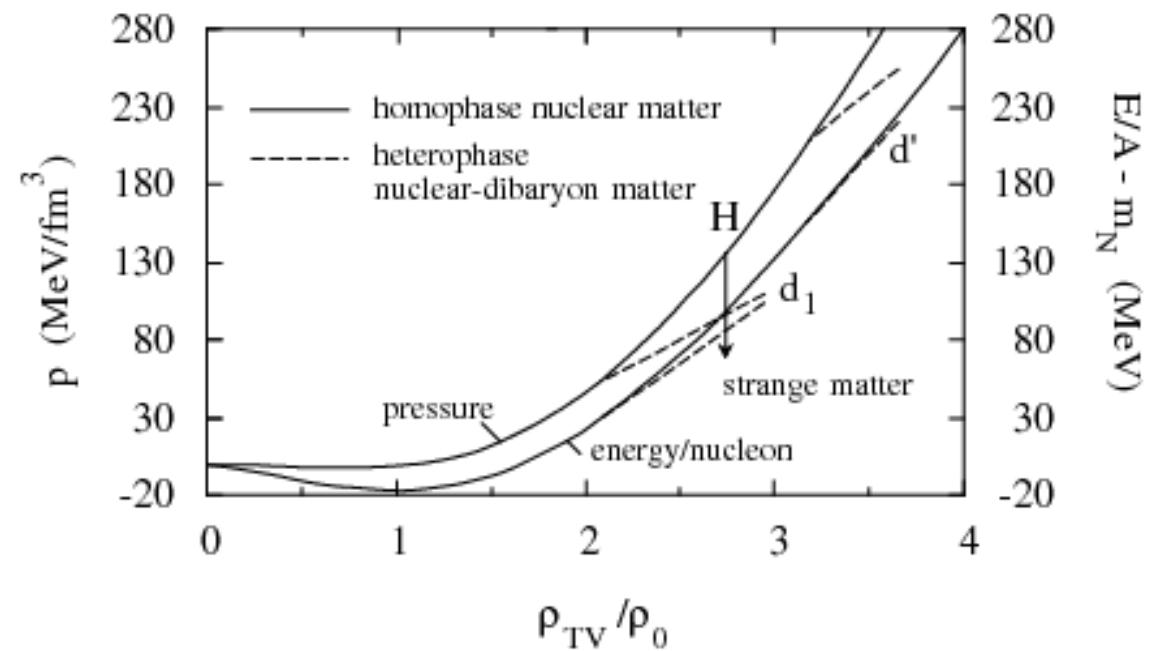
$$h_\sigma / (2g_\sigma) = 0.8$$

$$\rho_{TV} = \rho_{NV} + 2\rho_{DV}^c$$

Уравнение состояния ядерной материи с дибарионами (MFT)



Критическая плотность образования $\text{H}(2220)$,
 $\text{d}'(2060)$, $\text{d}_1(1920)$



$$h_\omega = 2g_\omega$$

$$h_\sigma / (2g_\sigma) = 0.8$$

$$\rho_{TV} = \rho_{NV} + 2\rho_{DV}^c$$

ДИБАРИОНЫ В ЯДЕРНОЙ МАТЕРИИ В RHA

(= MFT + CASIMIR EFF.)

$$\rho_{NS} = \langle \bar{\Psi}(0)\Psi(0) \rangle = \gamma \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{m_N^*}{E^*(\mathbf{p})} \theta(p_F - |\mathbf{p}|)$$

$$- 4m_N^3 \zeta(m_N^*/m_N), \quad \begin{array}{c} \leftarrow \\ \searrow \end{array} \quad \text{Casimir effect}$$
$$2m_D^* \rho_{DS} = 2m_D^* \langle \varphi(0)^* \varphi(0) \rangle = m_D^3 \zeta(m_D^*/m_D)$$

$$\langle T_{\mu\nu}^N(0) \rangle_{\text{vac}} = -4g_{\mu\nu}m_N^4 \eta(m_N^*/m_N)$$

$$\langle T_{\mu\nu}^D(0) \rangle_{\text{vac}} = g_{\mu\nu}m_D^4 \eta(m_D^*/m_D)$$

$$\begin{cases} 4\pi^2 \zeta(x) = x^3 \ln x + 1 - x - \frac{5}{2} (1-x)^2 + \frac{11}{2} (1-x)^3 \\ 16\pi^2 \eta(x) = x^4 \ln x + 1 - x - \frac{7}{2} (1-x)^2 + \frac{13}{3} (1-x)^3 - \frac{25}{12} (1-x)^4 \end{cases}$$

RHA

RHA

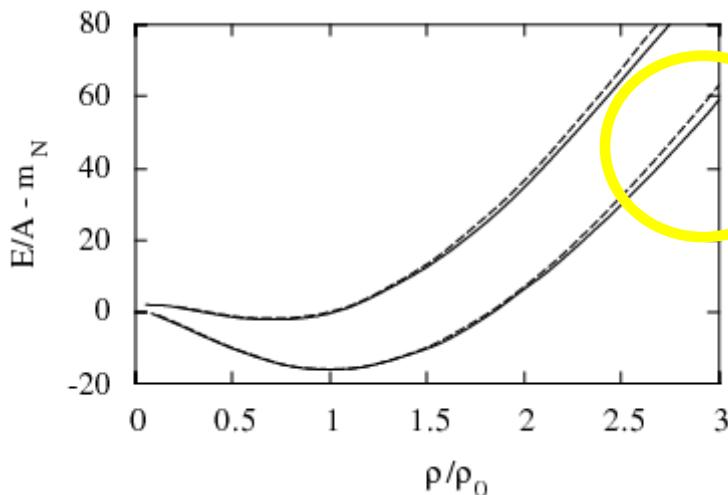
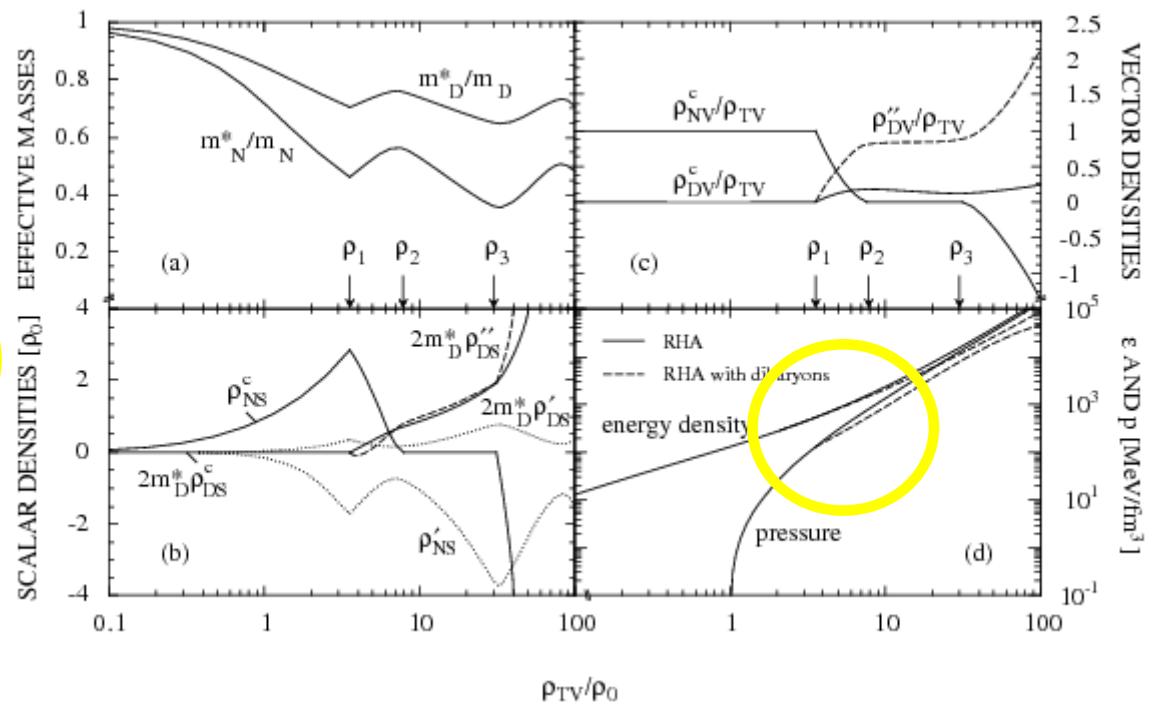


FIG. 2. Saturation curve for nuclear matter in the RHA: without dibaryons (solid line) and with the inclusion of H dibaryons (dashed line) for $h_\sigma/(2g_\sigma)=0.6$.



- a) m_*^N & m_*^D vs ρ_{TV}
- b) scalar density of nucleons
- c) fraction of nucleons and dibaryons
- d) energy density and pressure

$$m_D = 2060 \text{ MeV}$$

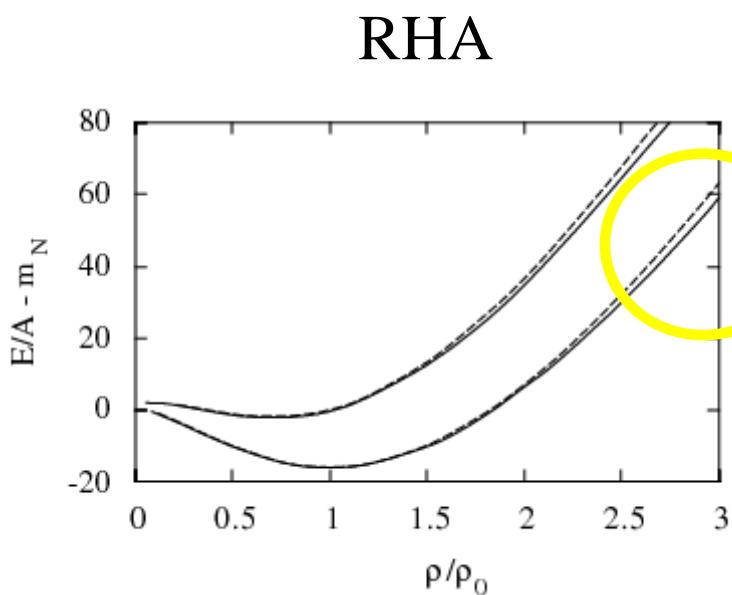
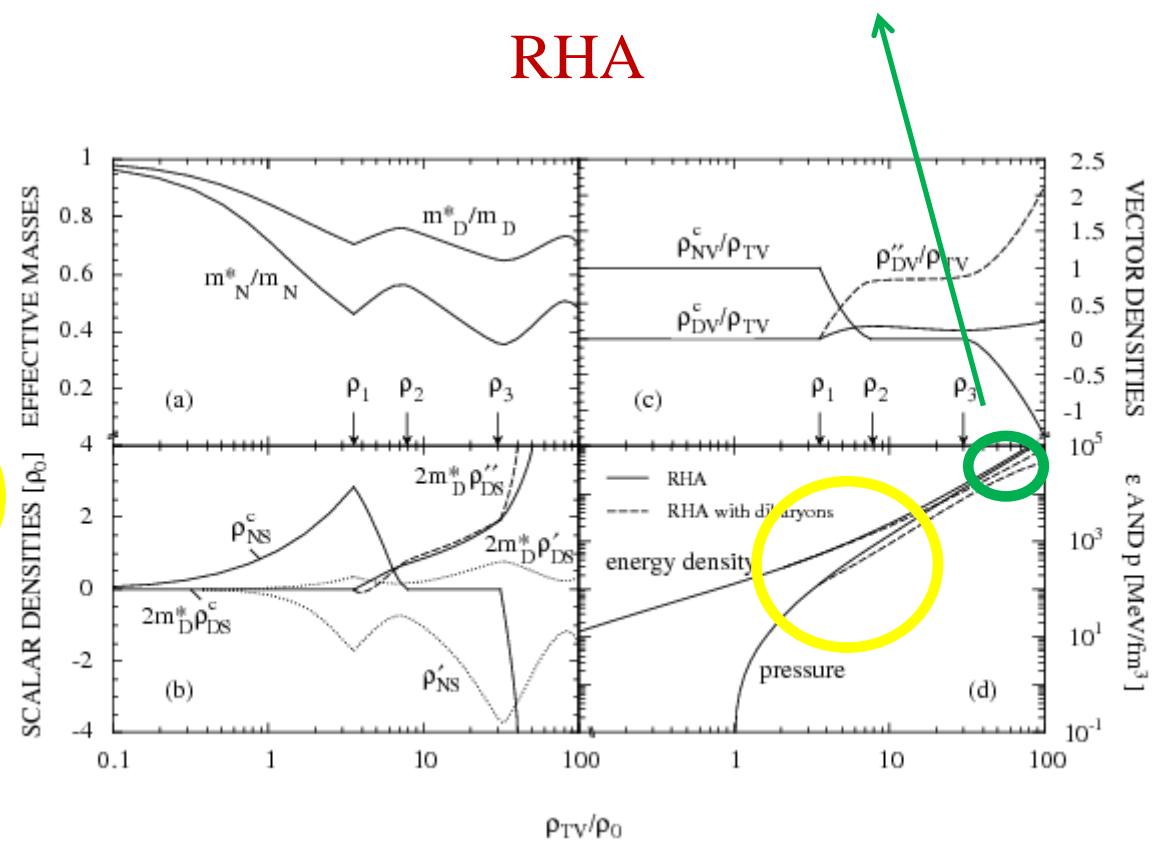
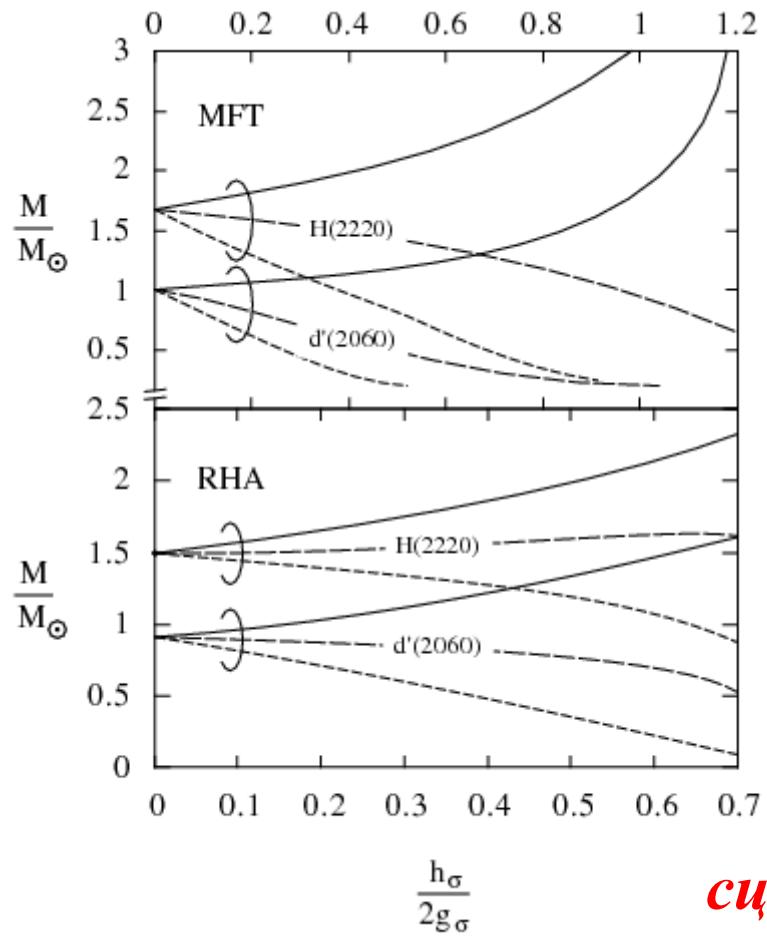


FIG. 2. Saturation curve for nuclear matter in the RHA: without dibaryons (solid line) and with the inclusion of H dibaryons (dashed line) for $h_\sigma/(2g_\sigma)=0.6$.



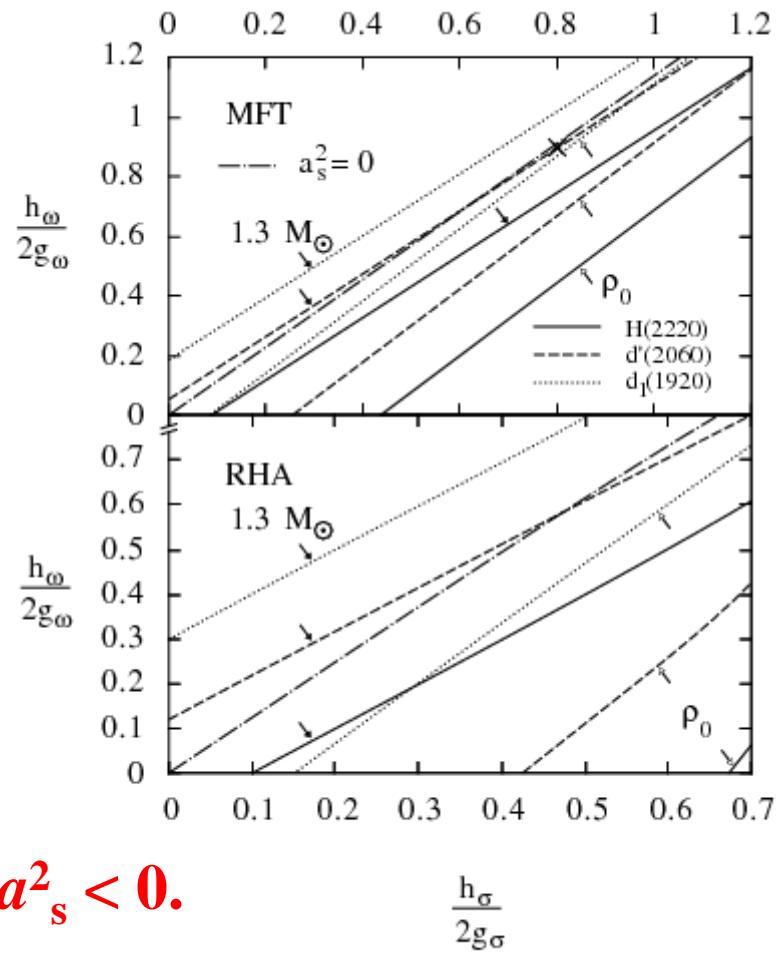
- a) m_*^N & m_*^D vs ρ_{TV}
- b) scalar density of nucleons
- c) fraction of nucleons and dibaryons
- d) energy density and pressure

$$m_D = 2060 \text{ MeV}$$



Lowest neutron star masses,
for which dibaryon formation
becomes energetically favorable
for $h_\omega/h_\omega^{\max} = 1, 0.8, 0.6$;

сценарий $a_s^2 < 0$.

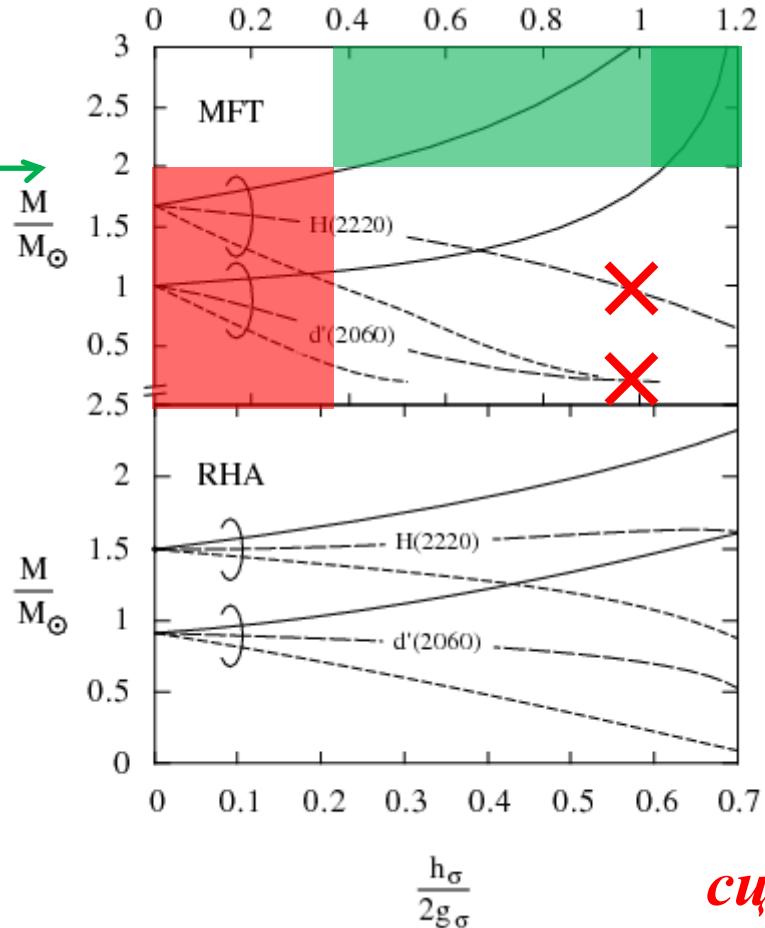


Parameter space for the dibaryon
coupling constants in MFT and RHA.

PSR J1614-2230 $M = 1.97 \pm 0.04 M_{\odot}$

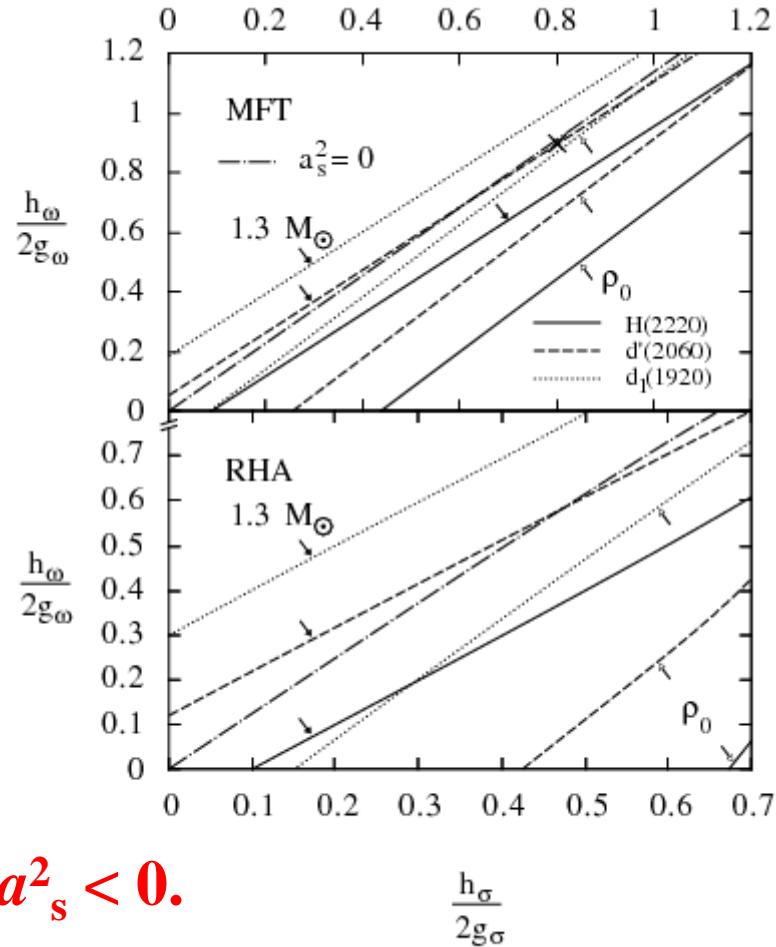
P. B. Demorest et al., Nature 467 (2010) 1081.

provided just neutron star



сценарий $a_s^2 < 0$.

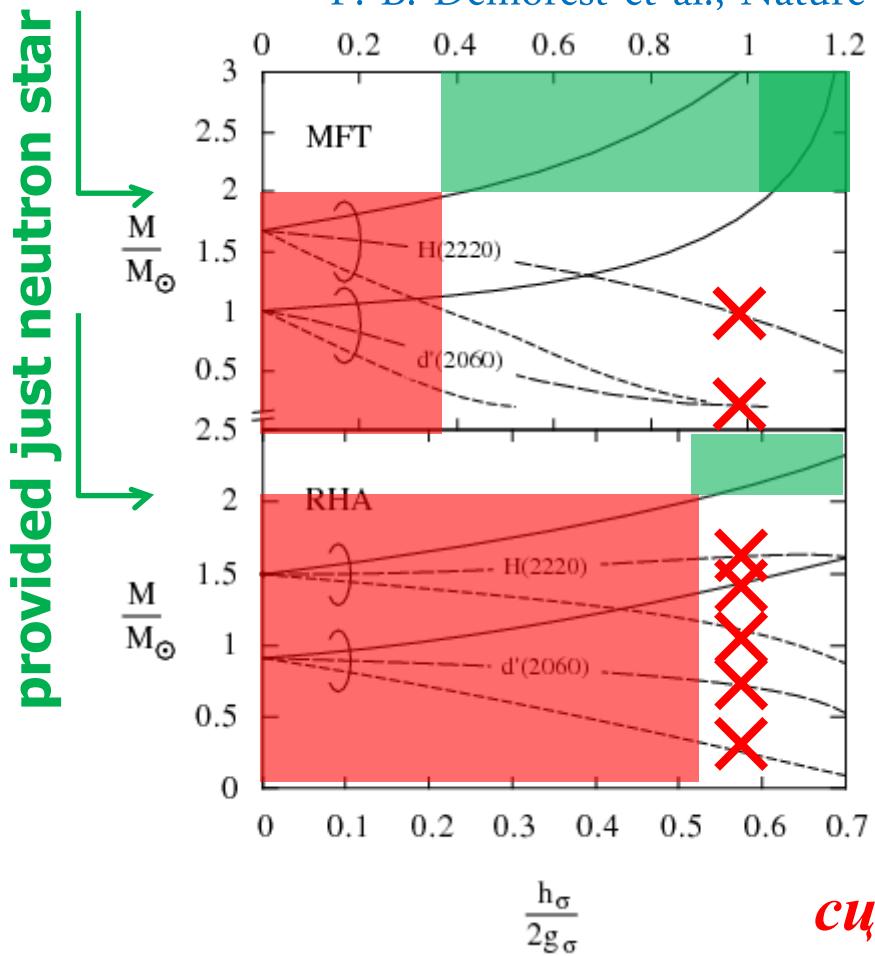
Lowest neutron star masses,
for which dibaryon formation
becomes energetically favorable
for $h_{\omega}/h_{\omega}^{max} = 1, 0.8, 0.6$; $a_s^2 < 0$.



Parameter space for the dibaryon coupling constants in MFT and RHA.

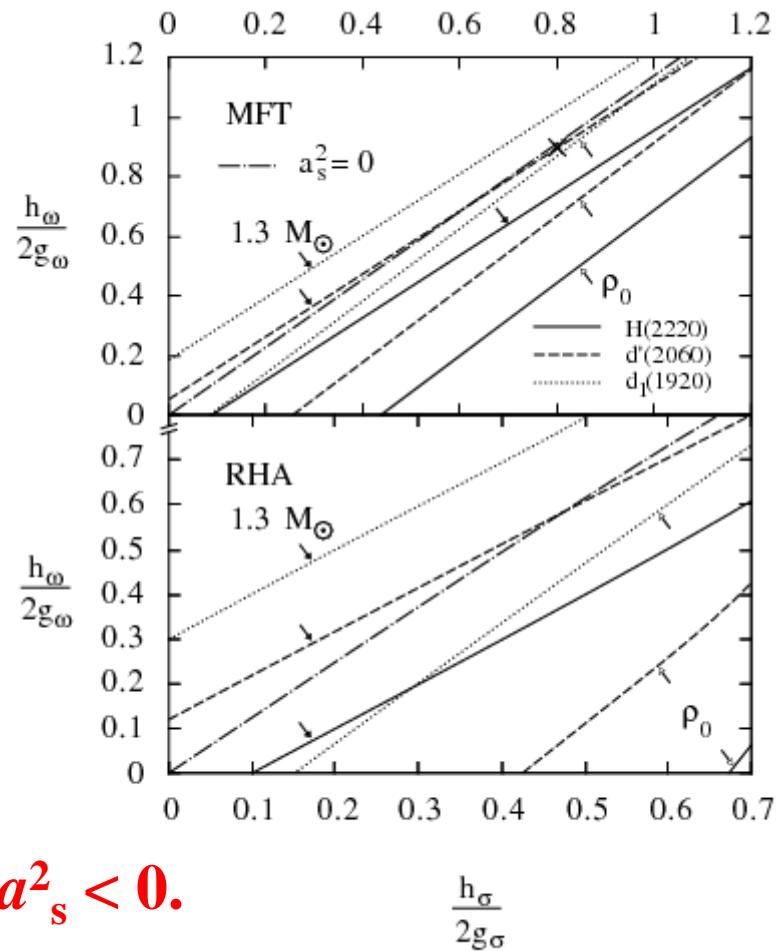
PSR J1614-2230 $M = 1.97 \pm 0.04 M_{\odot}$

P. B. Demorest et al., Nature 467 (2010) 1081.



Lowest neutron star masses,
for which dibaryon formation
becomes energetically favorable
for $h_{\omega}/h_{\omega}^{max} = 1, 0.8, 0.6$; $a_s^2 < 0$.

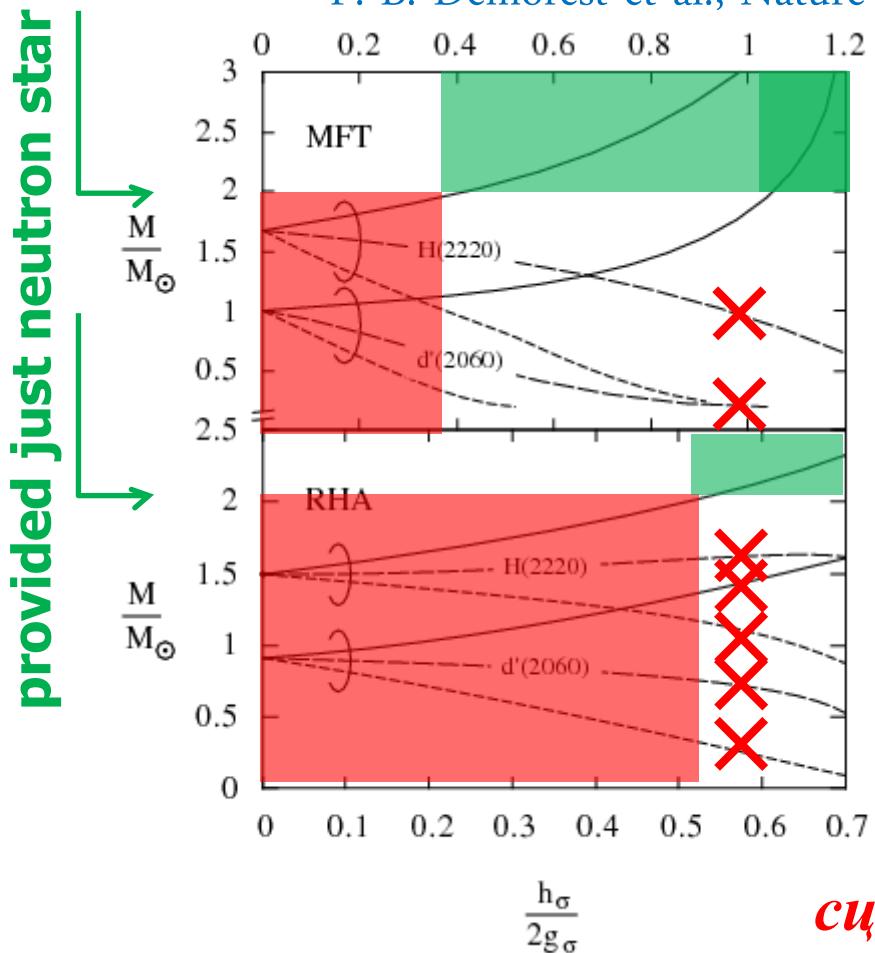
сценарий $a_s^2 < 0$.



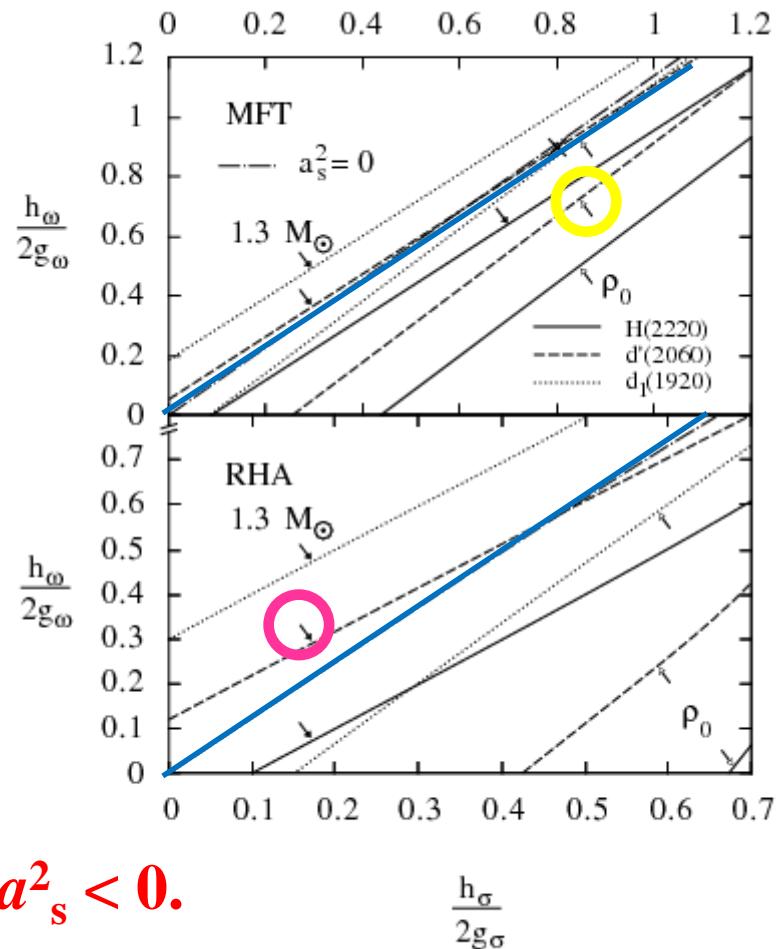
Parameter space for the dibaryon coupling constants in MFT and RHA.

PSR J1614-2230 $M = 1.97 \pm 0.04 M_{\odot}$

P. B. Demorest et al., Nature 467 (2010) 1081.



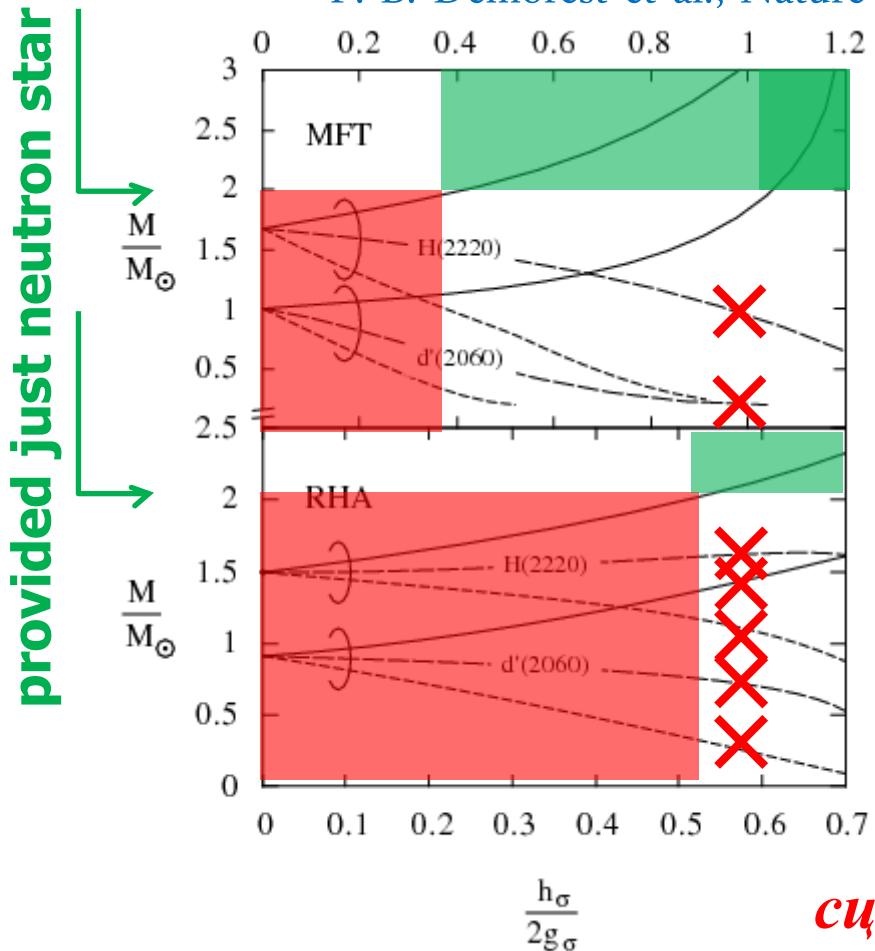
Lowest neutron star masses,
for which dibaryon formation
becomes energetically favorable
for $h_{\omega}/h_{\omega}^{max} = 1, 0.8, 0.6$; $a_s^2 < 0$.



Parameter space for the dibaryon coupling constants in MFT and RHA.

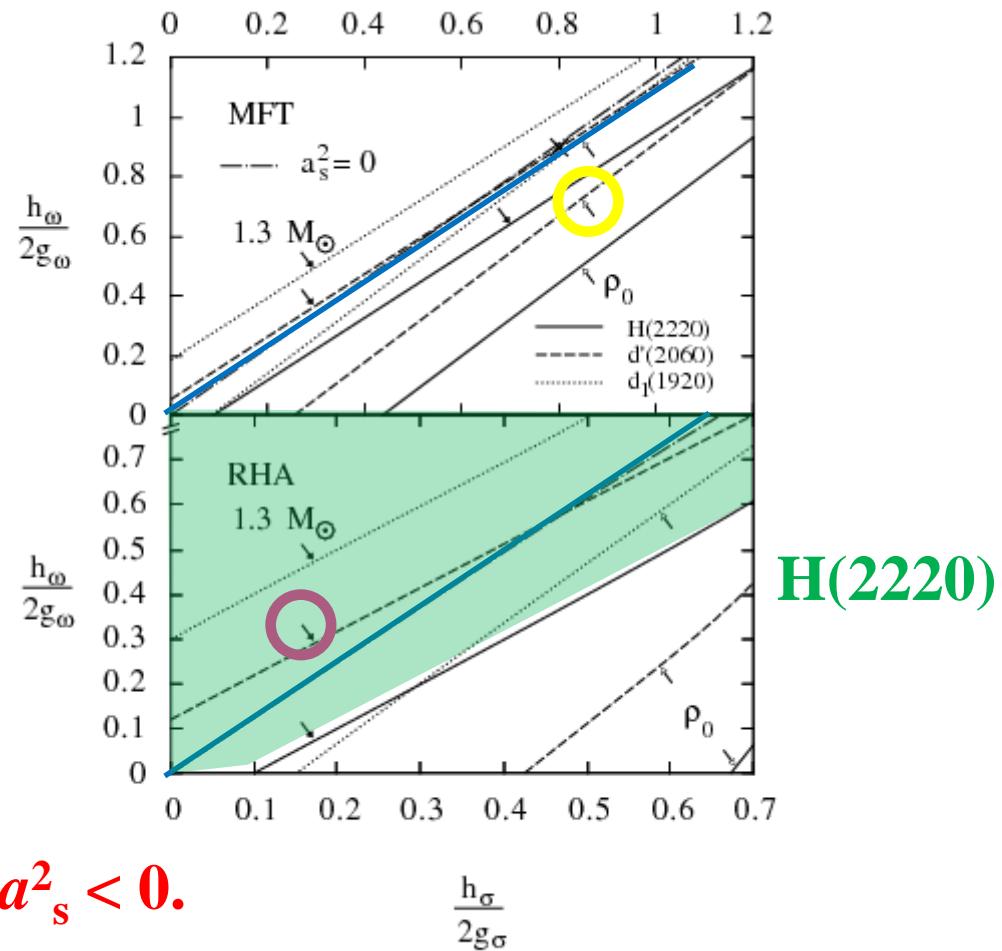
PSR J1614-2230 $M = 1.97 \pm 0.04 M_{\odot}$

P. B. Demorest et al., Nature 467 (2010) 1081.



сценарий $a_s^2 < 0$.

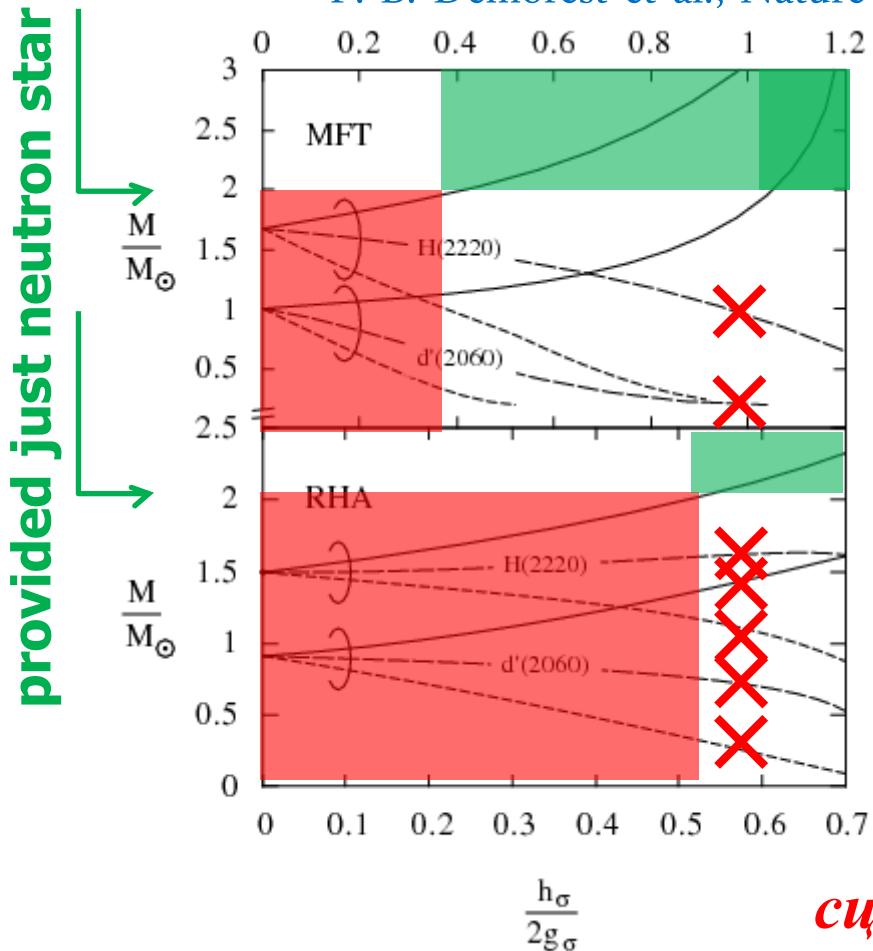
Lowest neutron star masses,
for which dibaryon formation
becomes energetically favorable
for $h_{\omega}/h_{\omega}^{max} = 1, 0.8, 0.6$; $a_s^2 < 0$.



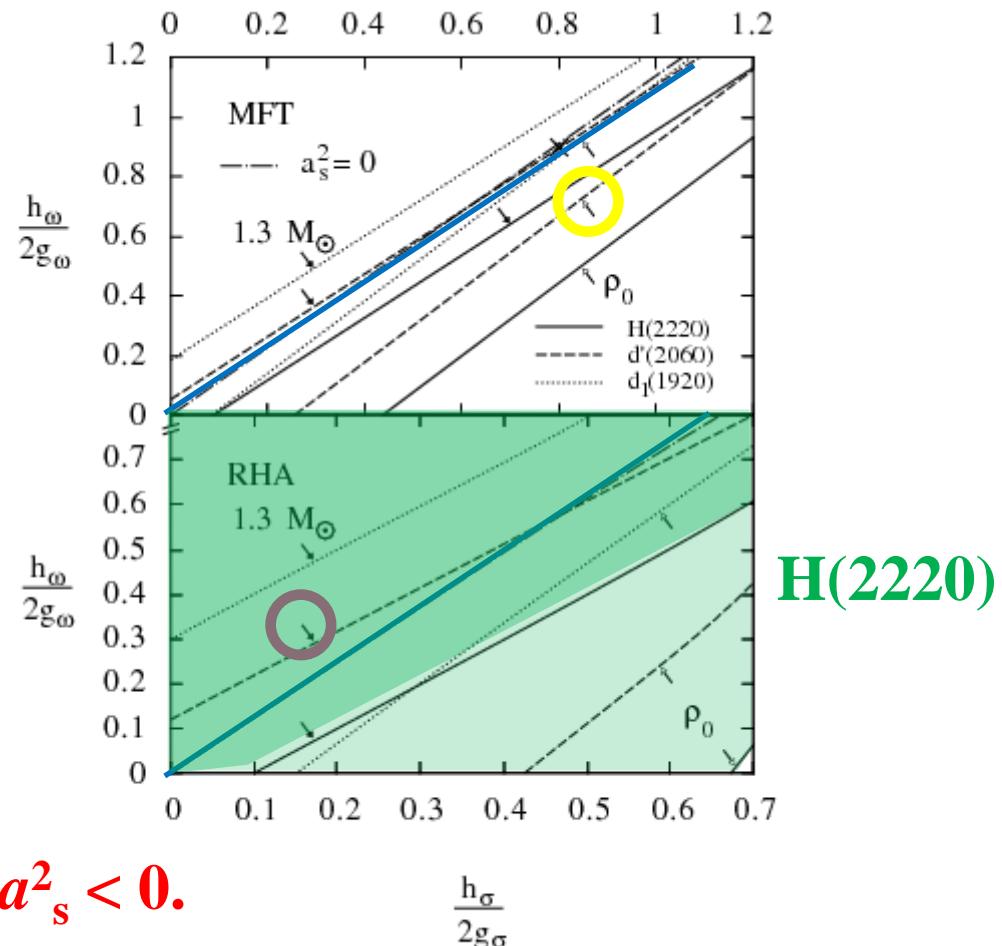
Parameter space for the dibaryon
coupling constants in MFT and RHA.

PSR J1614-2230 $M = 1.97 \pm 0.04 M_{\odot}$

P. B. Demorest et al., Nature 467 (2010) 1081.



Lowest neutron star masses,
for which dibaryon formation
becomes energetically favorable
for $h_{\omega}/h_{\omega}^{max} = 1, 0.8, 0.6$; $a_s^2 < 0$.



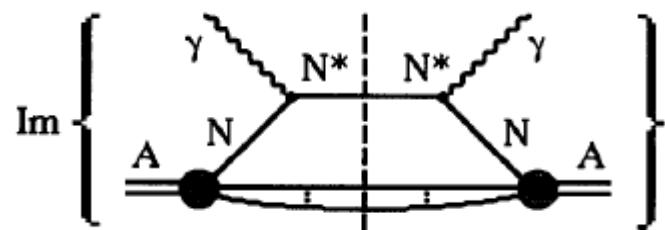
Parameter space for the dibaryon coupling constants in MFT and RHA.

1. Существует новый тип КДД полюсов, связанный с примитивами
2. Исследовано уравнение состояния ядерной материи с учетом Бозе конденсации дибарионов в MFT & RHA
3. Получены ограничения на массы и константы связи дибарионов из существования массивных нейтронных звезд
4. Бинарная смесь нуклонов и легких дибарионов устойчива ($a_s^2 > 0$).

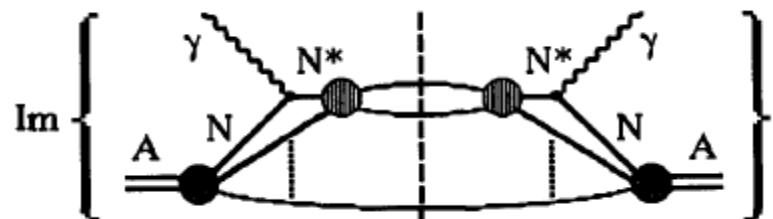
Рождение резонансов в ядрах

- ◆ L. A. Kondratyuk, M. I. K., N. Bianchi, E. De Sanctis and V. Muccifora, Nucl. Phys. A 579, 453 (1994);
- ◆ K. G. Boreskov, L. A. Kondratyuk, M. I. K. and J. H. Koch Nucl. Phys. A 619, 295 (1997).

Фраскати (1992) и Майнц (1994) измерили полные сечения фотопоглощения на некоторых ядрах для изучения поведения барионных резонансов в ядерной среде.

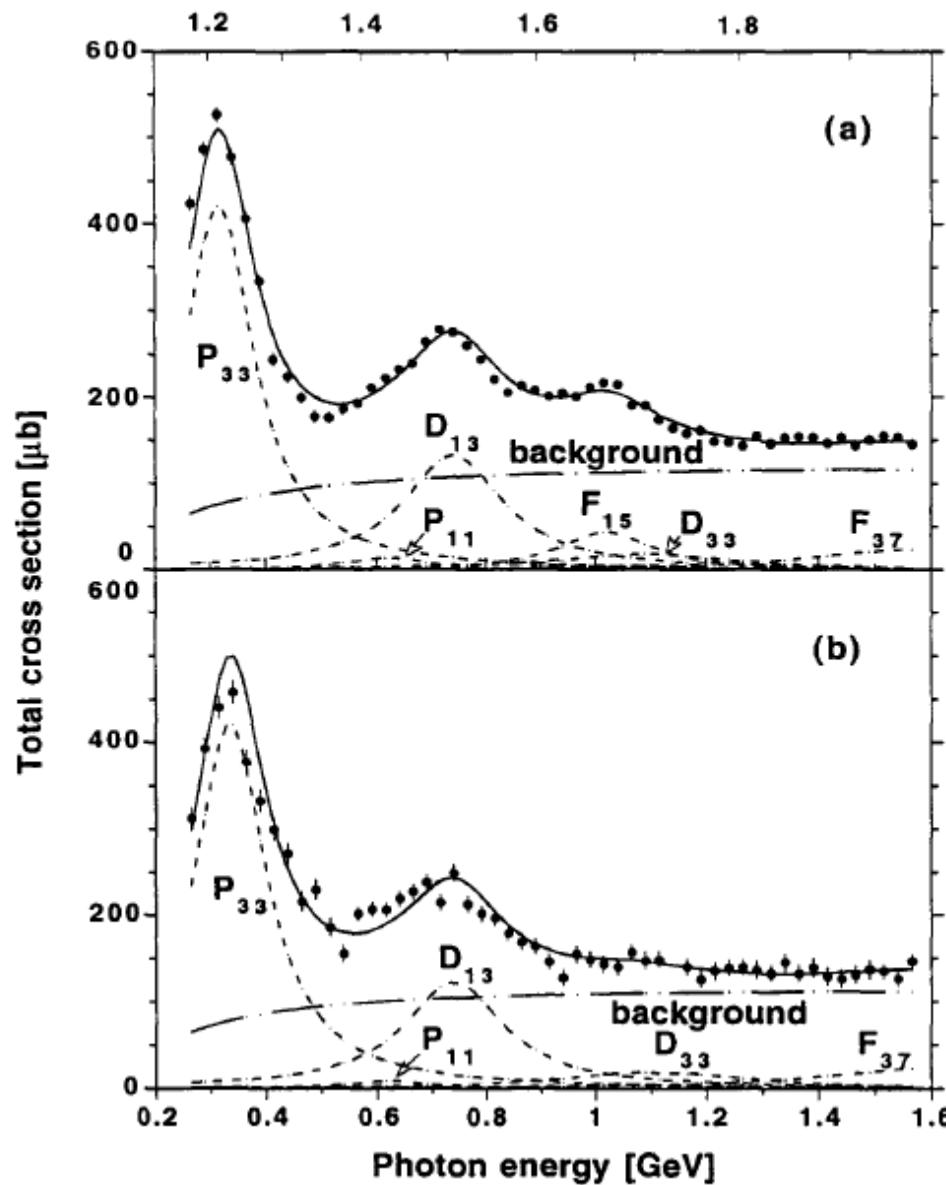


свободные нуклоны



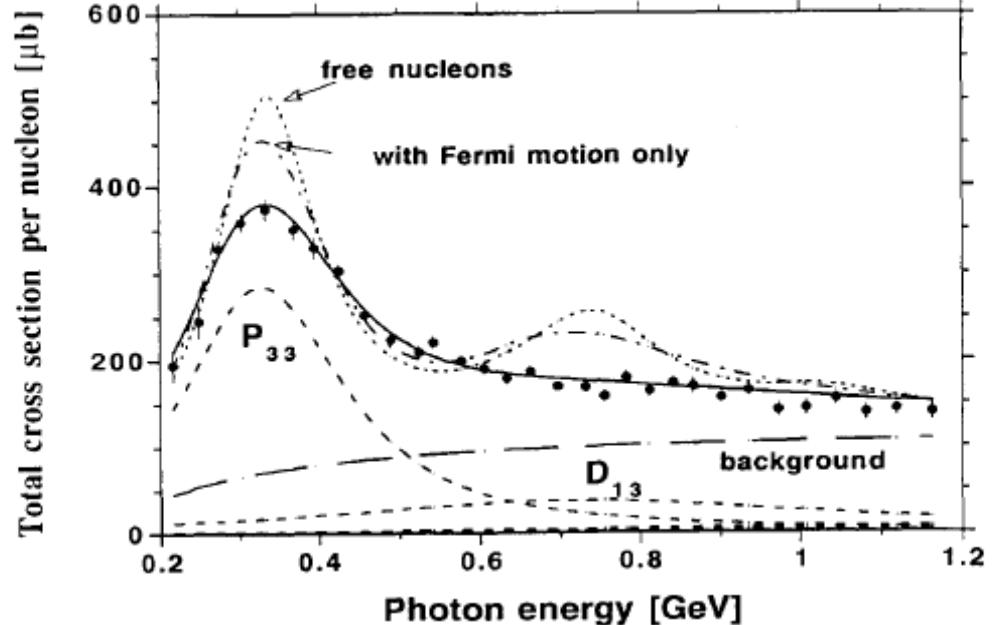
с учетом перерассеяния

Centre of mass energy [GeV]



(a)

(b)



$$\delta M = -\frac{2\pi}{M} \Re f_{NN^*}(\theta=0) \rho_0,$$

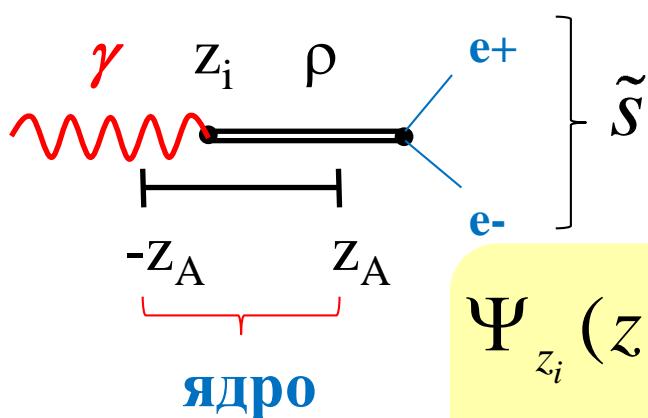
$$\Gamma^* = \frac{4\pi}{M} \Im f_{NN^*}(\theta=0) \rho_0.$$

$$\Im f_{NN^*}(\theta=0) = \frac{k}{4\pi} \sigma^*$$

фотопоглощение на нуклонах

фотопоглощение на уране

КОГЕРЕНТНОЕ РОЖДЕНИЕ РЕЗОНАНСОВ: ДВУХКОМПОНЕНТНАЯ ФОРМУЛА БРЕЙТА-ВИГНЕРА



$$P = \sqrt{E^2 - M_R^2 + iM_R\Gamma} \equiv p - Q_R,$$

$$P^* = \sqrt{E^2 - M_R^2 + iM_R\Gamma + 2M_RV^*} \equiv p - Q_R^*$$

$$\Psi_{z_i}(z, E) = N(z_i) \left[\exp(iP^*(z - z_A))\theta(z_i < z < z_A) + \exp(iP(z - z_A))\theta(z_A < z) \right].$$

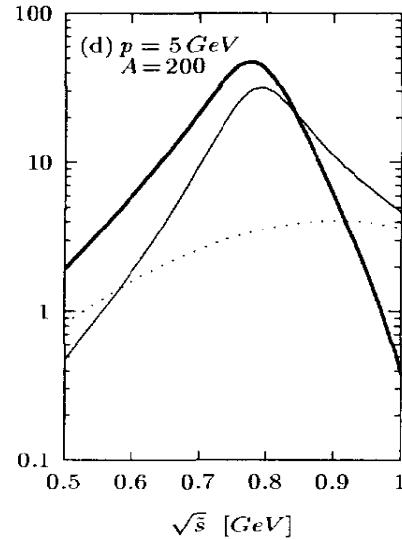
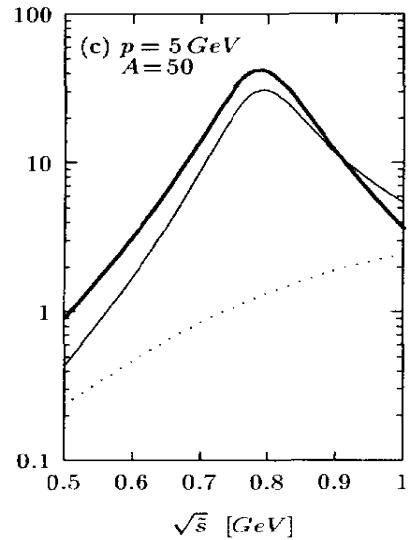
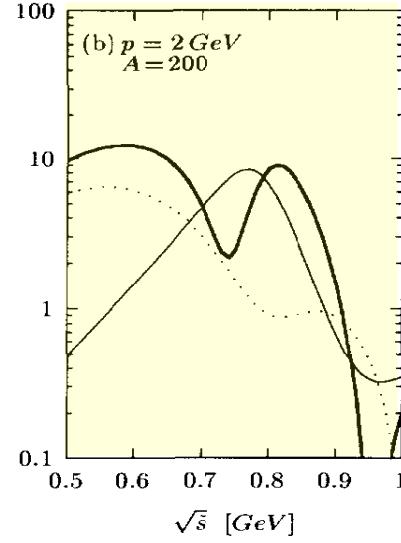
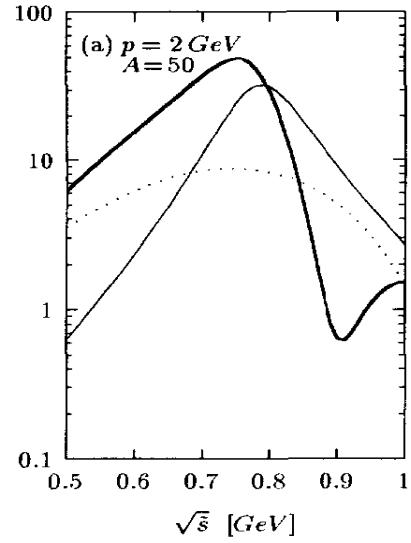
$$T(p, p') = \int_{-z_A}^{z_A} dz_i \int_{z_i}^{+\infty} dz_f e^{-ip'z_f} \Psi_{z_i}(z_f, E)$$

\leftarrow амплитуда

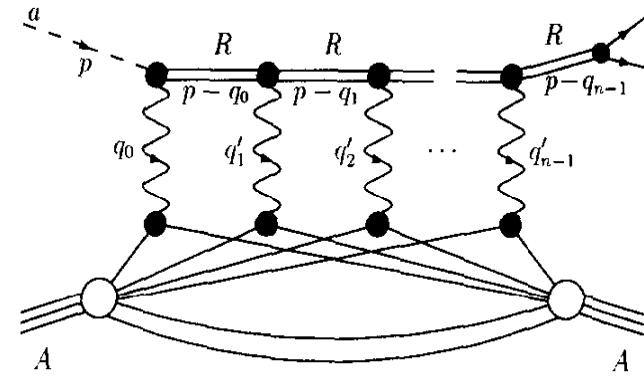
$$\propto \int_{-z_A}^{z_A} dz_i N(z_i) \left[\frac{1 - \exp(iD^*(z_A - z_i) / 2p)}{D^*} + \frac{\exp(iD(z_A - z_i) / 2p)}{D} \right].$$

↑ широкая компонента ↑ узкая компонента
 $D \quad \simeq \quad \tilde{s} - M_R^2 + iM_R\Gamma_R ,$
 $D^* \quad \simeq \quad \tilde{s} - M_R^{*2} + iM_R^*\Gamma_R^* .$ \leftarrow знаменатели

1D model \Leftarrow 3D model \Rightarrow diagrams for high energy



$$\gamma A \rightarrow \rho A \rightarrow (e^+ e^-) A$$

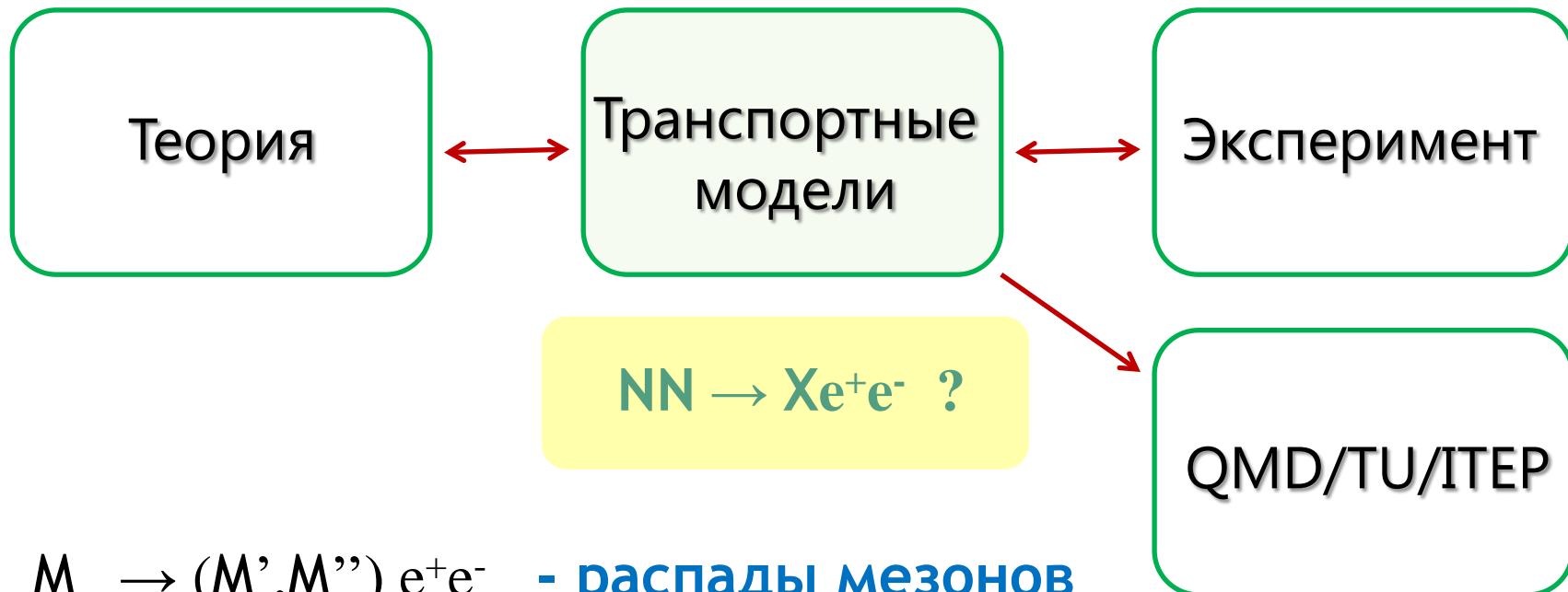


спектр $e^+ e^-$ пары
в распаде ρ -мезона

- 1. Столкновительная ширина,
описание данных Фраскати по фотопоглощению
на ядрах**
- 2. Двухкомпонентная формула Брейта-Вигнера**

Рождение e^+e^- пар в столкновениях тяжелых ионов

- ◆ A. Faessler, C. Fuchs and M. I. K., Phys. Rev. C 61, 035206 (2000);
- ◆ M. I. K. and A. Faessler, Phys. Rev. D 65, 017502 (2002);
- ◆ M. I. K., B. V. Martemyanov, A. Faessler and C. Fuchs, Annals Phys. 296, 299 (2002);
- ◆ A. Faessler, C. Fuchs, M. I. K. and B. V. Martemyanov, J. Phys. G 29, 603 (2003);
- ◆ C. Fuchs, M. I. K., H. L. Yadav, A. Faessler, B. V. Martemyanov and K. Shekhter, Phys. Rev. C 67, 025202 (2003);
- ◆ K. Shekhter, C. Fuchs, A. Faessler, M. I. K., B. Martemyanov, Phys. Rev. C 68, 014904 (2003);
- ◆ E. Santini, M. D. Cozma, A. Faessler, C. Fuchs, M. I. K., B. Martemyanov, Phys. Rev. C 78, 034910 (2008).



$M \rightarrow (M', M'') e^+e^-$ - распады мезонов

$R \rightarrow Ne^+e^-$ - распады нуклонных резонансов

$NN \rightarrow NR, R \rightarrow N(M, e^+e^-)$ - резонансная модель NRD

$M, M' = \rho, \omega, \pi, \eta, \dots$

сигнал фон

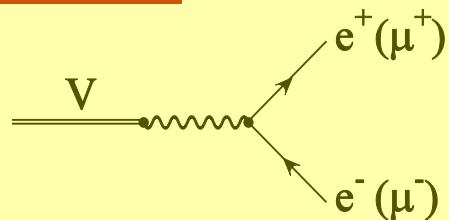
Дилептонные распады легких нестранных мезонов

$$M \rightarrow (M', M'') e^+ e^-$$

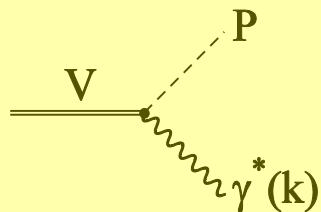
Вычислены γ и $e^+ e^-$ ширины всех легких нестранных мезонов с массой ниже $\phi(1020)$.

Некоторые примеры:

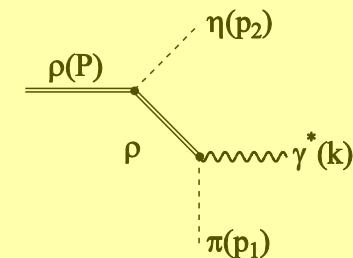
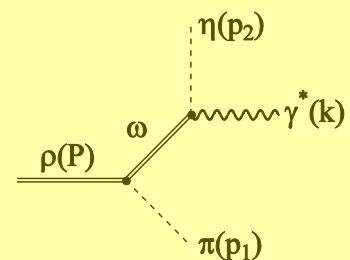
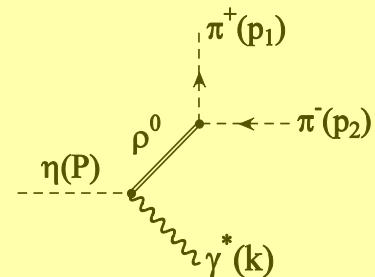
2-body



3-body



4-body



$$M \rightarrow (M', M'') e^+ e^-$$

TABLE III. The integral branchings ratios of the unflavored meson decays to electron-positron and muon-antimuon pairs. The experimental data are from Ref. [35].

Decay mode	$B_{e^+e^-}^{\text{theor}}$	$B_{e^+e^-}^{\text{expt}}$	$B_{\mu^+\mu^-}^{\text{theor}}$	$B_{\mu^+\mu^-}^{\text{expt}}$
$\rho^0 \rightarrow l^+l^-$	input	$(4.48 \pm 0.22) \times 10^{-5}$	4.5×10^{-5}	$(4.60 \pm 0.28) \times 10^{-5}$
$\rho \rightarrow \pi l^+l^-$	4.1×10^{-6}		4.6×10^{-7}	
$\rho^0 \rightarrow \eta l^+l^-$	2.7×10^{-6}		7.0×10^{-11}	
$\rho^\pm \rightarrow \pi^\pm \pi^0 l^+l^-$	5.4×10^{-5}		1.8×10^{-7}	
$\rho^0 \rightarrow \pi^+ \pi^- l^+l^-$	1.7×10^{-4}		6.7×10^{-7}	
$\rho^0 \rightarrow \pi^0 \pi^0 l^+l^-$	7.5×10^{-8}		2.4×10^{-9}	
$\rho \rightarrow \pi \eta l^+l^-$	1.9×10^{-12}			
$\omega \rightarrow l^+l^-$	input	$(7.15 \pm 0.19) \times 10^{-5}$	7.1×10^{-5}	$< 1.8 \times 10^{-4}$
$\omega \rightarrow \pi^0 l^+l^-$	7.9×10^{-4}	$(5.9 \pm 1.9) \times 10^{-4}$	9.2×10^{-5}	$(9.6 \pm 2.3) \times 10^{-5}$
$\omega \rightarrow \eta l^+l^-$	6.0×10^{-6}		1.8×10^{-9}	
$\omega \rightarrow \pi^+ \pi^- l^+l^-$	3.9×10^{-6}		2.9×10^{-8}	
$\omega \rightarrow \pi^0 \pi^0 l^+l^-$	2.0×10^{-7}		7.4×10^{-9}	
$\omega \rightarrow \pi^0 \eta l^+l^-$	8.7×10^{-10}			
$\phi \rightarrow l^+l^-$	input	$(3.00 \pm 0.06) \times 10^{-4}$	3.0×10^{-4}	$(2.48 \pm 0.34) \times 10^{-4}$
$\phi \rightarrow \pi^0 l^+l^-$	1.6×10^{-5}	$< 1.2 \times 10^{-4}$	4.8×10^{-6}	
$\phi \rightarrow \eta l^+l^-$	1.1×10^{-4}	$(1.3^{+0.8}_{-0.6}) \times 10^{-4}$	6.8×10^{-6}	
$\eta \rightarrow \gamma l^+l^-$	6.5×10^{-3}	$(4.9 \pm 1.1) \times 10^{-3}$	3.0×10^{-4}	$(3.1 \pm 0.4) \times 10^{-4}$
$\eta \rightarrow \pi^+ \pi^- l^+l^-$	3.6×10^{-4}	$(1.3^{+1.2}_{-0.8}) \times 10^{-3}$	1.2×10^{-8}	
$\eta' \rightarrow \gamma l^+l^-$	4.2×10^{-4}		8.1×10^{-5}	$(1.04 \pm 0.26) \times 10^{-4}$
$\eta' \rightarrow \omega l^+l^-$	2.0×10^{-4}			
$\eta' \rightarrow \pi^+ \pi^- l^+l^-$	1.8×10^{-3}		2.0×10^{-5}	
$f_0 \rightarrow \gamma l^+l^-$	2.2×10^{-7}		2.8×10^{-8}	
$f_0 \rightarrow \pi^+ \pi^- l^+l^-$	1.4×10^{-4}		4.1×10^{-7}	
$a_0^0 \rightarrow \gamma l^+l^-$	6.0×10^{-8}		7.4×10^{-9}	
$a_0 \rightarrow \pi \eta l^+l^-$	4.0×10^{-5}		1.4×10^{-9}	
$\pi^0 \rightarrow \gamma l^+l^-$	1.18×10^{-2}	$(1.198 \pm 0.032) \times 10^{-2}$		

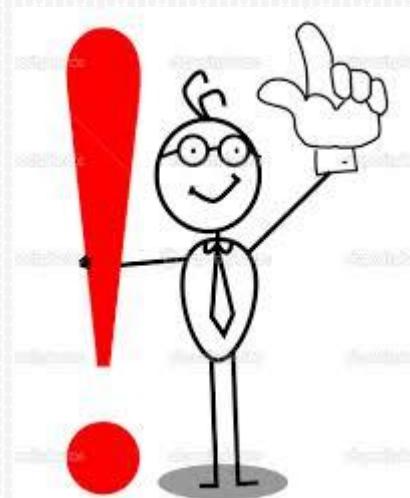
Ширины e^+e^- распадов

PDG/1998 - PDG/2013

TABLE III. The integral branchings ratios of the unflavored meson decays to electron-positron and muon-antimuon pairs. The experimental data are from Ref. [35].

Decay mode	$B_{e^+e^-}^{\text{theor}}$	$B_{e^+e^-}^{\text{expt}}$	$B_{\mu^+\mu^-}^{\text{theor}}$	$B_{\mu^+\mu^-}^{\text{expt}}$
$\rho^0 \rightarrow l^+l^-$	input	$(4.48 \pm 0.22) \times 10^{-5}$	4.5×10^{-5}	$(4.60 \pm 0.28) \times 10^{-5}$
$\rho \rightarrow \pi l^+l^-$	4.1×10^{-6}	$< 1.2 \times 10^{-5}$	4.6×10^{-7}	
$\rho^0 \rightarrow \eta l^+l^-$	2.7×10^{-6}		7.0×10^{-11}	
$\rho^\pm \rightarrow \pi^\pm \pi^0 l^+l^-$	5.4×10^{-5}		1.8×10^{-7}	
$\rho^0 \rightarrow \pi^+ \pi^- l^+l^-$	1.7×10^{-4}		6.7×10^{-7}	
$\rho^0 \rightarrow \pi^0 \pi^0 l^+l^-$	7.5×10^{-8}		2.4×10^{-9}	
$\rho \rightarrow \pi \eta l^+l^-$	1.9×10^{-12}			
$\omega \rightarrow l^+l^-$	input	$(7.15 \pm 0.19) \times 10^{-5}$	7.1×10^{-5}	$< 1.8 \times 10^{-4}$
$\omega \rightarrow \pi^0 l^+l^-$	7.9×10^{-4}	$(5.9 \pm 1.9) \times 10^{-4}$	9.2×10^{-5}	$(9.6 \pm 2.3) \times 10^{-5}$
$\omega \rightarrow \eta l^+l^-$	6.0×10^{-6}		1.8×10^{-9}	
$\omega \rightarrow \pi^+ \pi^- l^+l^-$	3.9×10^{-6}		2.9×10^{-8}	
$\omega \rightarrow \pi^0 \pi^0 l^+l^-$	2.0×10^{-7}		7.4×10^{-9}	
$\omega \rightarrow \pi^0 \eta l^+l^-$	8.7×10^{-10}			
$\phi \rightarrow l^+l^-$	input	$(3.00 \pm 0.06) \times 10^{-4}$	3.0×10^{-4}	$(2.48 \pm 0.34) \times 10^{-4}$
$\phi \rightarrow \pi^0 l^+l^-$	1.6×10^{-5}	$< 1.2 \times 10^{-4}$	4.8×10^{-6}	
$\phi \rightarrow \eta l^+l^-$	1.1×10^{-4}	$(1.3^{+0.8}_{-0.6}) \times 10^{-4}$	6.8×10^{-6}	
$\eta \rightarrow \gamma l^+l^-$	6.5×10^{-3}	$(4.9 \pm 1.1) \times 10^{-3}$	3.0×10^{-4}	$(3.1 \pm 0.4) \times 10^{-4}$
$\eta \rightarrow \pi^+ \pi^- l^+l^-$	3.6×10^{-4}	$(1.3^{+1.2}_{-0.8}) \times 10^{-3}$	1.2×10^{-8}	$< 3.6 \times 10^{-4}$
$\eta' \rightarrow \gamma l^+l^-$	4.2×10^{-4}	$< 9.0 \times 10^{-4}$	8.1×10^{-5}	$(1.04 \pm 0.26) \times 10^{-4}$
$\eta' \rightarrow \omega l^+l^-$	2.0×10^{-4}			
$\eta' \rightarrow \pi^+ \pi^- l^+l^-$	1.8×10^{-3}	$(2.4 \pm 1.3) \times 10^{-3}$	2.0×10^{-5}	$< 2.2 \times 10^{-4}$
$f_0 \rightarrow \gamma l^+l^-$	2.2×10^{-7}		2.8×10^{-8}	
$f_0 \rightarrow \pi^+ \pi^- l^+l^-$	1.4×10^{-4}		4.1×10^{-7}	
$a_0^0 \rightarrow \gamma l^+l^-$	6.0×10^{-8}		7.4×10^{-9}	
$a_0 \rightarrow \pi \eta l^+l^-$	4.0×10^{-5}		1.4×10^{-9}	
$\pi^0 \rightarrow \gamma l^+l^-$	1.18×10^{-2}	$(1.198 \pm 0.032) \times 10^{-2}$		

Ширины e^+e^- распадов

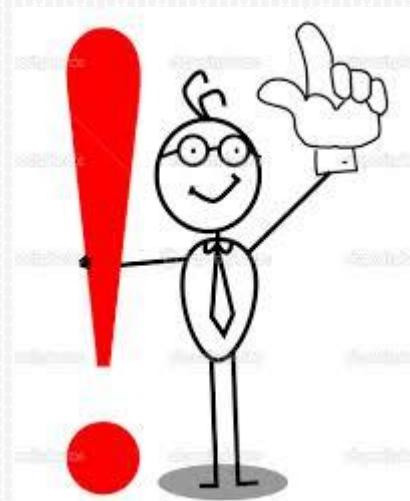


PDG/1998 - PDG/2013

TABLE III. The integral branchings ratios of the unflavored meson decays to electron-positron and muon-antimuon pairs. The experimental data are from Ref. [35].

Decay mode	$B_{e^+e^-}^{\text{theor}}$	$B_{e^+e^-}^{\text{expt}}$	$B_{\mu^+\mu^-}^{\text{theor}}$	$B_{\mu^+\mu^-}^{\text{expt}}$
$\rho^0 \rightarrow l^+l^-$	input	$(4.48 \pm 0.22) \times 10^{-5}$	4.5×10^{-5}	$(4.60 \pm 0.28) \times 10^{-5}$
$\rho \rightarrow \pi l^+l^-$	4.1×10^{-6}	$< 1.2 \times 10^{-5}$	4.6×10^{-7}	
$\rho^0 \rightarrow \eta l^+l^-$	2.7×10^{-6}		7.0×10^{-11}	
$\rho^\pm \rightarrow \pi^\pm \pi^0 l^+l^-$	5.4×10^{-5}		1.8×10^{-7}	
$\rho^0 \rightarrow \pi^+ \pi^- l^+l^-$	1.7×10^{-4}		6.7×10^{-7}	
$\rho^0 \rightarrow \pi^0 \pi^0 l^+l^-$	7.5×10^{-8}		2.4×10^{-9}	
$\rho \rightarrow \pi \eta l^+l^-$	1.9×10^{-12}			
$\omega \rightarrow l^+l^-$	input	$(7.15 \pm 0.19) \times 10^{-5}$	7.1×10^{-5}	$(9.0 \pm 3.1) \times 10^{-5}$
$\omega \rightarrow \pi^0 l^+l^-$	7.9×10^{-4}	$(5.9 \pm 1.9) \times 10^{-4}$	9.2×10^{-5}	$(9.6 \pm 2.3) \times 10^{-5}$
$\omega \rightarrow \eta l^+l^-$	6.0×10^{-6}		1.8×10^{-9}	
$\omega \rightarrow \pi^+ \pi^- l^+l^-$	3.9×10^{-6}		2.9×10^{-8}	
$\omega \rightarrow \pi^0 \pi^0 l^+l^-$	2.0×10^{-7}		7.4×10^{-9}	
$\omega \rightarrow \pi^0 \eta l^+l^-$	8.7×10^{-10}			
$\phi \rightarrow l^+l^-$	input	$(3.00 \pm 0.06) \times 10^{-4}$	3.0×10^{-4}	$(2.48 \pm 0.34) \times 10^{-4}$
$\phi \rightarrow \pi^0 l^+l^-$	1.6×10^{-5}	$< 1.2 \times 10^{-4}$	4.8×10^{-6}	
$\phi \rightarrow \eta l^+l^-$	1.1×10^{-4}	$(1.3^{+0.8}_{-0.6}) \times 10^{-4}$	6.8×10^{-6}	
$\eta \rightarrow \gamma l^+l^-$	6.5×10^{-3}	$(4.9 \pm 1.1) \times 10^{-3}$	3.0×10^{-4}	$(3.1 \pm 0.4) \times 10^{-4}$
$\eta \rightarrow \pi^+ \pi^- l^+l^-$	3.6×10^{-4}	$(1.3^{+1.2}_{-0.8}) \times 10^{-3}$	1.2×10^{-8}	$< 3.6 \times 10^{-4}$
$\eta' \rightarrow \gamma l^+l^-$	4.2×10^{-4}	$< 9.0 \times 10^{-4}$	8.1×10^{-5}	$(1.04 \pm 0.26) \times 10^{-4}$
$\eta' \rightarrow \omega l^+l^-$	2.0×10^{-4}			
$\eta' \rightarrow \pi^+ \pi^- l^+l^-$	1.8×10^{-3}	$(2.4 \pm 1.3) \times 10^{-3}$	2.0×10^{-5}	$< 2.2 \times 10^{-4}$
$f_0 \rightarrow \gamma l^+l^-$	2.2×10^{-7}		2.8×10^{-8}	
$f_0 \rightarrow \pi^+ \pi^- l^+l^-$	1.4×10^{-4}		4.1×10^{-7}	
$a_0^0 \rightarrow \gamma l^+l^-$	6.0×10^{-8}		7.4×10^{-9}	
$a_0 \rightarrow \pi \eta l^+l^-$	4.0×10^{-5}		1.4×10^{-9}	
$\pi^0 \rightarrow \gamma l^+l^-$	1.18×10^{-2}	$(1.198 \pm 0.032) \times 10^{-2}$		

Ширины e^+e^- распадов

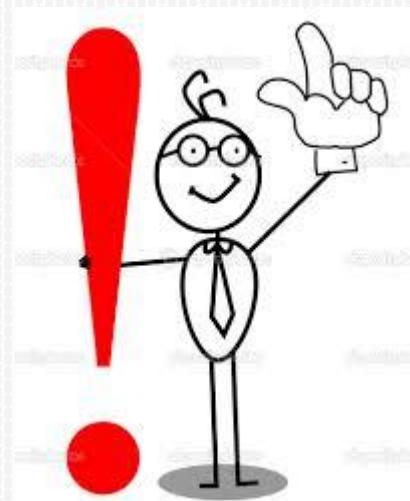


PDG/1998 - PDG/2013

TABLE III. The integral branchings ratios of the unflavored meson decays to electron-positron and muon-antimuon pairs. The experimental data are from Ref. [35].

Decay mode	$B_{e^+e^-}^{\text{theor}}$	$B_{e^+e^-}^{\text{expt}}$	$B_{\mu^+\mu^-}^{\text{theor}}$	$B_{\mu^+\mu^-}^{\text{expt}}$
$\rho^0 \rightarrow l^+l^-$	input	$(4.48 \pm 0.22) \times 10^{-5}$	4.5×10^{-5}	$(4.60 \pm 0.28) \times 10^{-5}$
$\rho \rightarrow \pi l^+l^-$	4.1×10^{-6}	$< 1.2 \times 10^{-5}$	4.6×10^{-7}	
$\rho^0 \rightarrow \eta l^+l^-$	2.7×10^{-6}		7.0×10^{-11}	
$\rho^\pm \rightarrow \pi^\pm \pi^0 l^+l^-$	5.4×10^{-5}		1.8×10^{-7}	
$\rho^0 \rightarrow \pi^+ \pi^- l^+l^-$	1.7×10^{-4}		6.7×10^{-7}	
$\rho^0 \rightarrow \pi^0 \pi^0 l^+l^-$	7.5×10^{-8}		2.4×10^{-9}	
$\rho \rightarrow \pi \eta l^+l^-$	1.9×10^{-12}			
$\omega \rightarrow l^+l^-$	input	$(7.15 \pm 0.19) \times 10^{-5}$	7.1×10^{-5}	$(9.0 \pm 3.1) \times 10^{-5}$
$\omega \rightarrow \pi^0 l^+l^-$	7.9×10^{-4}	$(7.7 \pm 0.6) \times 10^{-4}$	9.2×10^{-5}	$(1.3 \pm 0.4) \times 10^{-4}$
$\omega \rightarrow \eta l^+l^-$	6.0×10^{-6}		1.8×10^{-9}	
$\omega \rightarrow \pi^+ \pi^- l^+l^-$	3.9×10^{-6}		2.9×10^{-8}	
$\omega \rightarrow \pi^0 \pi^0 l^+l^-$	2.0×10^{-7}		7.4×10^{-9}	
$\omega \rightarrow \pi^0 \eta l^+l^-$	8.7×10^{-10}			
$\phi \rightarrow l^+l^-$	input	$(3.00 \pm 0.06) \times 10^{-4}$	3.0×10^{-4}	$(2.48 \pm 0.34) \times 10^{-4}$
$\phi \rightarrow \pi^0 l^+l^-$	1.6×10^{-5}	$< 1.2 \times 10^{-4}$	4.8×10^{-6}	
$\phi \rightarrow \eta l^+l^-$	1.1×10^{-4}	$(1.3^{+0.8}_{-0.6}) \times 10^{-4}$	6.8×10^{-6}	
$\eta \rightarrow \gamma l^+l^-$	6.5×10^{-3}	$(4.9 \pm 1.1) \times 10^{-3}$	3.0×10^{-4}	$(3.1 \pm 0.4) \times 10^{-4}$
$\eta \rightarrow \pi^+ \pi^- l^+l^-$	3.6×10^{-4}	$(1.3^{+1.2}_{-0.8}) \times 10^{-3}$	1.2×10^{-8}	$< 3.6 \times 10^{-4}$
$\eta' \rightarrow \gamma l^+l^-$	4.2×10^{-4}	$< 9.0 \times 10^{-4}$	8.1×10^{-5}	$(1.04 \pm 0.26) \times 10^{-4}$
$\eta' \rightarrow \omega l^+l^-$	2.0×10^{-4}			
$\eta' \rightarrow \pi^+ \pi^- l^+l^-$	1.8×10^{-3}	$(2.4 \pm 1.3) \times 10^{-3}$	2.0×10^{-5}	$< 2.2 \times 10^{-4}$
$f_0 \rightarrow \gamma l^+l^-$	2.2×10^{-7}		2.8×10^{-8}	
$f_0 \rightarrow \pi^+ \pi^- l^+l^-$	1.4×10^{-4}		4.1×10^{-7}	
$a_0^0 \rightarrow \gamma l^+l^-$	6.0×10^{-8}		7.4×10^{-9}	
$a_0 \rightarrow \pi \eta l^+l^-$	4.0×10^{-5}		1.4×10^{-9}	
$\pi^0 \rightarrow \gamma l^+l^-$	1.18×10^{-2}	$(1.198 \pm 0.032) \times 10^{-2}$		

Ширины e^+e^- распадов

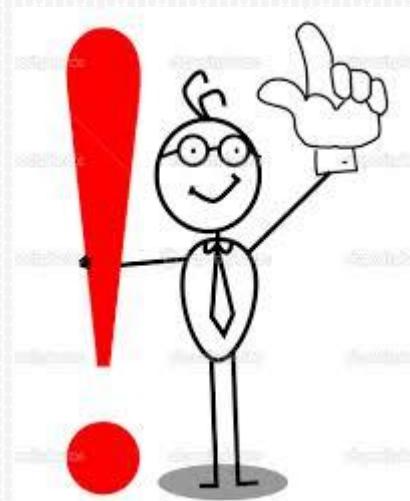


PDG/1998 - PDG/2013

TABLE III. The integral branchings ratios of the unflavored meson decays to electron-positron and muon-antimuon pairs. The experimental data are from Ref. [35].

Decay mode	$B_{e^+e^-}^{\text{theor}}$	$B_{e^+e^-}^{\text{expt}}$	$B_{\mu^+\mu^-}^{\text{theor}}$	$B_{\mu^+\mu^-}^{\text{expt}}$
$\rho^0 \rightarrow l^+l^-$	input	$(4.48 \pm 0.22) \times 10^{-5}$	4.5×10^{-5}	$(4.60 \pm 0.28) \times 10^{-5}$
$\rho \rightarrow \pi l^+l^-$	4.1×10^{-6}	$< 1.2 \times 10^{-5}$	4.6×10^{-7}	
$\rho^0 \rightarrow \eta l^+l^-$	2.7×10^{-6}		7.0×10^{-11}	
$\rho^\pm \rightarrow \pi^\pm \pi^0 l^+l^-$	5.4×10^{-5}		1.8×10^{-7}	
$\rho^0 \rightarrow \pi^+ \pi^- l^+l^-$	1.7×10^{-4}		6.7×10^{-7}	
$\rho^0 \rightarrow \pi^0 \pi^0 l^+l^-$	7.5×10^{-8}		2.4×10^{-9}	
$\rho \rightarrow \pi \eta l^+l^-$	1.9×10^{-12}			
$\omega \rightarrow l^+l^-$	input	$(7.15 \pm 0.19) \times 10^{-5}$	7.1×10^{-5}	$(9.0 \pm 3.1) \times 10^{-5}$
$\omega \rightarrow \pi^0 l^+l^-$	7.9×10^{-4}	$(7.7 \pm 0.6) \times 10^{-4}$	9.2×10^{-5}	$(1.3 \pm 0.4) \times 10^{-4}$
$\omega \rightarrow \eta l^+l^-$	6.0×10^{-6}		1.8×10^{-9}	
$\omega \rightarrow \pi^+ \pi^- l^+l^-$	3.9×10^{-6}		2.9×10^{-8}	
$\omega \rightarrow \pi^0 \pi^0 l^+l^-$	2.0×10^{-7}		7.4×10^{-9}	
$\omega \rightarrow \pi^0 \eta l^+l^-$	8.7×10^{-10}			
$\phi \rightarrow l^+l^-$	input	$(3.00 \pm 0.06) \times 10^{-4}$	3.0×10^{-4}	$(2.48 \pm 0.34) \times 10^{-4}$
$\phi \rightarrow \pi^0 l^+l^-$	1.6×10^{-5}	$< 1.2 \times 10^{-4}$	4.8×10^{-6}	
$\phi \rightarrow \eta l^+l^-$	1.1×10^{-4}	$(1.3^{+0.8}_{-0.6}) \times 10^{-4}$	6.8×10^{-6}	
$\eta \rightarrow \gamma l^+l^-$	6.5×10^{-3}	$(6.9 \pm 0.4) \times 10^{-3}$	3.0×10^{-4}	$(3.1 \pm 0.4) \times 10^{-4}$
$\eta \rightarrow \pi^+ \pi^- l^+l^-$	3.6×10^{-4}	$(1.3^{+1.2}_{-0.8}) \times 10^{-3}$	1.2×10^{-8}	$< 3.6 \times 10^{-4}$
$\eta' \rightarrow \gamma l^+l^-$	4.2×10^{-4}	$< 9.0 \times 10^{-4}$	8.1×10^{-5}	$(1.04 \pm 0.26) \times 10^{-4}$
$\eta' \rightarrow \omega l^+l^-$	2.0×10^{-4}			
$\eta' \rightarrow \pi^+ \pi^- l^+l^-$	1.8×10^{-3}	$(2.4 \pm 1.3) \times 10^{-3}$	2.0×10^{-5}	$< 2.2 \times 10^{-4}$
$f_0 \rightarrow \gamma l^+l^-$	2.2×10^{-7}		2.8×10^{-8}	
$f_0 \rightarrow \pi^+ \pi^- l^+l^-$	1.4×10^{-4}		4.1×10^{-7}	
$a_0^0 \rightarrow \gamma l^+l^-$	6.0×10^{-8}		7.4×10^{-9}	
$a_0 \rightarrow \pi \eta l^+l^-$	4.0×10^{-5}		1.4×10^{-9}	
$\pi^0 \rightarrow \gamma l^+l^-$	1.18×10^{-2}	$(1.198 \pm 0.032) \times 10^{-2}$		

Ширины e^+e^- распадов

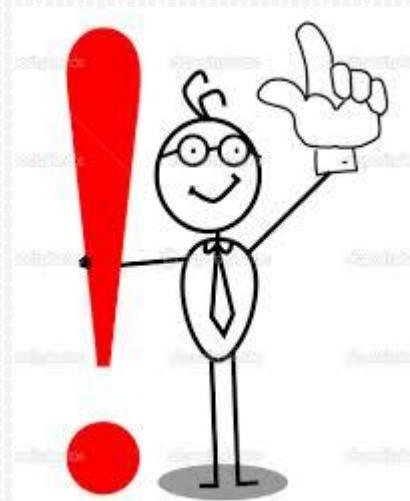


PDG/1998 - PDG/2013

TABLE III. The integral branchings ratios of the unflavored meson decays to electron-positron and muon-antimuon pairs. The experimental data are from Ref. [35].

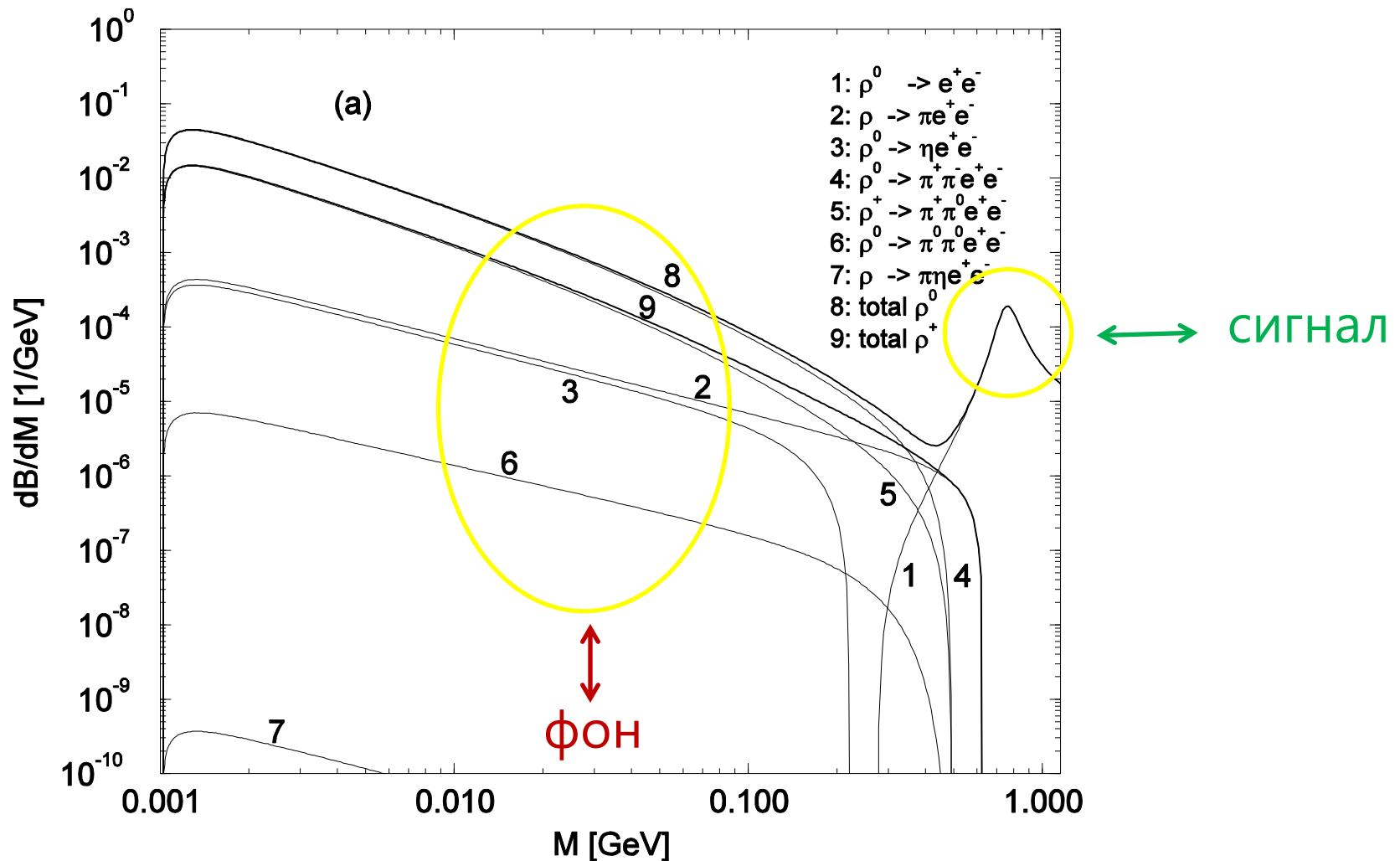
Decay mode	$B_{e^+e^-}^{\text{theor}}$	$B_{e^+e^-}^{\text{expt}}$	$B_{\mu^+\mu^-}^{\text{theor}}$	$B_{\mu^+\mu^-}^{\text{expt}}$
$\rho^0 \rightarrow l^+l^-$	input	$(4.48 \pm 0.22) \times 10^{-5}$	4.5×10^{-5}	$(4.60 \pm 0.28) \times 10^{-5}$
$\rho \rightarrow \pi l^+l^-$	4.1×10^{-6}	$< 1.2 \times 10^{-5}$	4.6×10^{-7}	
$\rho^0 \rightarrow \eta l^+l^-$	2.7×10^{-6}		7.0×10^{-11}	
$\rho^\pm \rightarrow \pi^\pm \pi^0 l^+l^-$	5.4×10^{-5}		1.8×10^{-7}	
$\rho^0 \rightarrow \pi^+ \pi^- l^+l^-$	1.7×10^{-4}		6.7×10^{-7}	
$\rho^0 \rightarrow \pi^0 \pi^0 l^+l^-$	7.5×10^{-8}		2.4×10^{-9}	
$\rho \rightarrow \pi \eta l^+l^-$	1.9×10^{-12}			
$\omega \rightarrow l^+l^-$	input	$(7.15 \pm 0.19) \times 10^{-5}$	7.1×10^{-5}	$(9.0 \pm 3.1) \times 10^{-5}$
$\omega \rightarrow \pi^0 l^+l^-$	7.9×10^{-4}	$(7.7 \pm 0.6) \times 10^{-4}$	9.2×10^{-5}	$(1.3 \pm 0.4) \times 10^{-4}$
$\omega \rightarrow \eta l^+l^-$	6.0×10^{-6}		1.8×10^{-9}	
$\omega \rightarrow \pi^+ \pi^- l^+l^-$	3.9×10^{-6}		2.9×10^{-8}	
$\omega \rightarrow \pi^0 \pi^0 l^+l^-$	2.0×10^{-7}		7.4×10^{-9}	
$\omega \rightarrow \pi^0 \eta l^+l^-$	8.7×10^{-10}			
$\phi \rightarrow l^+l^-$	input	$(3.00 \pm 0.06) \times 10^{-4}$	3.0×10^{-4}	$(2.48 \pm 0.34) \times 10^{-4}$
$\phi \rightarrow \pi^0 l^+l^-$	1.6×10^{-5}	$< 1.2 \times 10^{-4}$	4.8×10^{-6}	
$\phi \rightarrow \eta l^+l^-$	1.1×10^{-4}	$(1.3^{+0.8}_{-0.6}) \times 10^{-4}$	6.8×10^{-6}	
$\eta \rightarrow \gamma l^+l^-$	6.5×10^{-3}	$(6.9 \pm 0.4) \times 10^{-3}$	3.0×10^{-4}	$(3.1 \pm 0.4) \times 10^{-4}$
$\eta \rightarrow \pi^+ \pi^- l^+l^-$	3.6×10^{-4}	$(2.68 \pm 0.11) \times 10^{-4}$	1.2×10^{-8}	$< 3.6 \times 10^{-4}$
$\eta' \rightarrow \gamma l^+l^-$	4.2×10^{-4}	$< 9.0 \times 10^{-4}$	8.1×10^{-5}	$(1.04 \pm 0.26) \times 10^{-4}$
$\eta' \rightarrow \omega l^+l^-$	2.0×10^{-4}			
$\eta' \rightarrow \pi^+ \pi^- l^+l^-$	1.8×10^{-3}	$(2.4 \pm 1.3) \times 10^{-3}$	2.0×10^{-5}	$< 2.2 \times 10^{-4}$
$f_0 \rightarrow \gamma l^+l^-$	2.2×10^{-7}		2.8×10^{-8}	
$f_0 \rightarrow \pi^+ \pi^- l^+l^-$	1.4×10^{-4}		4.1×10^{-7}	
$a_0^0 \rightarrow \gamma l^+l^-$	6.0×10^{-8}		7.4×10^{-9}	
$a_0 \rightarrow \pi \eta l^+l^-$	4.0×10^{-5}		1.4×10^{-9}	
$\pi^0 \rightarrow \gamma l^+l^-$	1.18×10^{-2}	$(1.198 \pm 0.032) \times 10^{-2}$		

Ширины e^+e^- распадов



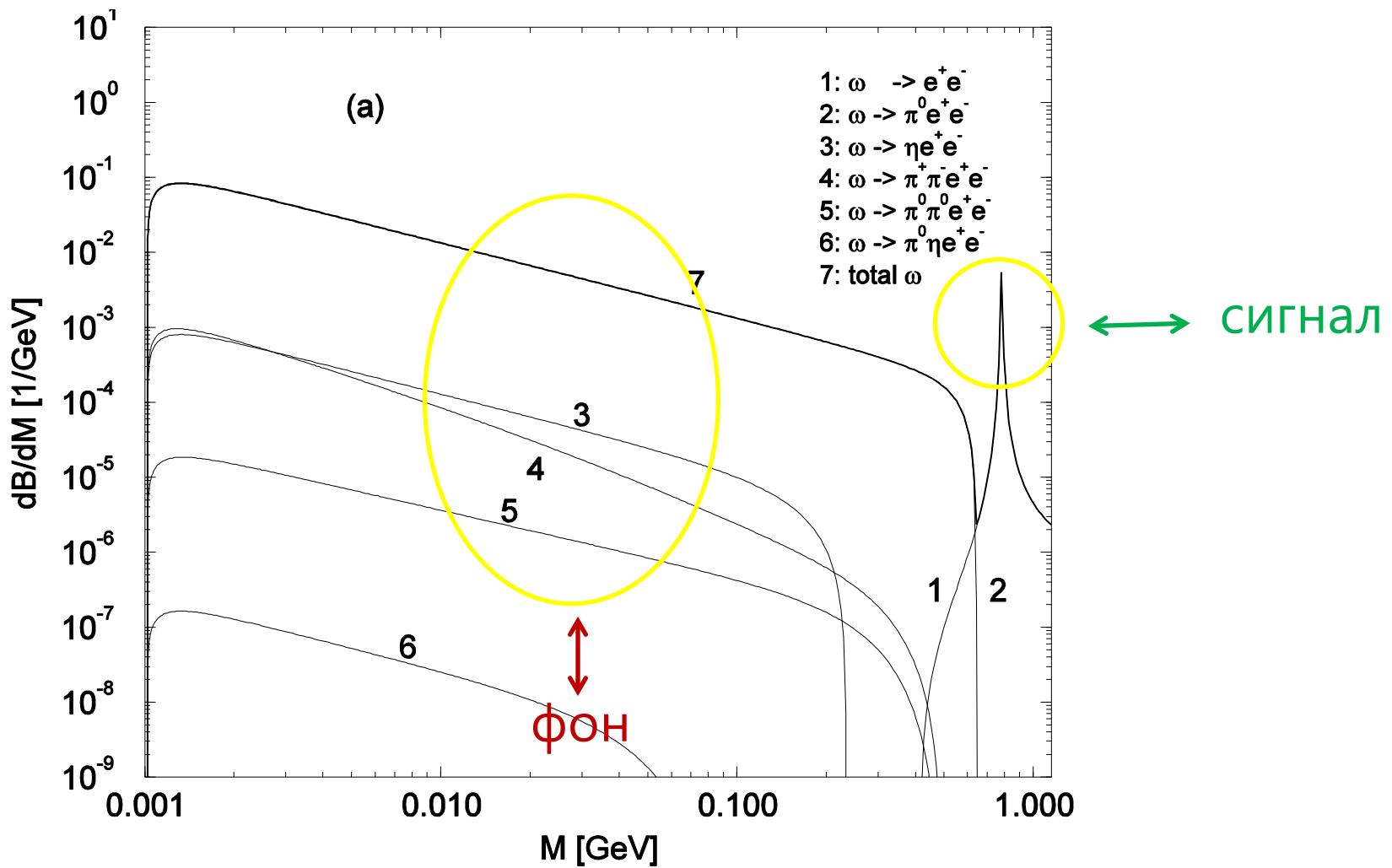
PDG/1998 - PDG/2013

Спектр дилептонов (ρ -мезон)



$M \rightarrow (M', M'') e^+ e^-$

Спектр дилептонов (ω -мезон)



$$M \rightarrow (M', M'') e^+ e^-$$

Дилептонные распады нуклонных резонансов

$$N^* \rightarrow N e^+ e^-$$

Распады $R \rightarrow N e^+ e^-$, $R = N^*, \Delta^*$, произвольная спин-четность ($J = l+1/2$):

$$\Gamma(N_{(\pm)}^* \rightarrow N \gamma^*) = \frac{9\alpha}{16} \frac{(l!)^2}{2^l (2l+1)!} \frac{m_\pm^2 (m_\mp^2 - M^2)^{l+1/2} (m_\pm^2 - M^2)^{l-1/2}}{m_*^{2l+1} m^2} \\ \left(\frac{l+1}{l} \left| G_{M/E}^{(\pm)} \right|^2 + (l+1)(l+2) \left| G_{E/M}^{(\pm)} \right|^2 + \frac{M^2}{m_*^2} \left| G_C^{(\pm)} \right|^2 \right).$$

$$d\Gamma(N^* \rightarrow Ne^+ e^-) = \Gamma(N^* \rightarrow N \gamma^*) M \Gamma(\gamma^* \rightarrow e^+ e^-) \frac{dM^2}{\pi M^4},$$

$$M \Gamma(\gamma^* \rightarrow e^+ e^-) = \frac{\alpha}{3} (M^2 + 2m_e^2) \sqrt{1 - \frac{4m_e^2}{M^2}}$$

eVMD модель

Правила кваркового счета:

$$F_1^{(\pm)}(M^2) = \frac{\sum_{j=0}^{n+1} C_{1j}^{(\pm)} M^{2j}}{\prod_{i=1}^{l+3+n} (1 - M^2/m_i^2)},$$

$$F_2^{(\pm)}(M^2) = \frac{\sum_{j=0}^n C_{2j}^{(\pm)} M^{2j}}{\prod_{i=1}^{l+3+n} (1 - M^2/m_i^2)},$$

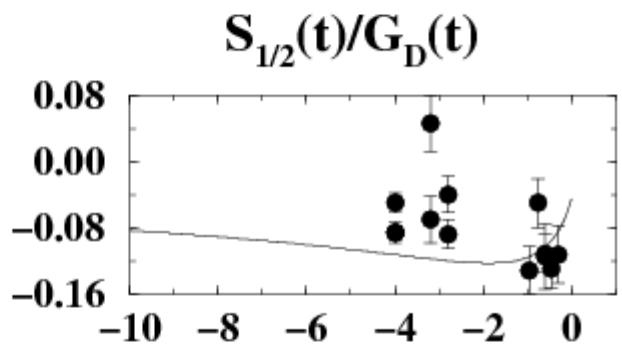
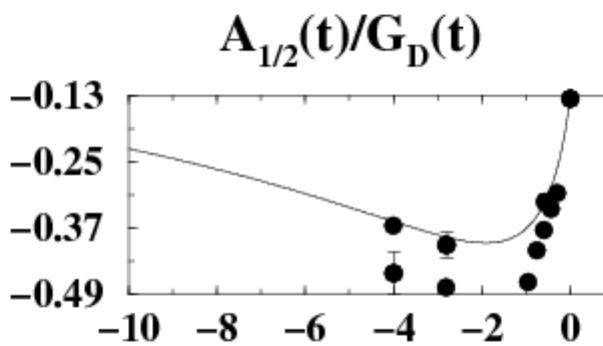
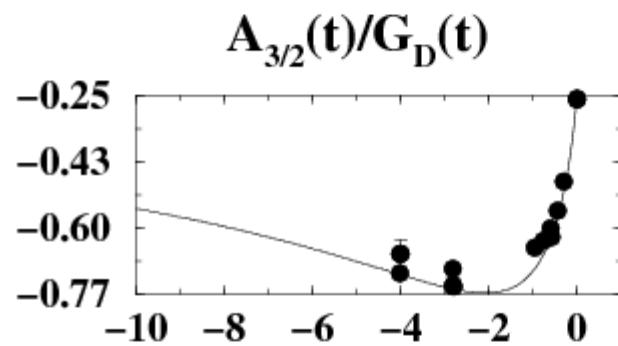
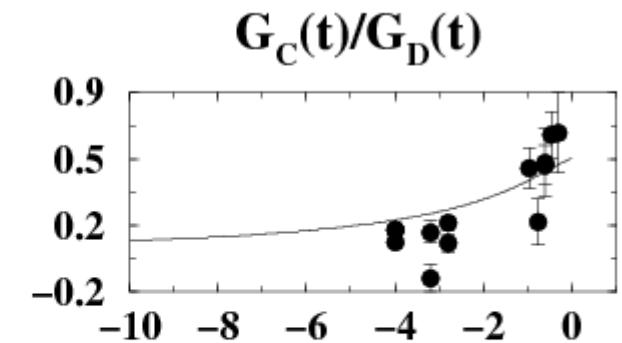
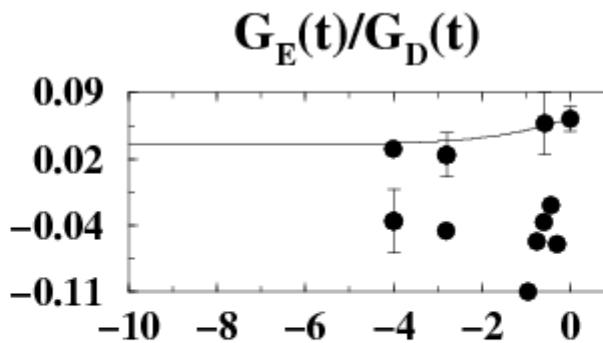
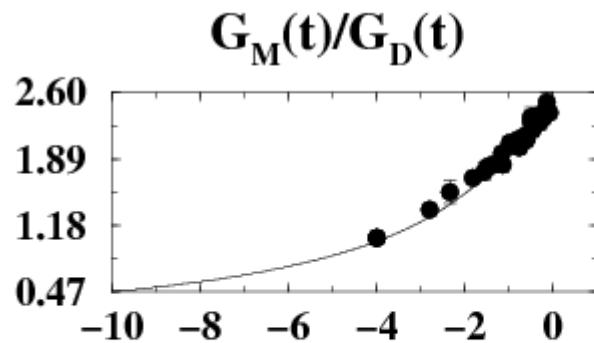
$$F_3^{(\pm)}(M^2) = \frac{\sum_{j=0}^n C_{3j}^{(\pm)} M^{2j}}{\prod_{i=1}^{l+3+n} (1 - M^2/m_i^2)}.$$

$$\left. \begin{array}{l} F_1^{(\pm)}(M^2) = O\left(\frac{1}{(-M^2)^{l+2}}\right), \\ F_2^{(\pm)}(M^2) = O\left(\frac{1}{(-M^2)^{l+3}}\right), \\ F_3^{(\pm)}(M^2) = O\left(\frac{1}{(-M^2)^{l+3}}\right). \end{array} \right\}$$

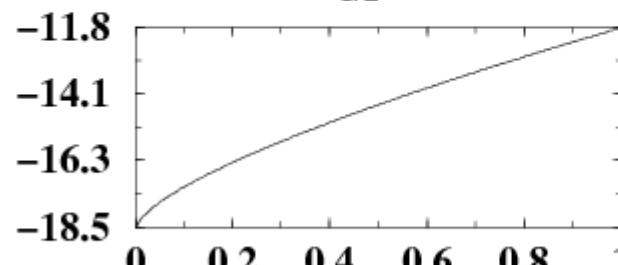
Форм-факторы в eVMD

$$G_D(t) = (1 - t/0.81)^{-2}.$$

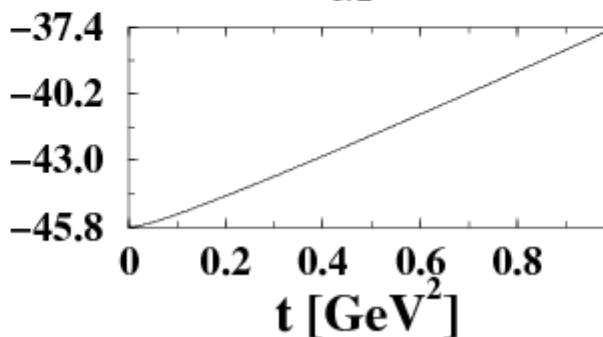
eVMD модель для $\Delta(1232)$



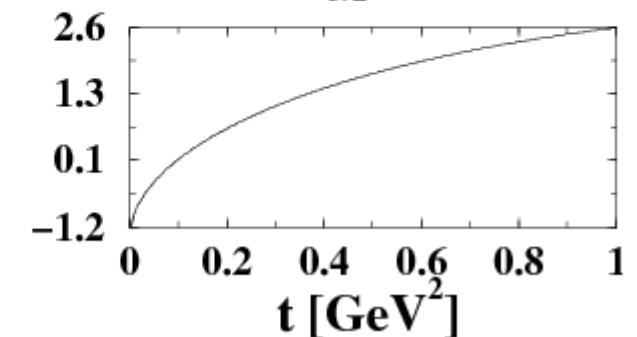
$p_{1/2}^\rho(t)$

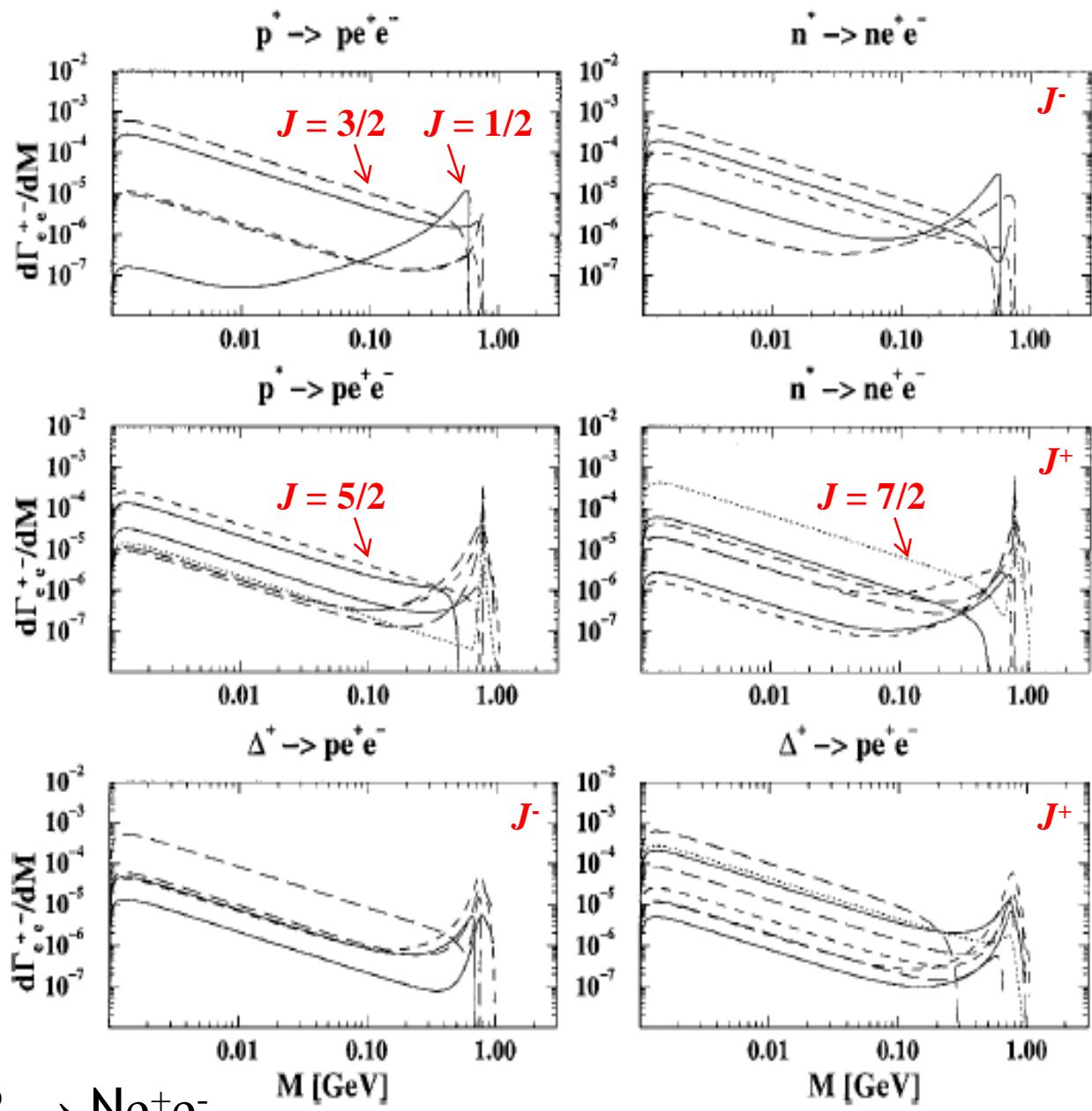


$p_{3/2}^\rho(t)$



$f_{3/2}^\rho(t)$





e^+e^- спектры $R \rightarrow N e^+e^-$
R с массой < 2 ГэВ

$R \rightarrow N e^+e^-$

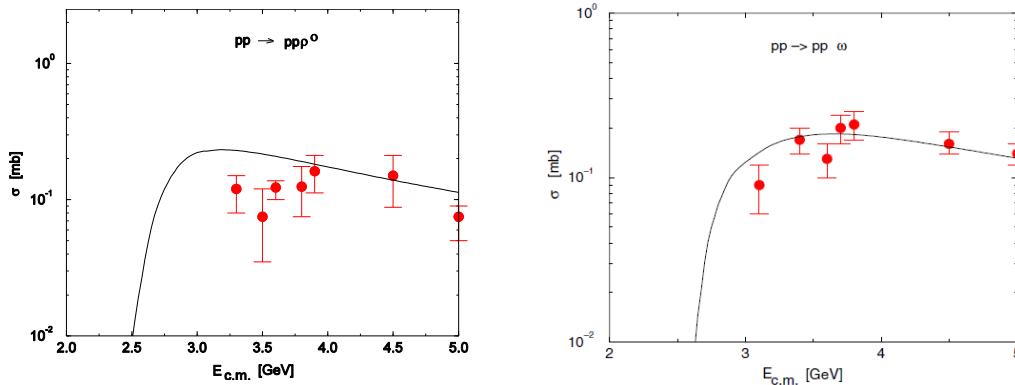
Рождение e^+e^- в pp соударениях. NRD model.

Сравнение с данными DLS (Bevalac)

Прямой канал $V = \rho^0, \omega, \phi$

$$\frac{d\sigma(s, M)^{pp \rightarrow e^+ e^- X}}{dM^2} = \sum_V \frac{d\sigma(s, M)^{pp \rightarrow V X}}{dM^2} B(M) \circled{V \rightarrow e^+ e^-}.$$

Нормировка на эксклюзивные сечения ρ и ω ($pp \rightarrow pN^*$, $N^* \rightarrow NV$)



Фон:

$$\frac{d\sigma(s, M)^{e^+ e^- X}_B}{dM^2} \Big|_{X \neq NN} = \sum_M \int d\mu^2 \frac{d\sigma(s, \mu)^{MX'}}{d\mu^2} \frac{dB(\mu, M)^{M \rightarrow e^+ e^- X''}}{dM^2} ..$$

NN → NR, R → N(M, e⁺e⁻)

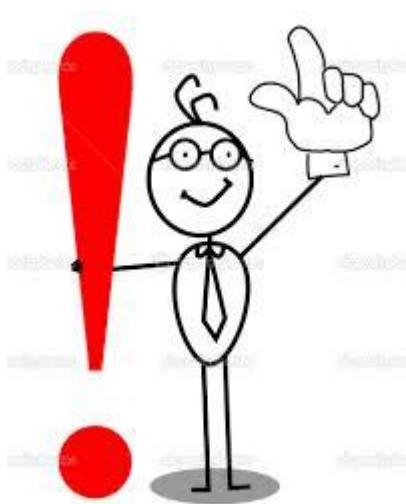
Сумма берется по мезонам

M = π, η, ρ, ω, φ

Константы связи $R N\rho$ из $R \rightarrow N\rho$ & $R \rightarrow N\gamma$ в VMD

R	N_{1440}	N_{1520}	N_{1535}	N_{1650}	N_{1680}	N_{1720}	Δ_{1232}	Δ_{1620}	Δ_{1700}	Δ_{1905}
J^P	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{5}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^+$
$f_{RN\rho}$	<26	7.0	<2.0	0.9	6.3	7.8	15.3	2.5	5.0	12.2
$f_{RN\rho}^\gamma$	1.3	3.8	1.8	<0.8	3.9	2.2	10.8	0.7	2.7	2.1

eVMD форм-фактор & правила кваркового счета

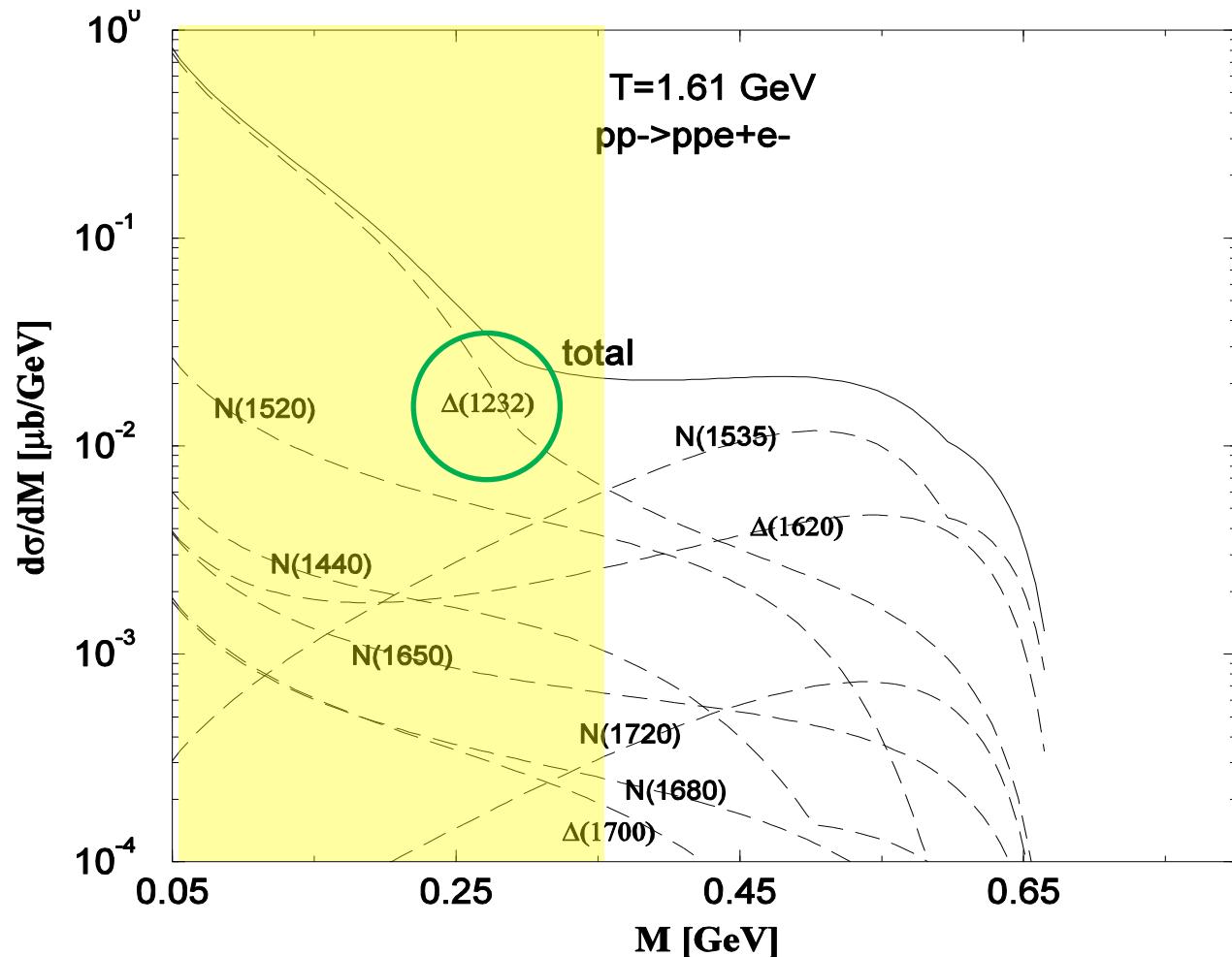


$$M(M^2) = \frac{f_{RN\rho}}{m_\rho} \frac{m_\rho^2}{g_\rho} \frac{1}{\tilde{m}_\rho^2 - M^2} \left(\frac{\tilde{m}_{\rho'}^2 - \tilde{m}_\rho^2}{\tilde{m}_{\rho'}^2 - M^2} \right) \left(\frac{\tilde{m}_{\rho''}^2 - \tilde{m}_\rho^2}{\tilde{m}_{\rho''}^2 - M^2} \right).$$

Поправочный
фактор:

$$F_\rho(M^2) = \left(\frac{\tilde{m}_{\rho'}^2 - \tilde{m}_\rho^2}{\tilde{m}_{\rho'}^2 - M^2} \right) \left(\frac{\tilde{m}_{\rho''}^2 - \tilde{m}_\rho^2}{\tilde{m}_{\rho''}^2 - M^2} \right)^2.$$

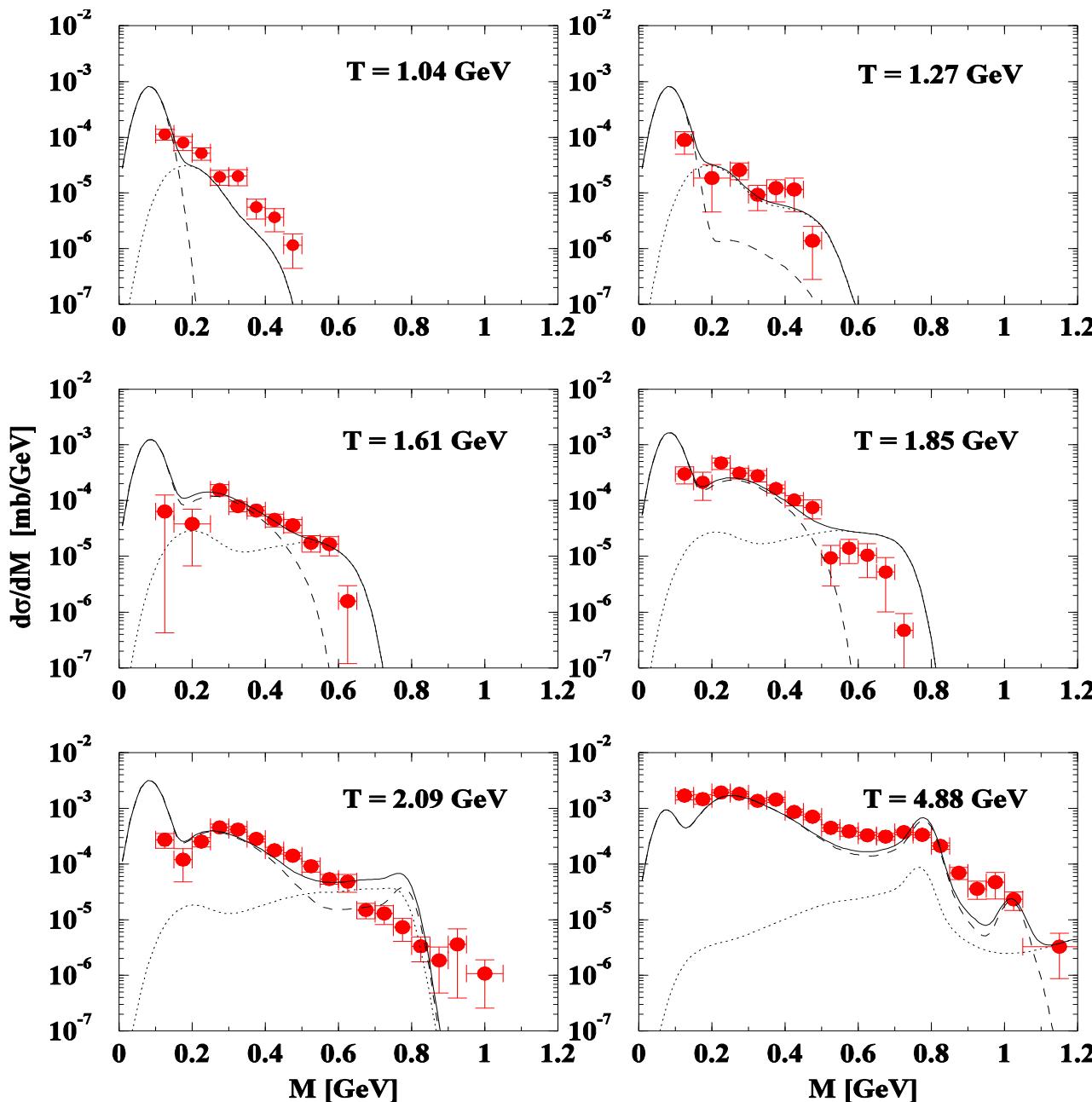
$$F_\rho(M^2 = m_\rho^2) = 1 \quad \& \quad F_\rho(M^2 = 0) = 0.56$$



Сечение рождения дилептонов в реакции $pp \rightarrow e^+e^-pp$
в схеме $pp \rightarrow Rp, R \rightarrow e^+e^-p$ при $T = 1.61 \text{ ГэВ}.$

Показаны вклады 10-и нуклонных резонансов.

$$NN \rightarrow NR, R \rightarrow N(M, e^+e^-)$$



Сечение
 $p p \rightarrow e^+ e^- X$ vs. $M_{e^+ e^-}$

в сравнении с данными DLS

$NN \rightarrow NR, R \rightarrow N(M, e^+ e^-)$

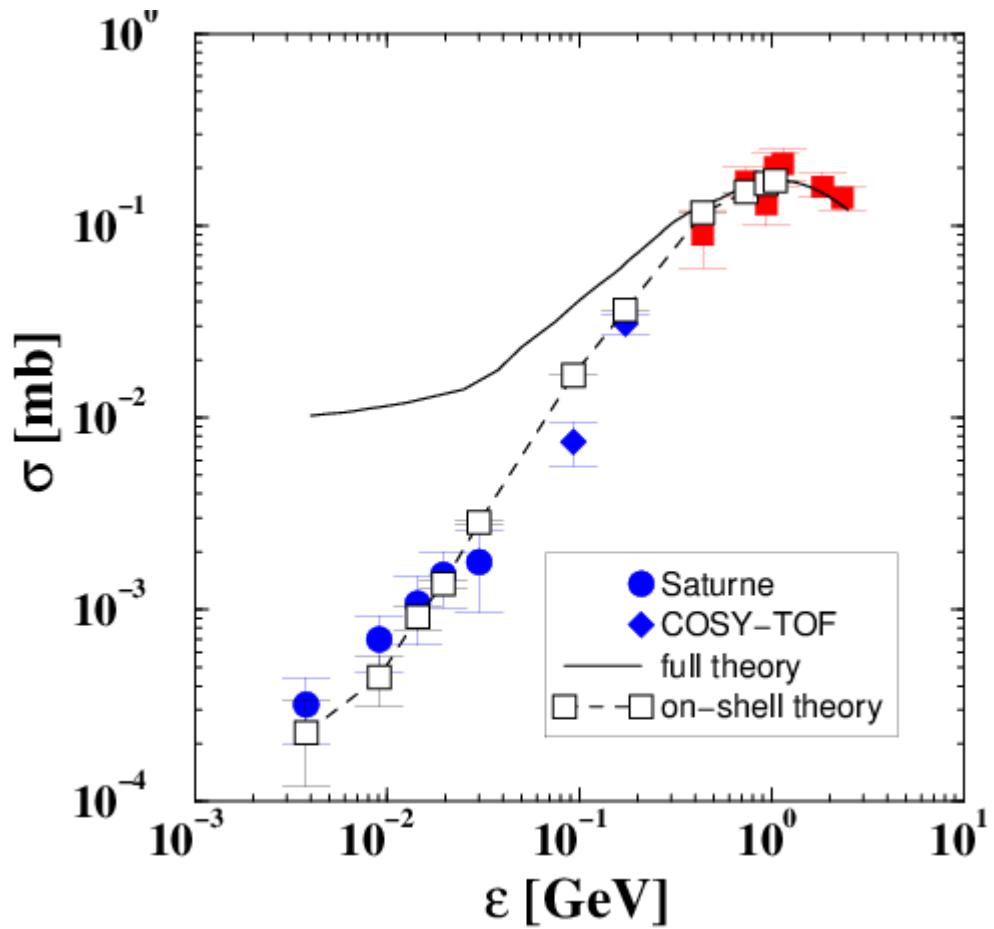
Рождение ω -мезона в NN столкновениях вблизи порога

сравнение с данными **Saturne & COSY-TOF**

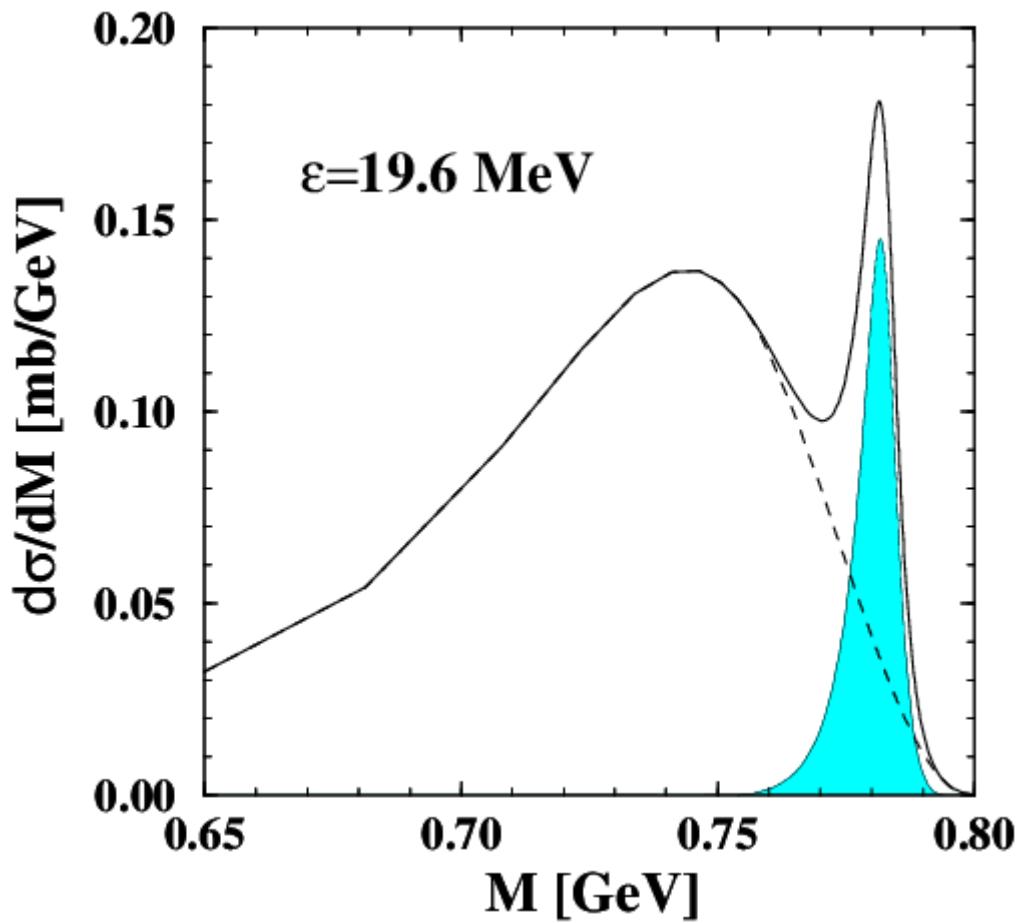
$$\frac{d\sigma(s, M)^{pp \rightarrow pp\omega}}{dM^2} = \sum_R \int_{(m_p + M)^2}^{(\sqrt{s} - m_p)^2} d\mu^2 \frac{d\sigma(s, \mu)^{pp \rightarrow pR}}{d\mu^2} \frac{dB(\mu, M)^{R \rightarrow p\omega}}{dM^2}.$$

$$d\sigma(s, \mu)^{pp \rightarrow pR} = \frac{|M_R|^2}{16 p_i \sqrt{s} \pi^2} \Phi_2(\sqrt{s}, \mu, m_p) dW_R(\mu)$$

$$dW_R(\mu) = \frac{1}{\pi} \frac{\mu \Gamma_R(\mu) d\mu^2}{(\mu^2 - m_R^2)^2 + (\mu \Gamma_R(\mu))^2}.$$



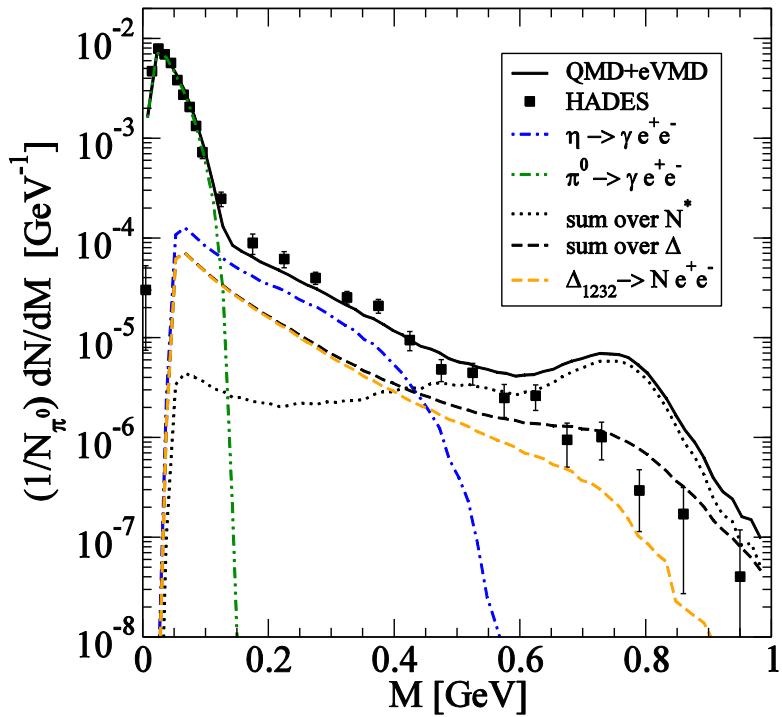
**Эксклюзивное сечение в
резонансной модели
vs. эксперимент;
 ε – энергия над порогом.**



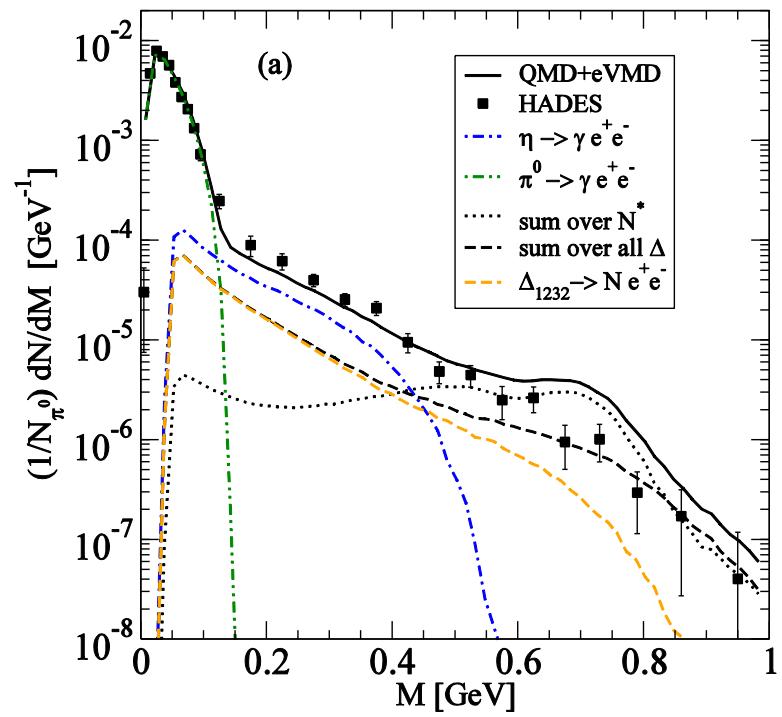
**Теоретический фон, обусловленный
подпороговым рождением ω -мезона,
вычитается из полного сечения.**

$$NN \rightarrow NR, R \rightarrow N(M, e^+e^-)$$

Дилептонный спектр в реакции C + C при энергии 2 АГэВ в сравнении с данными HADES.

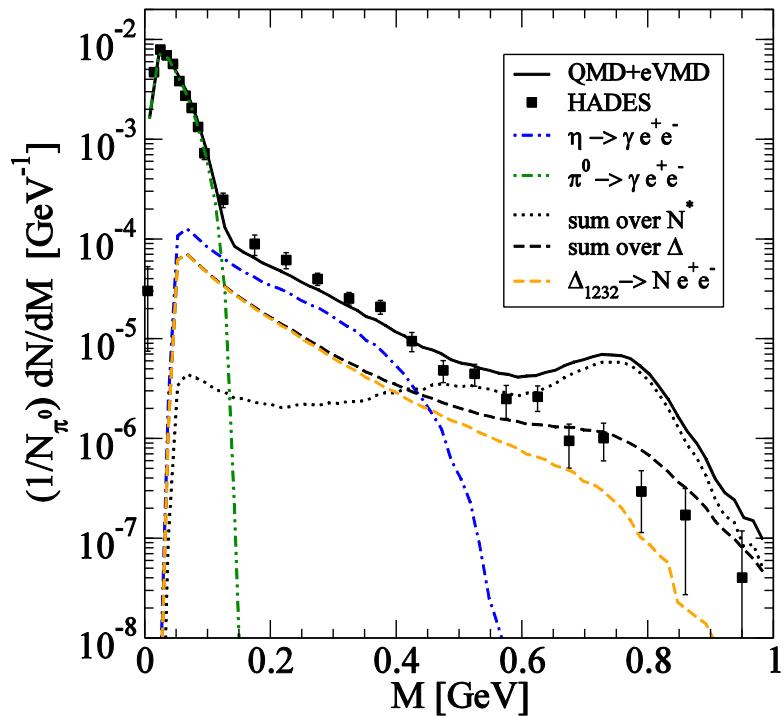


вакуумная версия NRD + eVMD

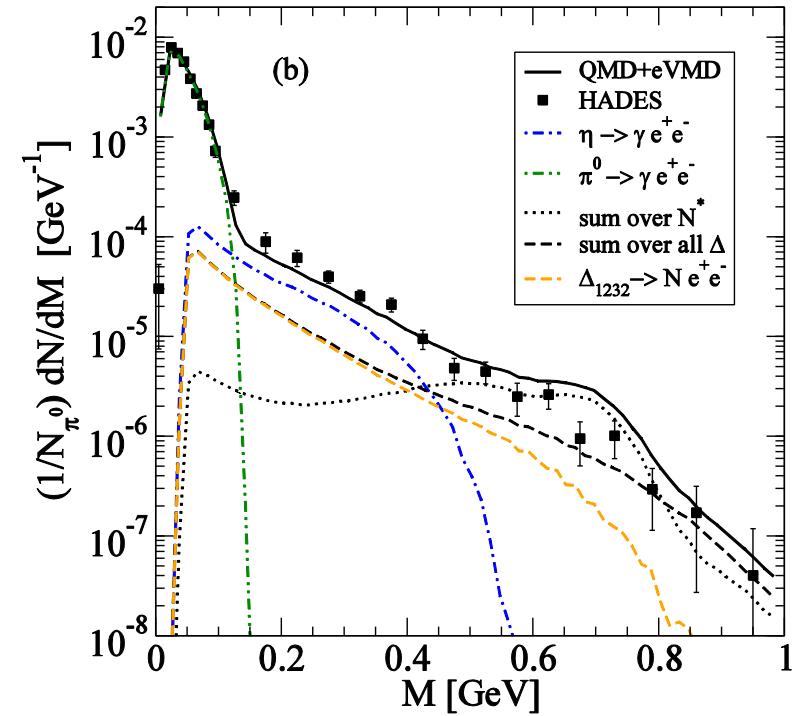


модифицированные ширины ρ и ω
(а) 200 и 60 МэВ

Дилептонный спектр в реакции C + C при энергии 2 АГэВ в сравнении с данными HADES.

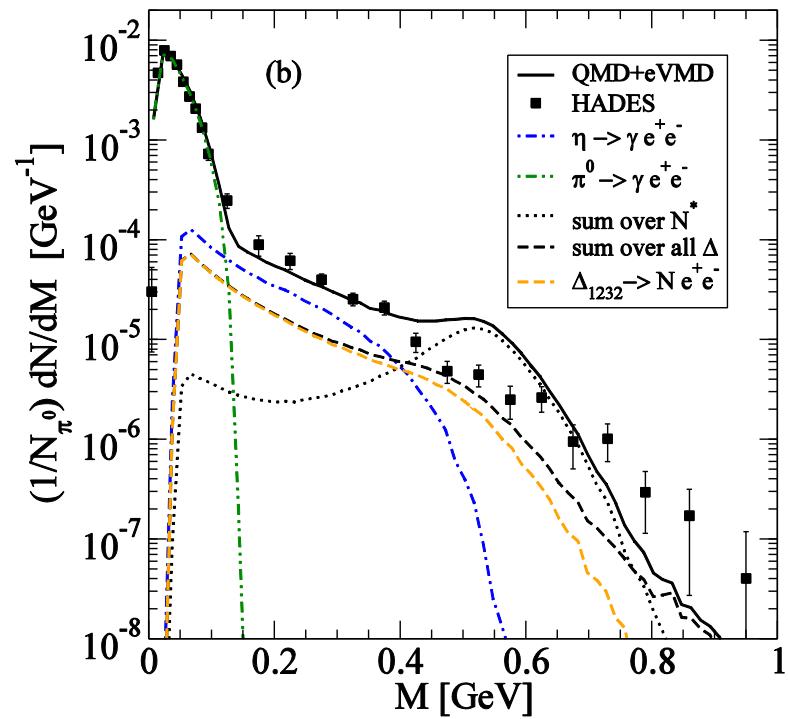
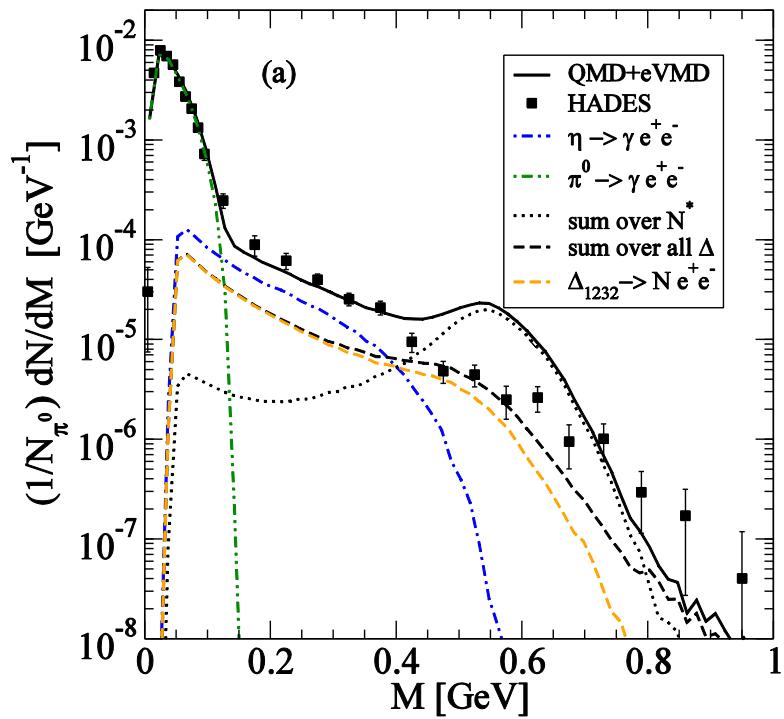


вакуумная версия NRD + eVMD



модифицированные ширины ρ и ω
(b) 250 и 120 МэВ

Дилептонный спектр в реакции C + C при энергии 2 АГэВ в сравнении с данными HADES.

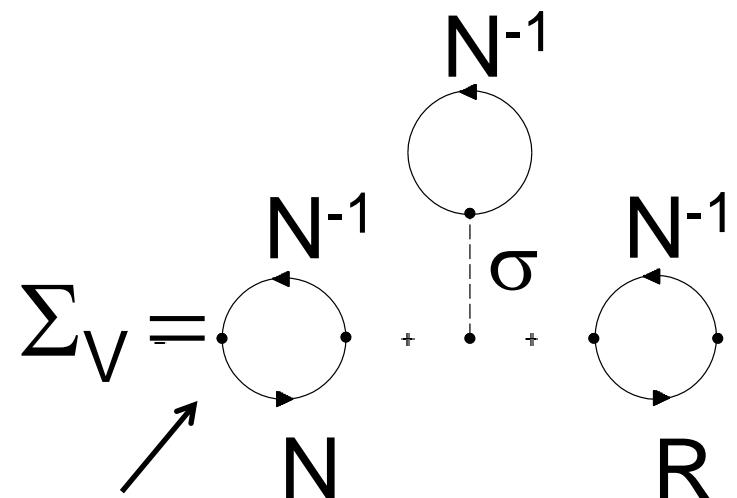


«Скейлинг Брауна-Ро»
ширины ρ и $\omega = 200$ и 60 МэВ

«Скейлинг Брауна-Ро»
ширины ρ и $\omega = 250$ и 120 МэВ

скейлинг Brown-Rho-Hatsuda-Lee

Массовые операторы $V = \rho$, ω и R в ядерной среде



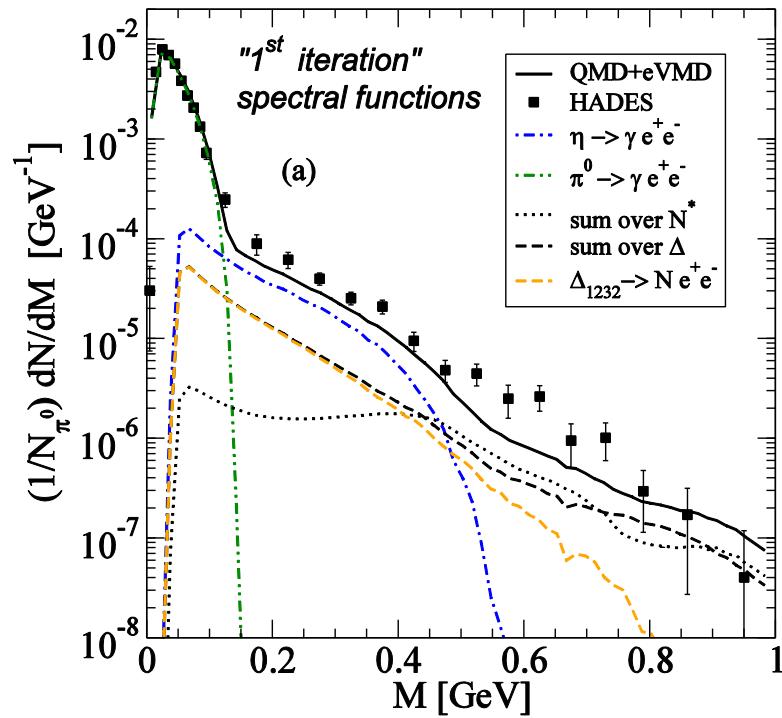
КОМПТОН

$$\Sigma_R = N(R) = V \circ \Sigma_V \circ V = V \circ N \circ V + \dots$$

$$R = \left[\begin{array}{l} N^*(1535)\frac{1}{2}^-, N^*(1650)\frac{1}{2}^-, N^*(1520)\frac{3}{2}^-, \\ N^*(1440)\frac{1}{2}^+, N^*(1720)\frac{3}{2}^+, N^*(1680)\frac{5}{2}^+ \\ \Delta(1620)\frac{1}{2}^-, \Delta(1700)\frac{3}{2}^-, \Delta(1232)\frac{3}{2}^+, \Delta(1905)\frac{5}{2}^+, \Delta(1950)\frac{7}{2}^+ \end{array} \right]$$

$\text{Im}\{\Sigma_{V,R}\} \sim \text{столкновительная ширина}$

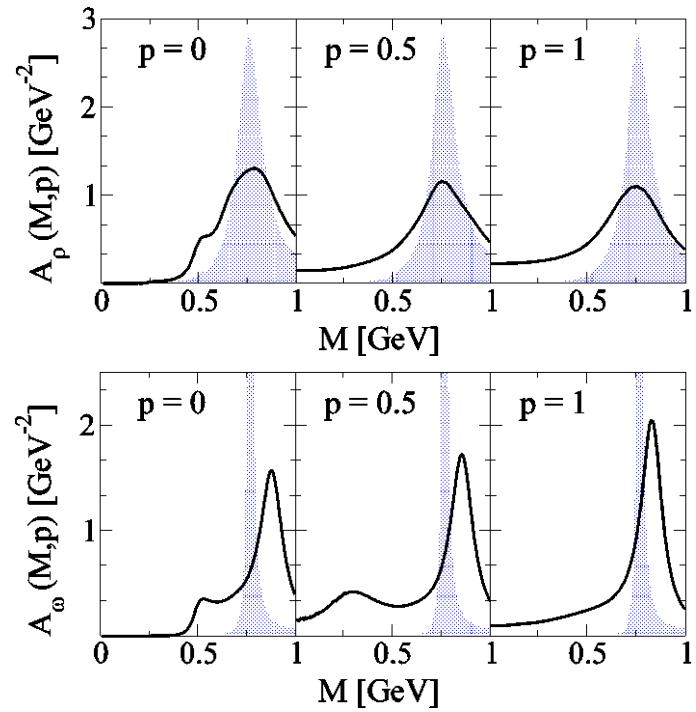
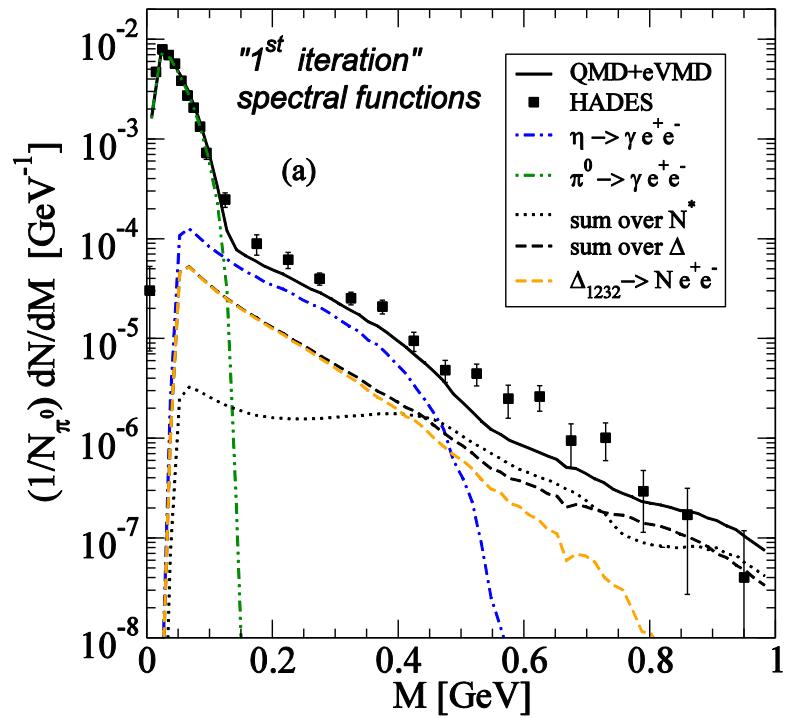
Дилептонный спектр в реакции C + C при энергии 2 АГэВ в сравнении с данными HADES.



$$\text{Im}\{\Sigma_R^*\} = 0$$

$$\text{Im}\{\Sigma_R\} \sim \text{selfconsistent}$$

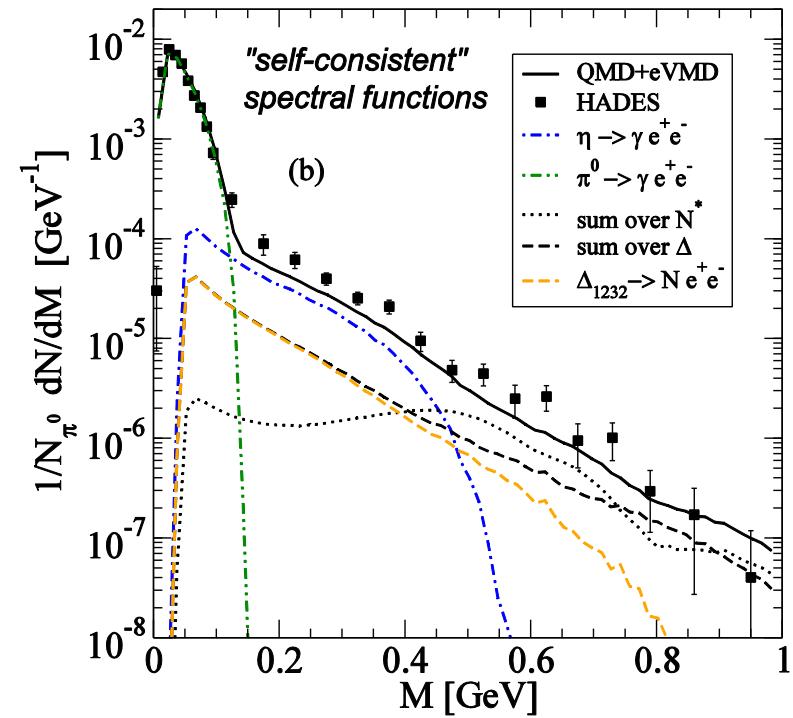
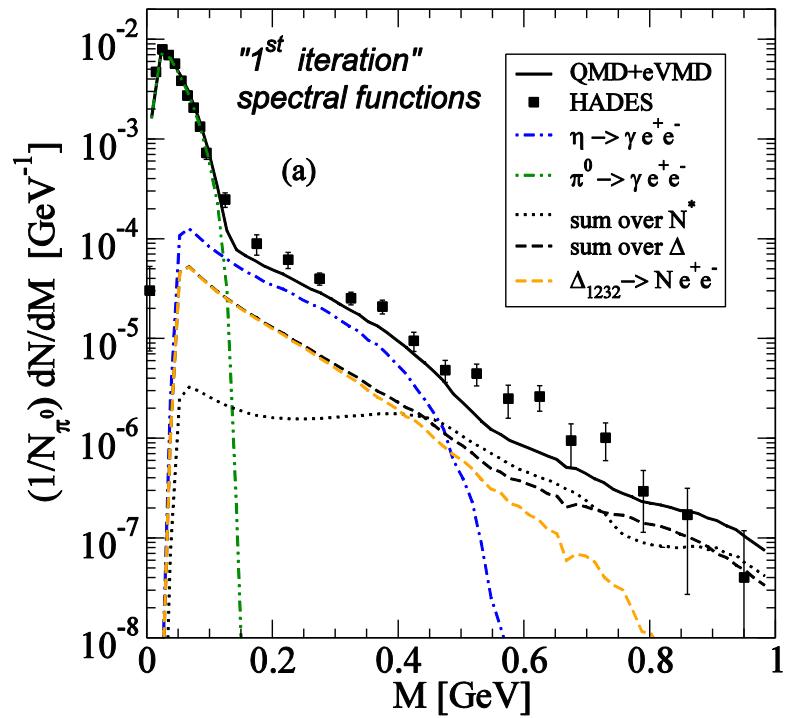
Дилептонный спектр в реакции C + C при энергии 2 АГэВ в сравнении с данными HADES.



$$\text{Im}\{\Sigma_R^*\} = 0$$

$\text{Im}\{\Sigma_R\} \sim \text{selfconsistent}$

Дилептонный спектр в реакции C + C при энергии 2 АГэВ в сравнении с данными HADES.



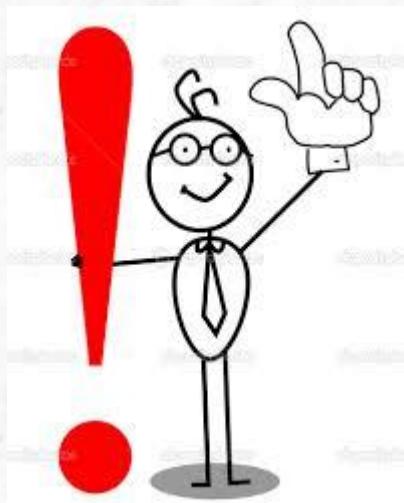
$$\text{Im}\{\Sigma_R^*\} = 0$$

$$\text{Im}\{\Sigma_R^*\} \sim \text{selfconsistent}$$

Анализ данных DLS/HADES по спектру дилептонов:

1. $\Gamma^{\text{coll}}_V \neq 0$ & $\Gamma^{\text{coll}}_R \neq 0$

2. $\Delta m_V \sim 0$, что согласуется с теоретическими моделями:
Бернард & Мейсснер Nucl. Phys. A 489, 647 (1988), $p \sim 0$
Елецкий & Иоффе Phys. Rev. Lett. 78, 1010 (1997), $p \sim 2$ ГэВ
Кондратюк и др., Phys. Rev. C 58 1078 (1998) $p \sim 0$



Quantum characteristics method for many – body scattering problem

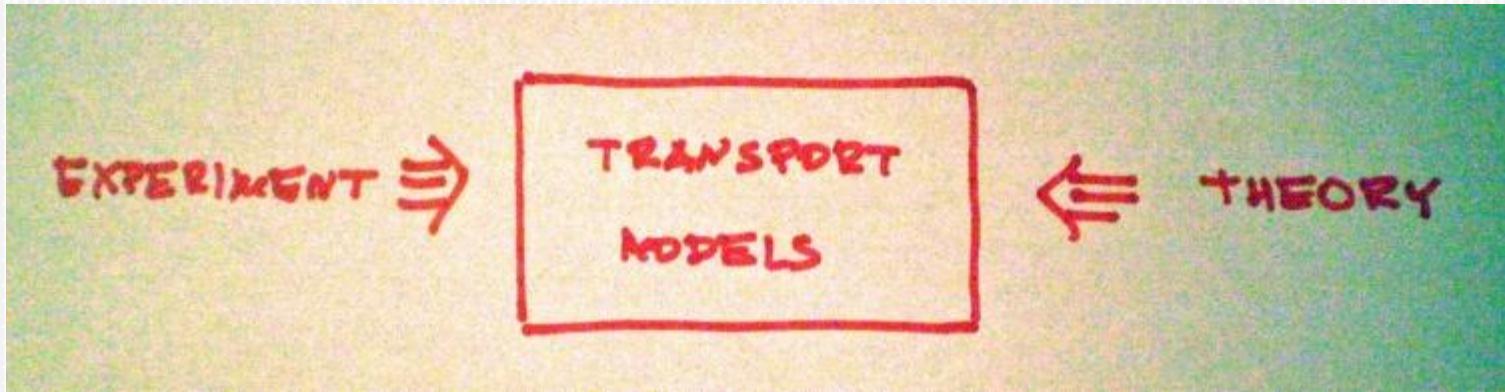
M. I. Krivoruchenko
ITEP, Moscow

October 12, 2009
Internal seminar

- ◆ Classical trajectories vs quantum transport. Why do we need trajectories?
- ◆ Weyl's correspondence & quantum characteristics.
- ◆ Quantum Liouville equation and quantum Hamilton's equations.
- ◆ Semiclassical expansion. A system of ODE for quantum transport.

- ◆ M. I. K., C. Fuchs, A. Faessler, Annalen der Physik **16**, 587-614 (2007);
- ◆ M. I. K., B. V. Martemyanov and C. Fuchs,
Phys. Rev. C **76**, 059801 (2007).

- + M. I. Krivoruchenko, *Some Applications of Quantum Mechanics*,
Ed. M. R. Pahlavani (InTech, Zagreb, 2012), pp. 67 – 90;
- + M. I. K. and A. Faessler, J. Math. Phys. **48**, 052107 (2007);



Trajectories is fundamental feature of transport models

CLASSICAL

$N < \infty$

ODE

QUANTUM

$N = \infty$

PDE

Significant reduction of complexity of the problem: By means of trajectories the field-theoretic problem reduces to statistical-mechanical problem of computing an ensemble of classical trajectories.

CAN WE GO BEYOND \hbar^0 ?

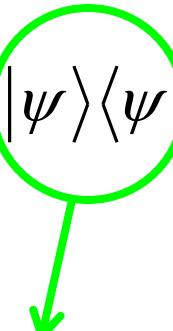
WIGNER FUNCTION & WIGNER MAP

For a pure state

$$W(p, x) = \int_{-\infty}^{+\infty} \frac{d\xi}{2\pi\hbar} e^{ip\xi/\hbar} \langle x - \xi/2 | \psi \rangle \langle \psi | x + \xi/2 \rangle$$

For a mixture

$$W(p, x) = \int_{-\infty}^{+\infty} \frac{d\xi}{2\pi\hbar} e^{ip\xi/\hbar} \langle x - \xi/2 | \hat{\rho} | x + \xi/2 \rangle$$



AND FOR ANY OPERATOR

$$f(p, x) = \int_{-\infty}^{+\infty} \frac{d\xi}{2\pi\hbar} e^{ip\xi/\hbar} \langle x - \xi/2 | \hat{f} | x + \xi/2 \rangle$$

TO: functions
in phase space



FROM: operators
in Hilbert space

AND FOR ANY OPERATOR

$$f(p, x) = \int_{-\infty}^{+\infty} \frac{d\xi}{2\pi\hbar} e^{ip\xi/\hbar} \langle x - \xi/2 | \hat{f} | x + \xi/2 \rangle$$

TO: functions
in phase space



FROM: operators
in Hilbert space

evolution
operator

$$U = e^{-i\hat{H}\tau}$$

$$\begin{aligned}\hat{x}(0) &= \hat{x} \\ \hat{p}(0) &= \hat{p}\end{aligned}$$

$$\begin{aligned}\hat{x}(\tau) &= U^\dagger \hat{x} U \\ \hat{p}(\tau) &= U^\dagger \hat{p} U\end{aligned}$$

Heisenberg picture

$$(\hat{x}, \hat{p}) \xrightarrow{W} (x, p)$$

$$(\hat{x}(\tau), \hat{p}(\tau)) \xrightarrow{W} (X(x, p, \tau),$$

+ initial conditions:

$$X(x, p, 0) = x \quad \& \quad P(x, p, 0) = p$$

Summarizing:

WE HAVE CONSTRUCTED PHASE SPACE TRAJECTORIES

$$\begin{aligned} & (\hat{x}(\tau), \hat{p}(\tau)) \\ & \downarrow W \end{aligned}$$

$$(X(x, p, \tau), P(x, p, \tau)): \quad X(x, p, 0) = x \quad \& \quad P(x, p, 0) = p$$

However,

- *do they play any role in the dynamics?*
- *If yes, how to find them?*

Do they play any role in the dynamics?

First,

$$\forall \hat{f} \quad \exists f(p, x) \xrightarrow{W} \hat{f} : \quad f(\hat{p}, \hat{x}) = \hat{f}$$

I.E. THE SET (\hat{p}, \hat{x}) IS COMPLETE FOR THE CONSTRUCTION OF OPERATORS

Evolution:

$$\begin{aligned}\hat{f} &\rightarrow \hat{f}(\tau) = e^{iH\tau} \hat{f} e^{-iH\tau} \\&= e^{iH\tau} f(\hat{p}, \hat{x}) e^{-iH\tau} \\&= f(e^{iH\tau} \hat{p} e^{-iH\tau}, e^{iH\tau} \hat{x} e^{-iH\tau}) \\&= f(\hat{p}(\tau), \hat{x}(\tau)).\end{aligned}$$



Given $(\hat{p}(t), \hat{x}(t))$ are known

THE EVOLUTION PROBLEM IS SOLVED

Next step: Wigner transformation

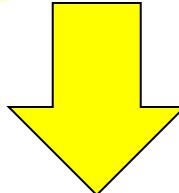
If yes, how to find them?

QUANTUM version of Hamilton equations:

$$\frac{\partial}{\partial \tau} u^i(\xi, \tau) = \{\zeta^i, H(\zeta)\} \Big|_{\zeta^i = *u^i(\xi, \tau)}$$

+ initial conditions:

$$u^i(\xi, 0) = \xi^i$$



RIGHT HAND SIDE BECOMES \star -FUNCTION

T. A. Osborn and F. H. Molzahn, Ann. Phys. 241, 79 (1995),
M.I.K., A. Faessler, J. Math. Phys. 48, 052107 (2007)

■ 4-th step: truncate the expansion and solve ODE:

Expanded quantum Hamilton's equations

$$\begin{aligned}\frac{d}{d\tau} u_s^i(\xi, \tau) &= F_s^i(u_0(\xi, \tau), \dots, J_{r,i_1 \dots i_t}^i(\xi, \tau), \dots), \\ \frac{d}{d\tau} J_{r,i_1 \dots i_t}^i(\xi, \tau) &= G_{r,i_1 \dots i_t}^i(u_0(\xi, \tau), \dots, J_{r,i_1 \dots i_t}^i(\xi, \tau), \dots).\end{aligned}$$

with initial conditions

$$\begin{aligned}u_0^i(\xi, 0) &= \xi^i, \quad J_{0,j}^i(\xi, 0) = \delta_j^i \\ u_s^i(\xi, 0) &= 0, \quad J_{s,i_1 \dots i_t}^i(\xi, 0) = 0 \quad (s \geq 1).\end{aligned}$$

To any fixed order in the Planck's constant, quantum characteristics can be found by solving a finite system of ordinary differential equations (ODE):

The quantum evolution problem can be approached using numerically efficient ODE integrators.

■ 1-st step: expand trajectories

$$\mathbf{u}^l(\xi, \tau) = \sum_{s=0}^{\infty} \hbar^{2s} \mathbf{u}_s^l(\xi, \tau),$$

where $\mathbf{u}_0^l(\xi, \tau)$ is the classical trajectory.

■ 2-nd step: define Jacobi fields

$$\mathbf{J}_{r, i_1 \dots i_t}^l(\xi, \tau) = \frac{\partial^t \mathbf{u}_r^l(\xi, \tau)}{\partial \xi^{i_1} \dots \partial \xi^{i_t}}$$

■ 3-rd step: expand r.h.s. of Hamilton's equations

$$\{\zeta^l, \mathcal{H}(\zeta)\}|_{\zeta=\star \mathbf{u}(\xi, \tau)} \equiv \mathbf{F}^l(\star \mathbf{u}(\xi, \tau)) = \sum_{s=0}^{\infty} \hbar^{2s} \mathbf{F}_s^l(\mathbf{u}_0(\xi, \tau), \dots, \mathbf{J}_{r, i_1 \dots i_t}^l(\xi, \tau), \dots).$$

To the lowest order in \hbar^2 ,

$$F_0^i = F^i,$$

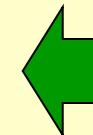
$$F_1^i = u_1^j F_{,j}^i - \frac{1}{24} J_{0,m}^j J_{0,n}^k J_0^{l,mn} F_{,jkl}^i - \frac{1}{16} J_{0,lm}^j J_0^{k,lm} F_{,jk}^i,$$

$$G_{0,k}^i = F_{,l}^i J_{0,k}^l,$$

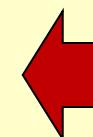
$$G_{0,kl}^i = F_{,mn}^i J_{0,k}^m J_{0,l}^n + F_{,m}^i J_{0,kl}^m.$$

Evolution equations:

$$\left. \begin{aligned} \frac{d}{d\tau} u_0^i &= F_0^i, \\ \frac{d}{d\tau} u_1^i &= F_1^i, \\ \frac{d}{d\tau} J_{0,k}^i &= G_{0,k}^i, \\ \frac{d}{d\tau} J_{0,mn}^i &= G_{0,mn}^i \end{aligned} \right\}$$



classical
Hamilton eqs.



1st quantum
correction to
classical
Hamilton eqs.

A closed system ..

Summary

- Quantum characteristics are existent.
 - ◆ Quantum characteristics allow to reduce the evolution problem of complex quantum systems to a statistical mechanical problem of computing an ensemble of phase space trajectories and their Jacobi fields. The method works at any fixed order of the semiclassical expansion over the Planck's constant.
 - ◆ Quantum characteristics allow to implement consistently quantum effects into transport models.
- In terms of **quantum characteristics & semiclassical expansion** the evolution problem reduces to:
solving an ODE system .

Свойства K мезонов в пионной материи

- ◆ B. V. Martemyanov, A. Faessler, C. Fuchs, M. I. Krivoruchenko,
Phys. Rev. Lett. **93**, 052301 (2004).

Ультрарелятивистские тяжелые ионы: По множественности в рождении доминируют пионы, Т ~ 200 МэВ.

PHENIX Collaboration at RHIC (2003):

$$\frac{\phi \rightarrow e^+ e^-}{\phi \rightarrow K^+ K^-} = \text{2-4 times of the vacuum value} \quad ?$$

Оператор собственной энергии К-мезонов определяется через πK амплитуду рассеяния вперед:

$$\begin{aligned} -\Sigma(p^2, E) = & \int A^+(s, 0, p^2)(dn_{s\pi^+} + dn_{s\pi^0} + dn_{s\pi^-}) \\ & + \int A^-(s, 0, p^2)(-dn_{s\pi^+} + dn_{s\pi^-}). \end{aligned}$$

πK амплитуда: $A^{1/2} = A^+ + 2A^-$, $A^{3/2} = A^+ - A^-$,

$$A^+ \propto K^\dagger K \vec{\pi} \vec{\pi}, \quad A^- \propto K^\dagger \vec{\sigma} K \vec{\pi} \times \vec{\pi},$$

Т $<$ m_π - киральная теория возмущений

$$A^\pm(s, t, p^2) = 8\pi\sqrt{s} \left(a_0^\pm + p^{*2} (b_0^\pm + 3a_1^\pm) + \frac{3}{2} t a_1^\pm \right) + c^\pm (p^2 - M_K^2)$$

длины рассеяния a_0 , a_1 , эффективный радиус b_0 и c из киральной теории возмущений

πK амплитуда:

Т $>$ m_π - феноменология, $\pi K \rightarrow K^* \rightarrow \pi K$

В киральной теории возмущений $T < m_\pi$:

$$\Sigma(M_K^2, M_K) = -4\pi n_\nu \frac{M_\pi + M_K}{M_\pi} a_0^+,$$

$$V_K = -\frac{2\pi n_\nu}{M_\pi + M_K} [a_0^+ + 2M_\pi M_K(b_0^+ + 3a_1^+)].$$

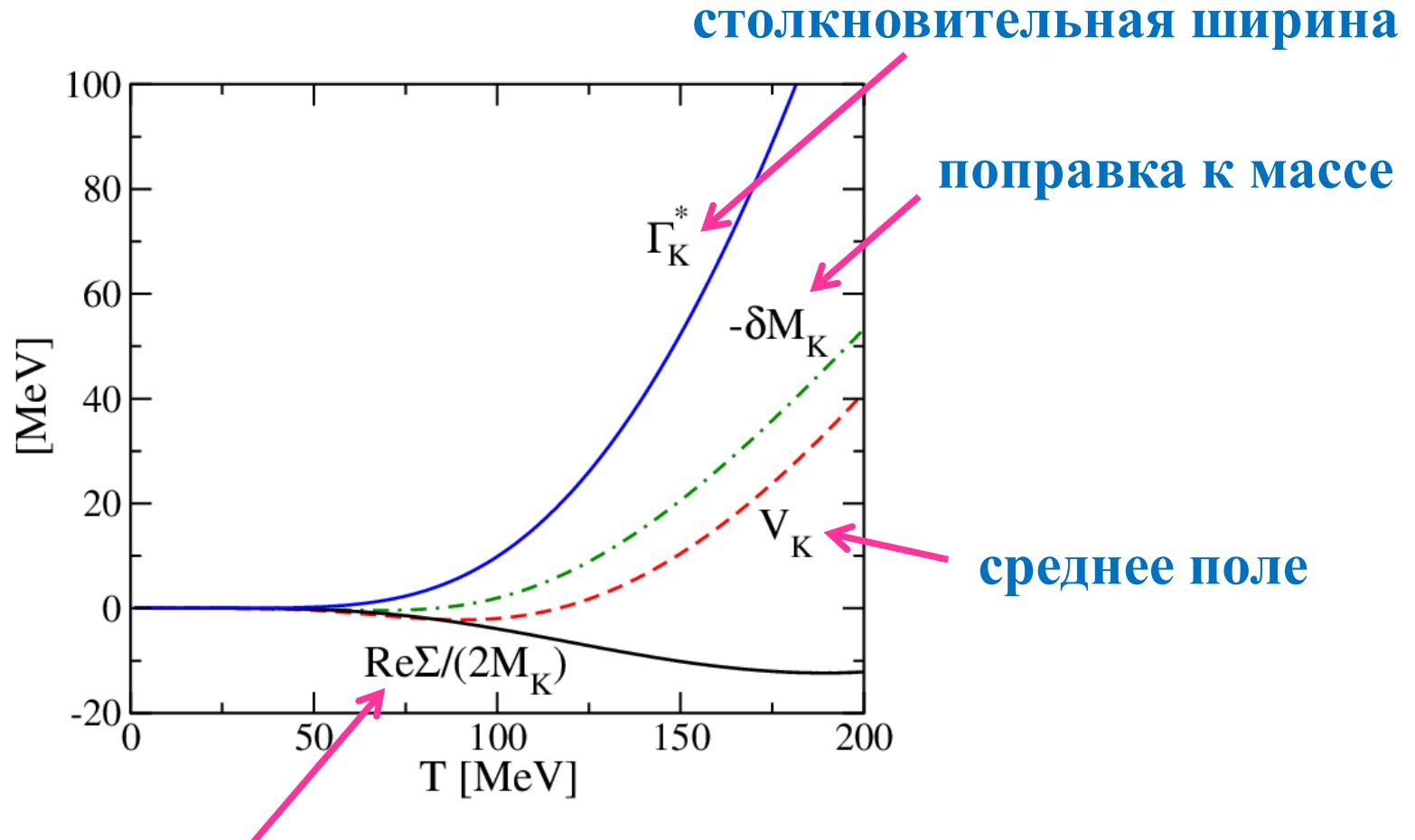
$$\delta M_K + V_K = \frac{\Sigma(M_K^2, M_K)}{2M_K}.$$

Феноменология $T > m_\pi$, с учетом $\pi K \rightarrow K^* \rightarrow \pi K$:

$$a_0^I + b_0^I p^{*2} \rightarrow e^{i\delta_0^I(p^*)} \sin \delta_0^I(p^*) / p^*,$$

$$a_1^{1/2} \rightarrow a_1^{1/2} \frac{|(M_\pi + M_K)^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}|}{s - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}.$$

Модификация свойств K-мезонов в изотопически симметричной пионной материи



$\phi(1020)$ -мезон распадается в зоне реакции, К-мезоны выходят без перерассеяния на пионах. **ВЕРОЯТНОСТЬ:**

$$w \sim \int_0^\tau (e^{-\Gamma_K^*(\tau-t)})^2 e^{-\Gamma_\phi^* t} \Gamma_\phi dt.$$

Пары К-мезонов с инвариантной массой $\phi(1020)$:

$$N_{K\bar{K}} \sim e^{-\Gamma_\phi^* \tau} + \frac{\Gamma_\phi}{2\Gamma_K^* - \Gamma_\phi^*} (e^{-\Gamma_\phi^* \tau} - e^{-2\Gamma_K^* \tau}).$$

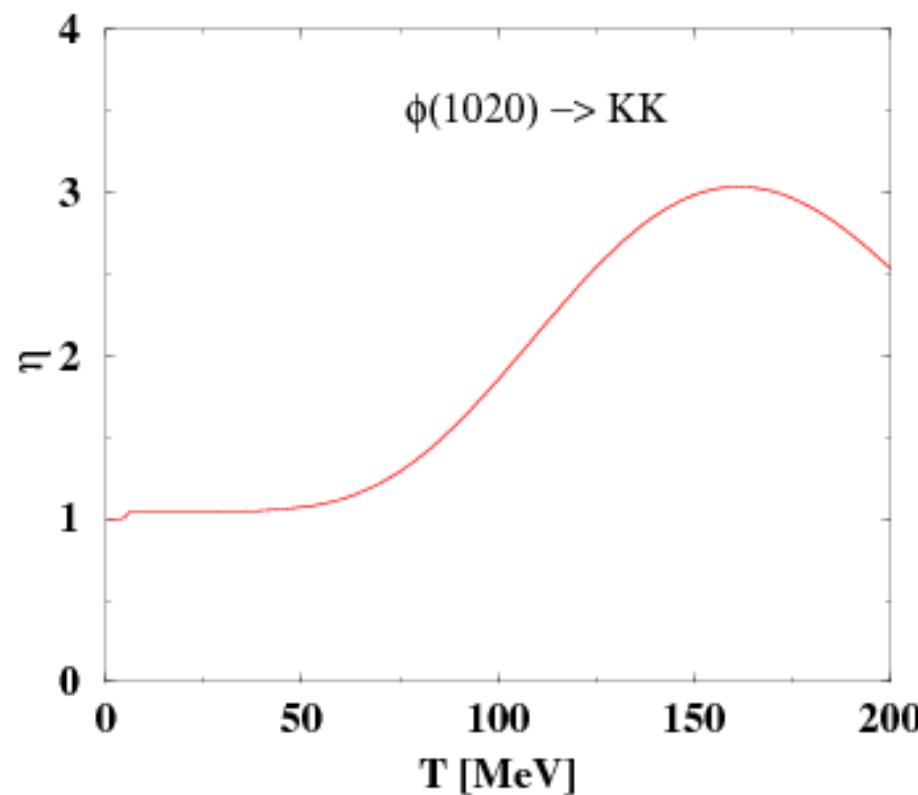
Выход e^+e^- пар:

$$N_{e^+e^-} \sim e^{-\Gamma_\phi^* \tau} B + (1 - e^{-\Gamma_\phi^* \tau}) B^*,$$

Наблюдаемое отношение ширин:

$$B^{\text{app}} = B \frac{1 + (e^{\Gamma_\phi^* \tau} - 1) \Gamma_\phi / \Gamma_\phi^*}{1 + \frac{\Gamma_\phi}{2\Gamma_K^* - \Gamma_\phi^*} (1 - e^{-(2\Gamma_K^* - \Gamma_\phi^*) \tau})}.$$

According to transport calculations $e^{-\tau\Gamma_\phi} \sim 1/2$ at RHIC

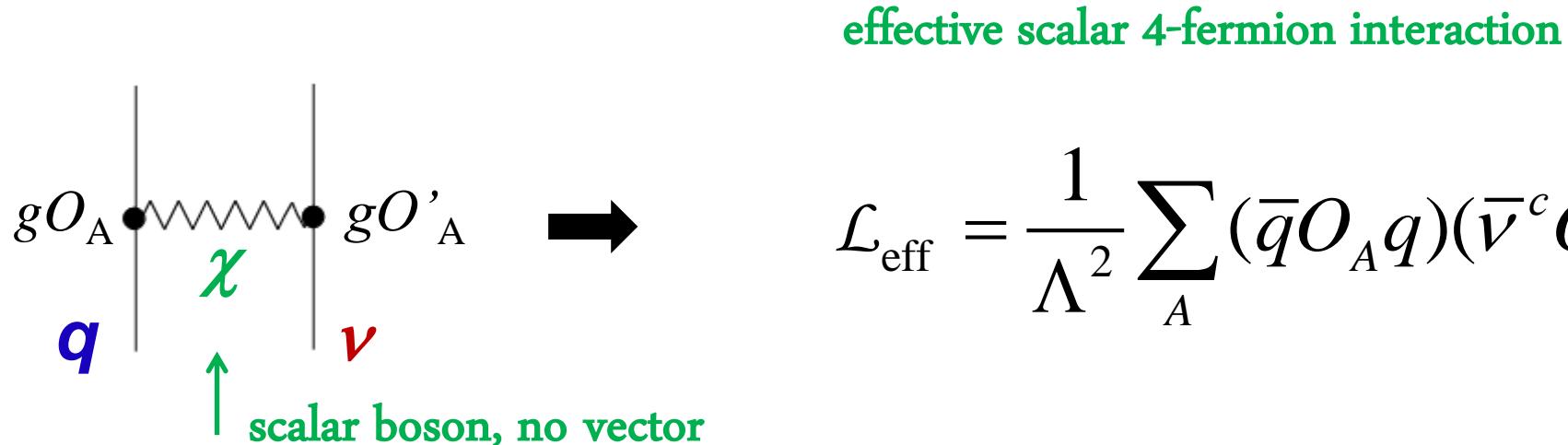


$T = 120 \div 170$ for thermal and chemical freeze-out

$\eta = B^{\text{app}}/B \sim 2\text{-}3$, in accord with the PHENIX data.

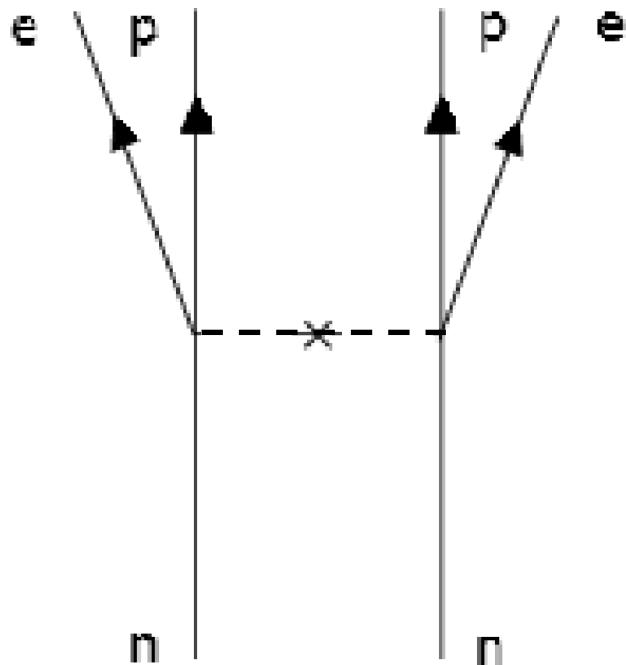
Майорановское нейтрино в ядерной материи

- ◆ S. Kovalenko, M. I. K., F. Simkovic,
Phys. Rev. Lett. **112**, 142503 (2014).

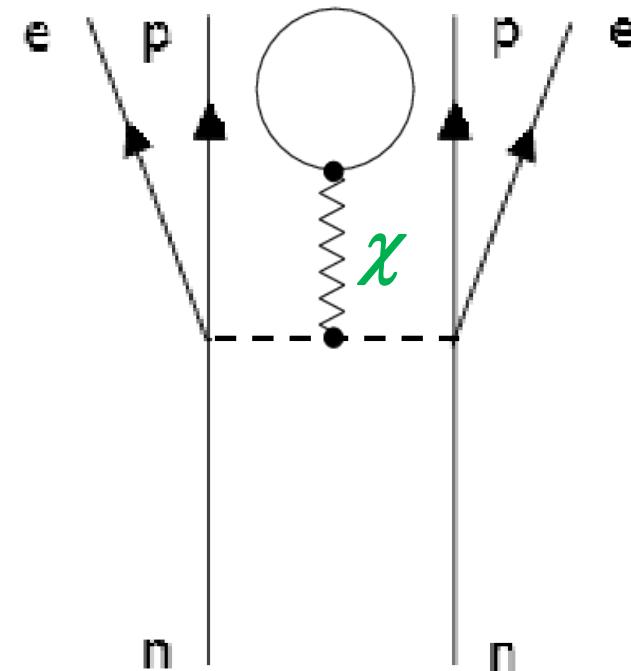


In-medium Majorana mass of neutrino

Induced $0\nu\beta\beta$ decay mechanism:

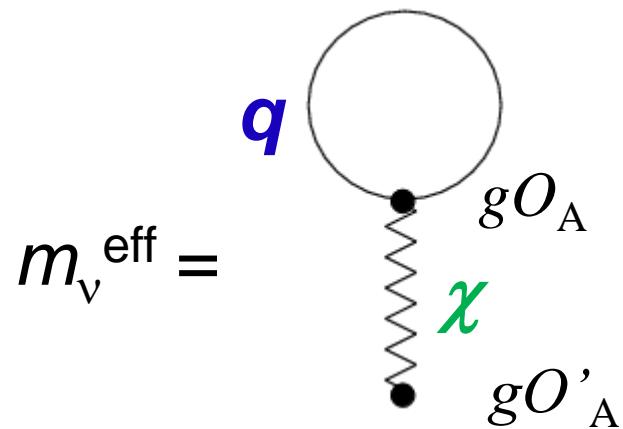


vacuum diagram

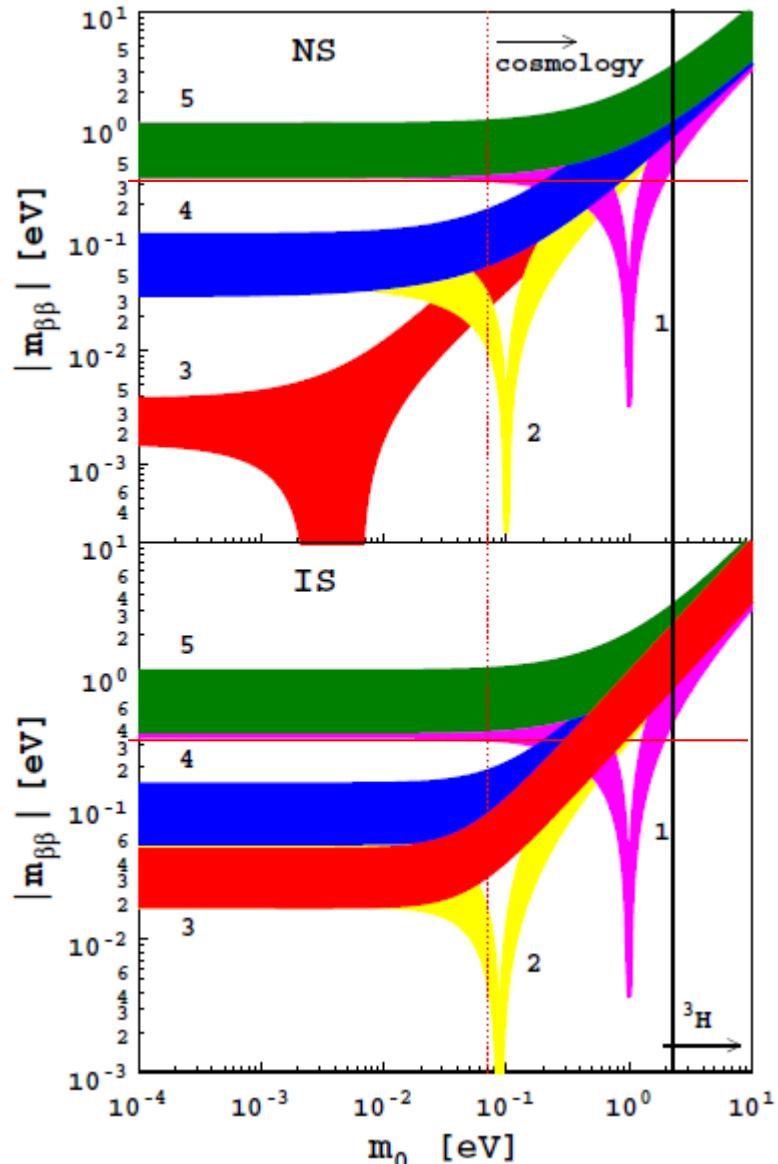


in nuclei + scalar

1. Classification of the vertices gO_A and gO'_A
2. Evaluation of the in-medium Majorana mass of neutrino:



3. Derive constraints for Λ_{NLV} from the $0\nu\beta\beta$ decay limits (+ космология + бета-распад трития)



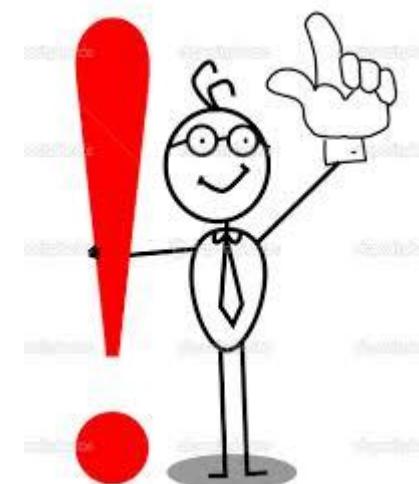
Area $\langle \chi \rangle g_1$
 1 -1
 2 -0.1
 3 0
 4 0.1
 5 1
 &
 $g_2 = g_4 = 0$

$$m_{\beta\beta} = \sum_{i=1}^n U_{ei}^2 \xi_i m_i.$$



$$m_{\beta\beta} = \sum_{i=1}^n U_{ei}^2 \xi_i \frac{\sqrt{(m_i + \langle \chi \rangle g_1)^2 + (\langle \chi \rangle g_2)^2}}{(1 - \langle \chi \rangle g_4)^2}.$$

- ◆ Total lepton number violation can be associated with an exotic scalar-type interaction
- ◆ The mean scalar field in nuclei generates an additional contribution to the Majorana neutrino mass, which leads to a modification of the $0\nu\beta\beta$ decay probability
- ◆ Observation of $0\nu\beta\beta$ decay VS. more stringent constraints on m_ν from cosmology and 3H β -decay
indication of an exotic interaction →
- ◆ Constraint for the scale parameter $\Lambda_{NLV} > 2.4 \text{ TeV}$ derived



Содержание:

- 1. Сверхпроводимость в кварковой материи и ядрах**
- 2. Бозе конденсация дибарионов в ядерной материи**
- 3. Рождение резонансов на ядрах**
- 4. Рождение e^+e^- пар в столкновениях тяжелых ионов**
- 5. Квантовый транспорт в формализме деформационного квантования**
- 6. Свойства K мезонов в пионной материи**
- 7. Майорановское нейтрино в ядерной материи**