Spontaneous breaking of general relativity as an origin of the gravity dark matter and dark energy

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Abstract

The spontaneous breaking of the general gauge invariance is proposed as a source of the gravity dark matter and dark energy in the framework of the extended gravity, with the dynamical fields of metric and a scalar quartet. In the weak-field limit, the Higgs mechanism for gravity is explicitly demonstrated in a generally covariant form. In the linearized approximation, under the natural (in the t'Hooft's sens) restrictions, the theory describes the massless tensor and massive scalar gravitons, being unitary and free of ghosts.

1 Introduction

The unification of the superficially unrelated phenomena in nature seems to be the main trend in the contemporary fundamental physics. Among such the formally unrelated phenomena there are the so-called dark matter (DM) and dark energy (DE). Nevertheless, being extremely elusive, DM and DE may naturally have its common origin in a modification/extension of gravity.

Presently, a firm basis for a theory of gravity is General Relativity (GR). The latter is well-known to be the generally covariant (GC) metric theory of gravity describing in vacuum one physical gravity mode – the massless two-component transverse-tensor graviton. This property is ensured by the conventionally adopted exact general gauge invariance/relativity of GR. At that, a scalar gravity mode, contained a priori in metric, gets unphysical as a by-product. However, to insure the generic property of the masslessness of tensor graviton it would suffice for a gravity theory to possess just the transverse gauge invariance/relativity [1]. Given this, there could appear in metric one more physical mode – the scalar graviton. As an extension to GR, the gravity theory based on the transverse relativity in the explicitly non-generally covariant form, with an extra scalar mode contained in metric, was proposed in [2, 3] and further elaborated in [4]. In the generally covariant form, such a theory was proposed in [5] and developed in [6]. At that, the GR violation was proposed as a raison d'être for appearance of the gravitational DM. To such interpretation, GC preservation proves to be crucial. The gravity DM possesses the generic properties conventionally assigned to DM. In particular, there was obtained a halo-type solution immanent to the theory. In a sense, such a solution may serve as a signature for the extended GR as the black-hole solution is the signature for GR itself. Inevitably, for the explicit GR violation, under GC preservation, one should introduce a nondynamical scalar density. This is the simplest theory realizing the scenario of the explicit GR violation with the gravity DM.

A more detailed study of DM may though require an extension of the scenario beyond the minimal one. This would imply, in turn, an extension of the GR violation. Irrespective of DM, the general second-order effective Lagrangian with the explicit GR violation in the non-covariant form was proposed in [7]. In the GC form, such a Lagrangian for the gravity DM was elaborated in [8] in the context of a nonlinear model. There, as before, to maintain GC under explicit GR violation, it is necessary to introduce a set of the nondynamical quantities, minimally, a quartet of the scalar fields. Such a proliferation of the uncontrollable nondynamical quantities in a half-dynamical theory makes one uneasy, both theoretically and phenomenologically. It would thus be desirable to make the theory completely dynamical by making the nondynamical quantities, minimally the scalar quartet, dynamical, as well. The dynamical scalar quartet in the context of the dynamical four-volume element in gravity was introduced in [9]. It was used for an implementation of the Higgs mechanism for gravity in [10]–[14].

The present paper is an extension and development of two preceding papers of the author [6, 8]. Now, the explicit GR violation with the nondynamical background fields as an origin of the gravity DM is substituted by the spontaneous GR braking with a dynamical scalar quartet. Simultaneously, this proves to serve as an origin of the gravity DE. In Section 2, the general framework for the unified description of the gravity, DM and DE by means of the spontaneous breaking of the general gauge invariance/relativity is worked out. The coupled classical equations for metric and the scalar quartet are presented. An implementation of the Higgs mechanism for gravity in the WF limit is then demonstrated in an explicitly generally covariant form. A consistency condition on a background, for the spontaneous symmetry breaking (SSB) in the given background to take place, is exposed. Finally, the linearized theory in a reduced form is shown to describes the massless tensor and massive scalar gravitons, being unitary and free of ghosts. The prospects for such a unified metric-quartet theory of the extended gravity, DM and DE are shortly discussed in Conclusion.

2 Dark unification

2.1 Metric-quartet gravity

An underlying theory of the gravity and space-time, whichever it might be, should inevitably manifest itself on an observable level as an emergent/effective field theory to match with the conventional field theory for the ordinary matter. In searching for such a fundamental theory, one should, conceivably, look first for the respective effective theory. The latter is to be characterized generically by a set of fields and symmetries ruling the interactions of the fields. Assume thus that the effective field theory of the extended gravity superseding GR is described by the dynamical fields of metric $g_{\mu\nu}$ and a scalar quartet X^a, $a = 0, \ldots, 3$, with an action

$$S = \int L_G(g_{\mu\nu}, X^a)\sqrt{-g}\, d^4x, \tag{1}$$

where $g = \det(g_{\mu\nu})$ and L_G is an effective Lagrangian. The latter is assumed to be generally covariant and invariant under a global Lorentz symmetry $SO(1,3)$ acting on the indices a, b, etc. The physical meaning of the extra variables X^a will be clarified later on. The most general L_G may generically be partited into three pieces depending on the appearance of the derivatives:[1,2]

$$L_G = L_g(\partial_\lambda g_{\mu\nu}) + K(\partial_\lambda g_{\mu\nu}, \partial_\lambda X^a) - V(\partial_\lambda X^a), \tag{2}$$

where L_g is a Lagrangian of the pure metric gravity, with K and V meaning, respectively, the "hard" (kinetic) and "soft" (potential) admixtures of the scalar quartet to metric gravity. Collectively, these admixtures are attributed to the gravity DM and DE, with the latter ones considered as the two facets of a common gravity *dark substance* (DS).[3] The phenomenon of the unification of the pure metric gravity and DS in the framework of the metric-quartet extended gravity may be referred to as the *dark unification.*

[1]At that, an additional dependence directly on $g_{\mu\nu}$ is tacitly allowed, too.

[2]One more a priori conceivable term without any derivatives, V_Λ, is omitted here due to the assumed shift symmetry $X^a \to X^a + C^a$, with an arbitrary constant C^a.

[3]In the present approach, the division of DS onto DM and DE is rather conventional, depending mainly on their spacial clusterization ability to comply with observations.

2.2 Pure metric gravity

Similarly to GR, one may take for the proper gravity the minimal generally covariant Lagrangian of the second order in derivatives of metric:[4]

$$L_g = -\frac{\kappa_g^2}{2} R. \tag{3}$$

Here R is the Ricci scalar curvature, $\kappa_g = 1/(8\pi G_N)^{1/2}$ the Planck mass and G_N the Newton's constant. In what follows, we put $\kappa_g = 1$. The gravity *per see* is taken to preserve the general gauge invariance/relativity. The GR breaking is attributed entirely to the gravity DS.

[4]A modified generally covariant L_g dependent only on metric, e.g., $f(R)$, is a priori conceivable, too. Though, acting as an origin of the gravity DE, such modifications could hardly produce the gravity DM.

2.3 Hard breaking

To preserve GC, the kinetic effective Lagrangian K should dependent on a difference of the Christoffel connection $\Gamma^\lambda_{\mu\nu}$ for the metric $g_{\mu\nu}$ and an auxiliary affine connection $\gamma^\lambda_{\mu\nu}$ (taken to be symmetric):

$$B^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \gamma^\lambda_{\mu\nu}. \tag{4}$$

Being given by the difference between the similarly transforming quantities, the field $B^\lambda_{\mu\nu}$ is a tensor and, as such, may serve to construct the scalar Lagrangian. In these terms, one can decompose K as

$$K = \frac{1}{2}\sum_{i=1}^{5} \varepsilon_i \mathcal{O}_i, \tag{5}$$

through a complete set of the independent two-derivative bilinear in B operators:[5]

$$\begin{aligned} &\mathcal{O}_1 = g^{\mu\nu} B^\kappa_{\mu\kappa} B^\lambda_{\nu\lambda}, \qquad \mathcal{O}_2 = g_{\mu\nu} g^{\kappa\lambda} g^{\rho\sigma} B^\mu_{\kappa\lambda} B^\nu_{\rho\sigma}, \\ &\mathcal{O}_3 = g^{\mu\nu} B^\kappa_{\mu\nu} B^\lambda_{\kappa\lambda}, \qquad \mathcal{O}_4 = g_{\mu\nu} g^{\kappa\lambda} g^{\rho\sigma} B^\mu_{\kappa\rho} B^\nu_{\lambda\sigma}, \\ &\mathcal{O}_5 = g^{\mu\nu} B^\lambda_{\mu\kappa} B^\kappa_{\nu\lambda}, \end{aligned} \tag{6}$$

with some free parameters ε_i, $i = 1, \ldots, 5$, presumably small, $|\varepsilon_i| \ll 1$.[6] eh auxiliary connection may be taken as follows [8]:

$$\gamma^\lambda_{\mu\nu} = \frac{\partial^2 X^a}{\partial x^\mu \partial x^\nu} \frac{\partial x^\lambda}{\partial X^a}\Big|_{X^a = X^a(x)} = X^\lambda_a \partial_\mu X^a_\nu, \tag{7}$$

with $X \equiv \det(\partial X^a/\partial x^\mu) \neq 0$ for the invertibility, $x^\mu = x^\mu(X^a)$. Assign to the extra variables X^a the dimension of length. The coordinates $x^\alpha \equiv \delta^\alpha_a X^a$, wherein $\gamma^\gamma_{\alpha\beta}(x^\alpha) \equiv 0$, define the distinguished observer's coordinates coinciding with the *affine* coordinates for the (piece-wise) flat affine background.[7] Relative to the arbitrary x^μ, X^a are the dynamical quantities. At that, $X^\alpha_\mu = \partial x^\alpha/\partial x^\mu$ and $X^\mu_\alpha = \partial x^\mu/\partial x^\alpha$ are the frames relating the distinguished x^α and arbitrary x^μ coordinates. Geometrically, one may present $\gamma^\lambda_{\mu\nu}$ as a Christoffel connection, $\gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu}(\gamma_{\mu\nu})$, for an auxiliary metric

$$\gamma_{\mu\nu} = X^a_\mu X^b_\nu \eta_{ab}, \tag{8}$$

with $\gamma = \det(\gamma_{\mu\nu}) = -X^2$ and the inverse metric $\gamma^{\mu\nu}$. By this token, $X^a_\mu = \gamma_{\mu\lambda}\eta^{ab}X^\lambda_b$ and $\gamma_{\mu\nu} = \gamma_{\mu\kappa}\gamma_{\mu\lambda}\gamma^{\kappa\lambda}$.[8,9] In these terms, as a minimal kinetic contribution to the spontaneous

[5] This is a fully dynamical generalization, with a dynamical $\gamma^\lambda_{\mu\nu}$ and the spontaneous GR breaking, of a half-dynamical description, with a nondynamical $\hat{\gamma}^\lambda_{\mu\nu}$ and the explicit GR violation [8].

[6] Two more second-derivative linear in $B^\lambda_{\mu\nu}$ terms, $g^{\mu\nu}\nabla_\lambda B^\lambda_{\mu\nu}$ and $g^{\mu\nu}\nabla_\mu B^\lambda_{\nu\lambda}$, with ∇_λ a covariant derivative, are omitted due to the imposed invariance under the reflection $B \to -B$.

[7] Henceforth the physical meaning of X^a. Namely, one may assume that the vacuum is a kind of a physical medium modelled by the affine space endowed with the *absolute* coordinate X^a undergoing the inhomogeneous Lorentz transformations. At that, GR deals only with the *relative* coordinates x^μ undergoing the arbitrary (smooth) transformations among themselves. The transition between the two kinds of coordinates may though be singular.

[8] The usage of $\gamma_{\mu\nu}$ is not obligatory but useful as explicating the geometrical meaning of the gravity.

[9] A priory, one could use $|X| = \sqrt{-\gamma}$ as a dynamical measure (four-volume density). We have retained to this end $\sqrt{-g}$ to maintain explicit baring to GR.

GR breaking, one can restrict himself by the operator

$$\mathcal{O}_1 = g^{\mu\nu}\partial_\mu \ln\frac{\sqrt{-g}}{\sqrt{-\gamma}}\partial_\nu \ln\frac{\sqrt{-g}}{\sqrt{-\gamma}}. \tag{9}$$

where $\sqrt{-\gamma} = |X|$.This minimal case presents a fully dynamical generalization of the semi-dynamical theory for the scalar-graviton DM due to the explicit GR violation, with a nondynamical $\hat{\gamma}$ [5, 6]. In what follows, we will refer to the scalar graviton as the *systolon* [6] referring to the tensor one conventionally just as graviton.[10]

[10]Choosing, in the spirit of the unimodular relativity, as an integration measure $\sqrt{-\gamma}$ instead of $\sqrt{-g}$ one could consider a *dynamical* unimodular theory of gravity, with metric $g_{\mu\nu}$ restricted by the condition $g = \gamma$. In this case, though, the kinetic term $\mathcal{O}_1$ for the scalar graviton/systolon would be lost.

2.4 Soft breaking

To account for the gravity SSB take as the scalar variables [13, 14]:

$$H^{ab} = g^{\mu\nu} X^a_\mu X^b_\nu, \tag{10}$$

as well as [5, 6]:

$$\sigma = -\frac{1}{2} \ln |\det(H^{ab})| = \ln \sqrt{-g}/|X| = \ln \sqrt{-g}/\sqrt{-\gamma}. \tag{11}$$

Choose then the potential term generically as follows:

$$V = \frac{1}{8} m_t^2 H^{ab} H_{ab} + \frac{1}{2} m_x^2 æ(H^2) + \frac{1}{2} m_s^2 \sigma^2 + \Delta V(H^{ab}, \sigma), \tag{12}$$

with m_t, m_x and m_s some mass parameters, $H \equiv H^{ab}\eta_{ab}$, $æ(H^2)$ a scalar function to be properly chosen, and ΔV a rest of the potential containing the higher degrees of H^{ab} and σ. The parameters m_t and m_s, with m_x and $æ(H^2)$ properly chosen, prove to be the masses, respectively, of the tensor and scalar gravitons.[11,12]

Due to GC, the theory being fully dynamical is also gauge invariant under the general diffeomorphisms (Diff's), or, the Lie derivatives, defined as

$$\begin{aligned} \text{Diff}: \quad \Delta_\xi g_{\mu\nu} &= g_{\lambda\nu}\partial_\mu \xi^\lambda + g_{\mu\lambda}\partial_\nu \xi^\lambda + \xi^\lambda \partial_\lambda g_{\mu\nu}, \\ \Delta_\xi X^a_\mu &= X^a_\lambda \partial_\mu \xi^\lambda + \xi^\lambda \partial_\lambda X^a_\mu, \end{aligned} \tag{13}$$

corresponding to $\Delta_\xi x^\mu = -\xi^\mu$, with ξ^λ an arbitrary gauge vector. It follows henceforth the Lie derivative of $\sqrt{-g}$:

$$\Delta_\xi \sqrt{-g} = \partial_\mu(\xi^\mu \sqrt{-g}), \tag{14}$$

and similarly for $\sqrt{-\gamma}$, so that σ transforms as a scalar:

$$\Delta_\xi \sigma = \xi^\mu \partial_\mu \sigma. \tag{15}$$

The Lie derivative of a quantity may explicitly be expressed in a tensor form through replacing ∂_μ by a covariant derivative ∇_μ. The general Diff reduces the number of the independent field components in Lagrangian at most to ten (vs. six in GR). To account for this, a gauge fixing Lagrangian, L_F (or the respective gauge fixing condition), appropriate for a problem at hand, is to be added.

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[11] A particular form of the tensor-graviton mass term may be justified through compliance with the Fiertz-Pauli Lagrangian in the linearized approximation (LA) (see, later).

[12] A priori, one could consider the additional terms dependent only on X^a_μ through $\gamma_{\mu\nu}$ such as $R(\gamma_{\mu\nu})$.

2.5 Classical equations

Supplementing the Lagrangian L by a matter one L_m and varying the total action with respect to $\delta g^{\kappa\lambda}$ ($\delta g^{\kappa\lambda} = -g^{\kappa\mu} g^{\lambda\nu} \delta g_{\mu\nu}$) and δX^a, so that, in particular:

$$
\begin{aligned}
\delta H^{ab} &= X^a_\kappa X^b_\lambda \delta g^{\kappa\lambda} + 2 g^{\kappa\lambda} X^a_\kappa \delta X^b_\lambda, \\
\delta H &= \eta_{ab} \delta H^{ab}, \\
\delta \gamma_{\mu\nu} &= \eta_{ab} (X^a_\mu \delta X^b_\nu + X^a_\nu \delta X^b_\mu), \\
\delta \sqrt{-\gamma} &= (1/2) \sqrt{-\gamma} \gamma^{\kappa\lambda} \delta \gamma_{\kappa\lambda} \\
\delta \sqrt{-g} &= -(1/2) \sqrt{-g} g_{\kappa\lambda} \delta g^{\kappa\lambda}, \\
\delta \sigma &= \delta \sqrt{-g} / \sqrt{-g} - \delta \sqrt{-\gamma} / \sqrt{-\gamma},
\end{aligned} \tag{16}
$$

we get a pair of the coupled field equations (FEs) for the extended gravity in a generic form as follows:

$$
\begin{aligned}
G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T^m_{\mu\nu} + T^K_{\mu\nu} + T^V_{\mu\nu} &\equiv T^m_{\mu\nu} + T^D_{\mu\nu}, \\
\partial_\kappa \Big(\frac{\delta K}{\delta \gamma_{\kappa\lambda}} X^a_\lambda - \frac{\partial V}{\partial H^{ab}} g^{\kappa\lambda} X^b_\lambda + \frac{1}{2} \frac{\partial V}{\partial \sigma} \gamma^{\kappa\lambda} X^a_\lambda \Big) &= 0,
\end{aligned} \tag{17}
$$

where $\delta / \delta \gamma_{\kappa\lambda}$ is a total variational derivative with respect to $\gamma_{\kappa\lambda}$. To eliminate the gauge ambiguity, in the real solving FEs one should first fix the coordinates by imposing an appropriate gauge condition. This will tacitly be understood. The l.h.s. in the upper line of (17) is the gravity tensor $G_{\mu\nu}$ due to L_g, with $T^D_{\mu\nu} \equiv T^K_{\mu\nu} + T^V_{\mu\nu}$ in the r.h.s. treated as the energy-momentum tensor of DS.[13]

[13]This is, in essence, the raison d'être for associating the admixtures with DS.

The kinetic contribution to $T^D_{\mu\nu}$ is

$$T^K_{\mu\nu} \equiv \frac{1}{2}\sum_{i=1}^{5} \varepsilon_i T_{i\,\mu\nu}(g_{\mu\nu}, B^\lambda_{\mu\nu}), \tag{18}$$

with $T_{i\mu\nu}$ the partial contributions due to L_i:

$$T_{i\,\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{O}_i)}{\delta g^{\mu\nu}}. \tag{19}$$

with $\delta/\delta g_{\mu\nu}$ designating a total variational derivative. A similar expression holds for the canonical energy-momentum tensor $T^m_{\mu\nu}$ of the conventional matter. Likewise, the potential contribution to $T^D_{\mu\nu}$ is

$$T^V_{\mu\nu} = -2\frac{\partial V}{\partial H^{ab}} X^a_\mu X^b_\nu + (\frac{\partial V}{\partial \sigma} + V)g_{\mu\nu}. \tag{20}$$

Due to the Bianchi identity, $\nabla_\mu G^{\mu\nu} = 0$, the total energy-momentum tensor, $T_{\mu\nu}$, should be covariantly conserved:

$$\nabla_\mu T^{\mu\nu} \equiv \nabla_\mu(T^{\mu\nu}_m + T^{\mu\nu}_D) = 0. \tag{21}$$

Assuming L_m to be independent of X^a, one conventionally gets that $\nabla_\mu T^{\mu\nu}_m = 0$. In this case (or in the matter vacuum), the DS contribution should separately be covariantly conserved, too.

It may be said that the system (17) determines in a self-consistent dynamical manner the two world *strata*: the metric *structure* and the affine *texture*. At that, the first equation determines the metric structure at a given affine texture. This equation is the same as in the half-dynamical theory. The second equation acts v.v., determining the affine texture appropriate for a given metric structure. At least, this may be looked for by means of the consecutive approximations starting from a putative texture (or, v.v.). The account for the full dynamics should, first, restrict a prior freedom of choosing a nondynamical background in the framework of the half-dynamical description and, second, introduce to the latter some corrections to be controlled. E.g., under the minimal GR breaking only through $\mathcal{O}_1$ and $V(\sigma)$, the second line in FE's (17) looks like

$$\partial_\kappa((\varepsilon_1 \nabla^\lambda\nabla_\lambda\sigma + \partial V/\partial\sigma)X^\kappa_a) = 0. \tag{22}$$

It is satisfied, in particular, at $\varepsilon_1\nabla^\lambda\nabla_\lambda\sigma + \partial V/\partial\sigma = 0$, which explicitly reproduces the analogous quasi-harmonic solution in the semi-dynamical approach [6]. This indicates that the adopted there nondynamical $\hat\gamma$ for the solution is correct. Other solutions of the semi-dynamical approach need the similar self-consistency verification and, conceivably, modification.

2.6 Weak-field limit

The physics content of a nonlinear field theory, as the quantum one, is governed by the weak-field (WF) limit. To this end, consider the expansion

$$\begin{aligned} g_{\mu\nu} &= \bar g_{\mu\nu} + h_{\mu\nu}, \\ X^a &= \hat X^a + \chi^a, \\ X^a_\mu &= \hat X^a_\mu + \partial_\mu \chi^a, \end{aligned} \tag{23}$$

where $\hat X^a$, $\hat X^a_\mu \equiv \partial_\mu \hat X^a$ and $\bar g_{\mu\nu}$ are some background (vacuum) values; $g^{\mu\nu} = \bar g^{\mu\nu} - h^{\mu\nu}$, $h^{\mu\nu} = \bar g^{\mu\kappa} \bar g^{\nu\lambda} h_{\kappa\lambda}$, and $|h_{\mu\nu}|, |\chi^a| \ll 1$.[14] At that, the background coordinates are assumed to be fixed by a suitable gauge condition. By this token, one has

$$H^{ab} = \hat X^a_\kappa \hat X^b_\lambda (\bar g^{\kappa\lambda} - h^{\kappa\lambda}) + \bar g^{\kappa\lambda} (\hat X^a_\kappa \partial_\lambda \chi^b + \hat X^b_\kappa \partial_\lambda \chi^a). \tag{24}$$

It is more convenient to work in the fully space-time notation. To this end, introducing $H^{\mu\nu} \equiv \hat X^\mu_a \hat X^\nu_b H^{ab}$ one first gets:

$$H^{\mu\nu} = \bar g^{\mu\nu} - h^{\mu\nu} + \bar g^{\mu\lambda} \hat X^\nu_c \partial_\lambda \chi^c + \bar g^{\nu\lambda} \hat X^\mu_c \partial_\lambda \chi^c \equiv \bar g^{\mu\nu} - h'^{\mu\nu}, \tag{25}$$

For $h'_{\mu\nu} = \bar g_{\kappa\mu} \bar g_{\lambda\nu} h'^{\kappa\lambda}$ there fulfills then:[15]

$$h'_{\mu\nu} = h_{\mu\nu} - \bar g_{\mu\lambda} (\hat\nabla_\nu \chi^\lambda + \bar g_{\nu\lambda} \hat\nabla_\mu \chi^\lambda), \tag{26}$$

where $\chi^\mu \equiv \hat X^\mu_a \chi^a$ and $\hat\nabla_\mu$ is a covariant derivative with respect to $\hat\gamma^\lambda_{\mu\nu}$, so that $\hat\nabla_\lambda \hat\gamma_{\mu\nu} = 0$. With account for

$$\sigma = -\frac{1}{2} \ln |\det(H^{\mu\nu}) / \det(\hat\gamma^{\mu\nu})| \tag{27}$$

one gets in the WF limit

$$\sigma = \ln \sqrt{-\bar g} / \sqrt{-\hat\gamma} + \frac{1}{2} h', \tag{28}$$

where $\bar g \equiv \det(\bar g_{\mu\nu})$, $\hat\gamma \equiv \det(\hat\gamma_{\mu\nu})$ and $h' \equiv \bar g^{\kappa\lambda} h'_{\kappa\lambda}$.

Further, accounting in the WF limit for the relations

$$\begin{aligned} \Gamma^\lambda_{\mu\nu} &= \bar\Gamma^\lambda_{\mu\nu} + \frac{1}{2} \bar g^{\lambda\kappa} (\bar\nabla_\mu h_{\nu\kappa} + \bar\nabla_\nu h_{\mu\kappa} - \bar\nabla_\kappa h_{\mu\nu}), \\ \gamma^\lambda_{\mu\nu} &= \hat\gamma^\lambda_{\mu\nu} + \hat\nabla_\mu \hat\nabla_\nu \chi^\lambda, \end{aligned} \tag{29}$$

where $\bar\Gamma^\lambda_{\mu\nu}$ and $\hat\gamma^\lambda_{\mu\nu}$ are the background connections corresponding to the background metrics $\bar g_{\mu\nu}$ and $\hat\gamma_{\mu\nu}$, respectively, one can see that[16]

[14] Ultimately, $\hat X^a$ and χ^a may be associated, respectively, with a mean value and a quantum fluctuation of some absolute coordinates X^a. It may, conceivably, be said that DM and DE are a manifestation of such a "trembling" of the absolute coordinates relative to the smoothed observer's ones.

[15] To escape ambiguities, the indeces are raised and lowered in the WF limit exclusively by $\bar g^{\mu\nu}$ and $\bar g_{\mu\nu}$, respectively.

[16] In other words, $B^\lambda_{\mu\nu}$ reduces in the WF limit to a redefined dynamical connection in the nondynamical background.

$$B^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu}(h'_{\mu\nu}) - \hat{\gamma}^{\lambda}_{\mu\nu}. \tag{30}$$

Hence the kinetic term K is also expressible in the WF limit exclusively in terms of $h'_{\mu\nu}$. Now, make in L_g the field substitution $h_{\mu\nu} \to h'_{\mu\nu}$. Due to GR, L_g is left invariant, $L_g(h'_{\mu\nu}) \equiv L_g(h_{\mu\nu})$, if there exists a *fictitious* change of coordinates $\Delta_\zeta x^\mu = -\zeta^\mu$, with

$$\Delta_\zeta h_{\mu\nu} = \bar{\nabla}_\mu \zeta_\nu + \bar{\nabla}_\nu \zeta_\mu, \tag{31}$$

$(\zeta_\mu = \bar{g}_{\mu\lambda}\zeta^\lambda)$, so that there fulfills[17]

$$\bar{\nabla}_\mu \zeta_\nu + \bar{\nabla}_\nu \zeta_\mu = -(\bar{g}_{\mu\lambda}\hat{\nabla}_\nu \chi^\lambda + \bar{g}_{\nu\lambda}\hat{\nabla}_\mu \chi^\lambda). \tag{32}$$

The integrability condition for (32) is

$$(\bar{\nabla}^\nu \bar{\nabla}_\nu)\zeta_\mu + \bar{\nabla}_\mu(\bar{\nabla}^\nu \zeta_\nu) = -(\bar{g}_{\mu\lambda}\bar{\nabla}^\nu \hat{\nabla}_\nu \chi^\lambda + \bar{\nabla}_\lambda \hat{\nabla}_\mu \chi^\lambda). \tag{33}$$

Eq. (32) is the consistency equation for the gravity SSB in the WF limit: if in the given backgrounds with $\bar{g}_{\mu\nu}$ and $\hat{\gamma}_{\mu\nu}$ there exists for each χ^μ a respective ζ^μ, then SSB may take place. At the very least, this is possible in the flat metric and affine backgrounds (see, later). In this case, a gauge fixing Lagrangian should be added depending on a residual gauge invariance. In an opposite case, the WF limit depends explicitly on χ^μ, too. Anyhow, reducing the two kinds of the dynamical variables, $h_{\mu\nu}$ and χ^μ, to just one its combination $h'_{\mu\nu}$ is exclusively a WF (or, low-energy) phenomenon which dilutes in the higher orders, with the two variables contributing, in general, independently. By construction, general gauge invariance should perturbatively recover in the strong fields (or, at the high energies).

Altogether, under the choice of $h'_{\mu\nu}$ in the WF limit as a new dynamical variable, it totally absorbs χ^μ to produce ultimately four additional gravity degrees of freedom (d.o.f.'s). This is an implementation of the Higgs mechanism for gravity. Most generally thus, the WF theory describes ten d.o.f.'s, including, possibly, ghosts. Further reduction of the number of d.o.f.'s and its status (ghost or physical) depend on a residual gauge invariance/relativity in the WF limit. While a general case with the arbitrary parameters ε_i and the arbitrary (classically compatible) backgrounds $\bar{g}_{\mu\nu}$ and $\hat{\gamma}_{\mu\nu}$ requires special investigation, we present below a simplest, but still physically meaningful, case illustrating the reasonability of the theory.

[17]The same is true for a matter Lagrangian L_m, in the case if it is as in GR.

2.7 Linearized approximation

Consider the minimal GR breaking case when the metric and affine backgrounds are flat and, moreover, its respective Minkowskian coordinates coincide with each other. In other words, let $x^\alpha - \delta^\alpha_a \hat{X}^a$ be the distinguished observer's coordinates where there simultaneously fulfills

$$\hat{X}^a_\alpha = \partial_\alpha \hat{X}^a = \delta^a_\alpha, \quad \bar{g}_{\alpha\beta} = \hat{\gamma}_{\alpha\beta} = \eta_{\alpha\beta}, \tag{34}$$

with $\eta_{\alpha\beta} = \mathrm{diag}(1, -1, -1, -1)_{\alpha\beta}$ the Minkowski symbol. The indices are manipulated by means of $\eta_{\alpha\beta}$ and $\eta^{\alpha\beta}$. This gives

$$h'_{\alpha\beta} = h_{\alpha\beta} - (\partial_\alpha \chi_\beta + \partial_\beta \chi_\alpha), \quad h' = h - 2\partial_\gamma \chi^\gamma. \tag{35}$$

In such a flat background the consistency equation (32) fulfills with $\zeta^\alpha = -\chi^\alpha$. Hence, all χ^a can be absorbed in LA by the metric field. With all the connections being trivial, the WF limit becomes, in fact, the linearized approximation (LA). Expanding the Lagrangian we get in LA in an obvious notation[18]

$$L_G = (1 + \varepsilon_t) L_g + \Delta L_{vs} + \frac{1}{8} m_t^2 ((h'_{\alpha\beta})^2 - h'^2) + \frac{1}{8} m_s^2 h'^2, \tag{36}$$

where

$$\begin{aligned} L_g &= \frac{1}{8}(\mathcal{O}_t - 2\mathcal{O}_v + 2\mathcal{O}_x - \mathcal{O}_s), \\ \Delta L_{vs} &= \frac{1}{8}(\varepsilon_v \mathcal{O}_v + \varepsilon_x \mathcal{O}_x + \varepsilon_s \mathcal{O}_s), \end{aligned} \tag{37}$$

in terms of a complete (up to the total derivatives) set of the partial operators

$$\begin{aligned} \mathcal{O}_t &= (\partial_\gamma h'_{\alpha\beta})^2, \qquad \mathcal{O}_s = (\partial_\alpha h')^2, \\ \mathcal{O}_v &= (\partial^\beta h'_{\alpha\beta})^2, \qquad \mathcal{O}_x = \partial^\alpha h'_{\alpha\beta} \partial^\beta h', \end{aligned} \tag{38}$$

[18]Here and in what follows, we intently preserve the prime to stress that the fields at hand are the redefined ones containing the absorbed Goldstone bosons.

and the respective partial constants as follows:

$$\begin{aligned}
\varepsilon_t &= 3\varepsilon_4 - \varepsilon_5, \\
\varepsilon_v &= 4(\varepsilon_2 + \varepsilon_4), \\
\varepsilon_x &= -2\varepsilon_2 + \varepsilon_3 - 3\varepsilon_4 + \varepsilon_5, \\
\varepsilon_s &= \varepsilon_1 + \varepsilon_2 - \varepsilon_3 + 3\varepsilon_4 - \varepsilon_5.
\end{aligned} \tag{39}$$

At that, we have used a proper $m_x æ(h)$ in (12) to obtain in (36) the conventional Fiertz-Pauli mass term for the scalar graviton. Under the general Diff's:

$$\Delta_\xi h'_{\alpha\beta} = \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha, \quad \Delta_\xi h' = 2\partial_\alpha \xi^\alpha \tag{40}$$

one gets (up to the total derivatives)

$$\begin{aligned}
\Delta_\xi \mathcal{O}_t &= -2(\partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha)\partial^2 h'^{\alpha\beta}, \\
\Delta_\xi \mathcal{O}_v &= -(\partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha)\partial^2 h'^{\alpha\beta} - \partial_\gamma \xi^\gamma \partial_\alpha \partial_\beta h'^{\alpha\beta}, \\
\Delta_\xi \mathcal{O}_x &= -\partial_\gamma \xi^\gamma (\partial^2 h' + \partial_\alpha \partial_\beta h'^{\alpha\beta}), \\
\Delta_\xi \mathcal{O}_s &= -2\partial_\gamma \xi^\gamma \partial^2 h'. \\
\Delta_\xi (h'_{\alpha\beta})^2 &= 2(\partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha) h'^{\alpha\beta}, \\
\Delta_\xi h'^2 &= 2\partial_\gamma \xi^\gamma h'.
\end{aligned} \tag{41}$$

It follows henceforth that L_g is always Diff-invariant, $\Delta_\xi L_g = 0$, whereas $\Delta \mathcal{O}_{vs}$ is, generally, Diff-variant. To be as close to GR as possible, we require the Diff invariance in LA to be violated in a minimal fashion. This may be achieved by imposing a residual transverse diffeomorphism (TDiff) invariance.

2.8 Transverse-diffeomorphism invariance

With the scalar-graviton/systolon DM in mind impose on L_G the constraints

$$\varepsilon_v = 4(\varepsilon_2 + \varepsilon_4) = 0, \;\; m_t = 0. \tag{42}$$

The Lagrangian in LA now becomes

$$L_G = (1 + \varepsilon_t)L_g + \Delta L_s + \frac{1}{8} m_s^2 h'^2, \tag{43}$$

$$\Delta L_s = \frac{1}{8}(\tilde{\varepsilon}_x \mathcal{O}_x + \tilde{\varepsilon}_s \mathcal{O}_s), \tag{44}$$

with the reduced partial constants as follows:

$$\begin{aligned} \tilde{\varepsilon}_x &= \varepsilon_3 - 2\varepsilon_4 + \varepsilon_5, \\ \tilde{\varepsilon}_s &= \varepsilon_1 - \varepsilon_3 + 2\varepsilon_4 - \varepsilon_5 \end{aligned} \tag{45}$$

and the same ε_t. It follows from (41) that the symmetry of L_G increases under such the constraints from no Diff up to the three-parameter transverse diffeomorphisms (TDiff's):

$$\text{TDiff} \;:\; \partial_\gamma \xi^\gamma = 0, \tag{46}$$

These two constraints define the most general gravity DM theory possessing in LA no explicit problems.[19] For removing the gauge ambiguities, one should thus impose a gauge condition, e.g.,

$$\partial_\alpha \partial^\gamma h'_{\beta\gamma} - \partial_\beta \partial^\gamma h'_{\alpha\gamma} = 0. \tag{47}$$

Decomposing $h'_{\alpha\beta}$ as

$$h'_{\alpha\beta} = \tilde{h}'_{\alpha\beta} + \frac{1}{4}\eta_{\alpha\beta} h', \tag{48}$$

where $\eta^{\alpha\beta}\tilde{h}'_{\alpha\beta} = 0$, one sees that h' is unrestricted by the condition. At the quantum level, one should add a proper gauge fixing Lagrangian, e.g., [2, 3]:

$$L_F = \lambda(\partial_\alpha \partial^\gamma h'_{\beta\gamma} - \partial_\beta \partial^\gamma h'_{\alpha\gamma})^2, \tag{49}$$

with λ a dimensionless gauge parameter. The term L_F effectively eliminates three field components out of L_G, leaving, generally, seven independent ones. At $\lambda \to \infty$, L_F ensures the classical restriction (47). For simplicity, let moreover $\tilde{\varepsilon}_x = 0$, so that $\tilde{\varepsilon}_s = \varepsilon_1$.[20] In this case, one gets

$$L_G = (1 + \varepsilon_t) L_g + \frac{1}{8}\kappa_s^2 (\partial_\alpha h')^2 + \frac{1}{8} m_s^2 h'^2, \tag{50}$$

where we have put $\kappa_s^2 \equiv \varepsilon_1 \geq 0$.[21] Such a particular case may be shown to present a consistent quantum field theory, unitary and free of ghosts [2, 3]. It describes a massless two-component transverse-tensor graviton and its massive scalar counterpart. With account for the higher-order corrections the TDiff invariance of LA is superseded in the full nonlinear theory by the general Diff invariance, conceivably, not spoiling the properties of the theory. This reduced case, with the three independent parameters ε_1, ε_4 and ε_5 (with the restrictions $\varepsilon_1 > 0$, $\varepsilon_2 = \varepsilon_4$ and $\varepsilon_3 = 2\varepsilon_4 - \varepsilon_5$), and $m_s^2 \neq 0$, is, in principle, sufficient to encompass both the gravity DM and DE. If insufficient, this may be extended to the more general case with $\tilde{\varepsilon}_x \neq 0$..

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[19] The case $\varepsilon_v \neq 0$ would lead to classical instabilities, while the simultaneous fulfillment of $m_t \neq 0$ and $m_s \neq 0$ would result in the appearance of ghosts [4]. Having the scalar-graviton/systolon DM in mind, we choose $\varepsilon_v = m_t = 0$, with the appearance of TDiff. V.v., TDiff ensures these two constraints to be "natural" in the sense of increasing the symmetry.

[20] However, not increasing the symmetry, such a restriction is not "natural" in the t'Hooft's sense. A more general TDiff case is treated in a higher-derivative gauge in [4].

[21] The limit $\kappa_s = 0$ would correspond to the non-propagating auxiliary scalar-gravity mode.

3 Conclusion

The unified dynamical description of the gravity, DM and DE, through the spontaneous breaking of the general gauge invariance/relativity may be brought to a consistent theory of dark unification, with the gravity DM and DE as the two kinds of a common dark substance. Being fully dynamical, the theory with the spontaneous GR breaking, in distinction with the explicit GR violation, eliminates the ambiguities related with the nondynamical quantities. In the strong fields, the general gauge invariance/relativity restores not spoiling the high-energy behaviour of the theory. Containing, in addition to metric, the dynamical fields of the absolute coordinates, the theory drastically changes the vision of the world as built of the two influencing each other strata: the metric structure and affine texture. The proposed metric-quartet theory of the dark unification by no means is, and is not intended to be, an ultimate theory of gravity. Being, as GR, just an effective field theory beyond GR, such an intermediate theory may, conceivably, pave the way from GR towards an ultimate/(more) fundamental theory of gravity and space-time. There remain, of course, many open problems but, with the clearly stated foundations of the theory, they are, in principle, liable to solving. Further theoretical studying of the theory and its phenomenological confirmation/limitation would thus be urgent.

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