Propagator in the SU(2) gluodynamics on a lattice at subcritical temperatures

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- Basics and definitions
- Scale fixing
- Results of simulations
- Electric mass and finite-volume effects
- Critical behavior
- Correct extrapolation to the infinite volume

$$\langle \mathcal{O} \rangle = Tr\left(e^{-\beta H}\mathcal{O}\right) \ / \ Tr\ e^{-\beta H}, \qquad \beta = \frac{1}{T}$$

- the temperature Green's functions

$$\frac{1}{\mathcal{Z}} \sum_n \langle n | e^{-\beta H} T \left[A^{a_1}_{\mu_1}(\tau_1, \vec{x}_1) \ldots A^{a_r}_{\mu_r}(\tau_r, \vec{x}_r) \right] | n \rangle \ ;$$

- the free energy of a quark–antiquark pair

$$\exp\left(-\beta F_{q(\vec{x}),\bar{q}(\vec{y})}\right) = \langle L(\vec{x}) L^{\dagger}(\vec{y}) \rangle$$

$$L(x) = \frac{1}{2}\ Pexp\left(\frac{ga}{2i}\int_0^{\beta} A^a_0(\tau, \vec{x})\sigma^a \, d\tau\right)$$

We compute the "gluon" propagator in the $SU(2)$ theory at the temperature $T = \frac{1}{\beta}$, defined (in general) as follows:

$$D^{bc}_{\mu\nu}(\tau,\vec{x}) = \frac{1}{\mathcal{Z}} \int_{A_\mu(0,\vec{x}) = A_\mu(\beta,\vec{x})} DA^a_\mu(x) A^b_\mu(\tau,\vec{x})\, A^c_\mu(0,0) e^{-S_E[A]} |\det\ M_{FP}(A)| \quad (1)$$

$$S_E[A] = \int_0^\beta d\tau \int_V d^3\vec{x}\ \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\alpha}(\partial_\mu A^a_\mu)^2\right) \quad (2)$$

$$\mathcal{Z} = \int DA^a_\mu(x) e^{-S_E[A]} |\det\ M_{FP}(A)| \quad (3)$$

$$D^{bc}_{\mu\nu}(\tau,\vec{x}) = \delta^{bc} D_{\mu\nu}(\tau,\vec{x})$$

$$(D(p))^{-1}{}_{\mu\nu} = (D^0(p))^{-1}{}_{\mu\nu}(p) - \Pi_{\mu\nu}(p)$$

$$p_\mu \, \Pi_{\mu\nu}(p) = 0 \qquad (4)$$

$$p_\mu p_\nu D_{\mu\nu}(p) = \alpha$$

$$\Pi_{\mu\nu} = G(p) P^T_{\mu\nu} + F(p) P^L_{\mu\nu} \qquad (5)$$

$$P^T_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \frac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P^L_{\mu\nu} = \frac{1}{p^2} \begin{pmatrix} |\vec{p}|^2 & p_4 \vec{p} \\ p_4 \vec{p} & \frac{p_4^2}{|\vec{p}|^2} p_i p_j \end{pmatrix}$$

$$p_\mu P^L_{\mu\nu} = p_\mu P^T_{\mu\nu} = 0$$

$$P^{L,T}_{\mu\nu} P^{L,T}_{\nu\lambda} = P^{L,T}_{\mu\lambda}, \qquad P^L_{\mu\nu} P^T_{\nu\lambda} = 0$$

$$P^L_{\mu\mu} = 1, \qquad P^T_{\mu\mu} = 2$$

$$D_{\mu\nu}(p) = D_L(p) P^L_{\mu\nu} + D_T(p) P^T_{\mu\nu} + \alpha \frac{p_\mu p_\nu}{p^4}$$

$$D_L(p) = \frac{1}{p^2 - F(p)}, \qquad D_T(p) = \frac{1}{p^2 - G(p)}$$

$$\Pi_{\mu\nu} = F(p) P^L_{\mu\nu} + G(p) P^T_{\mu\nu}$$

The quantities under study:

$$D_{ii}(|\vec{p}|^2) = 2 D_T(0, \vec{p}), \qquad D_{44}(|\vec{p}|^2) = D_L(0, \vec{p}),$$

We consider $\alpha = 0$ (the Landau gauge)

$$S = \frac{4}{g^2} \sum_{P=x,\mu,\nu} \left(1 - \frac{1}{2} \mathrm{Tr}\ U_P \right)$$

where

$$U_P = U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}$$

$$U_{x,\mu} \in SU(2),$$

$$U_{x,\mu} \; = \; u_0 \; + \; i \sum_{a=1}^{3} u_a \sigma_a, \quad (6)$$

$$A^a_\mu = - \frac{2u^a_\mu}{ga}, \qquad (7)$$

$$\Lambda : \quad U_{x,\mu} \to \Lambda^{\dagger}_x U_{x,\mu} \Lambda_{x+\hat{\mu}},$$

We fix the absolute Landau gauge by finding the global maximum of the functional

$$\mathcal{F}[\,\mathcal{U}\,] = \frac{1}{2} \sum_{x,\mu} \mathrm{Tr}\, U_{x,\mu}, \quad (8)$$

Stationarity condition:

$$\partial_\nu A^a_\nu = 0.$$

$$A_\mu \to A_\mu^\Lambda = (\Lambda Z)^\dagger A_\mu (\Lambda Z) + \frac{i}{g}\,(\Lambda Z)^\dagger \partial_\mu (\Lambda Z).$$

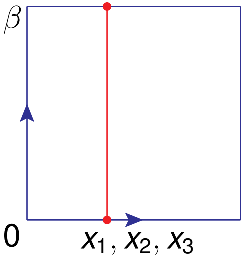
$$A_\mu \to A_\mu^\Lambda = \Lambda^\dagger A_\mu \Lambda + \frac{i}{g}\,\Lambda^\dagger \partial_\mu \Lambda.$$

In the case of $SU(2)$

$$Z \in \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$$

Gauge transformation is the same on both sides!

Center symmetry in the continuum case



We extend the gauge group by nonperiodic gauge transformations:

$$\Lambda(\beta, \vec{x}) = Z\Lambda(0, \vec{x}) \text{ etc.}$$

$$P \exp\left(ig \int_0^\beta A_0(\tau, \vec{x}) d\tau\right) =$$

$$= L(\vec{x}) \longrightarrow L(\vec{x})Z$$

Thus the Hilbert space is broken into 16 superselection sectors

$\mathbb{Z}_2^{D+1} = \mathcal{G}^E \,/\, \mathcal{G}$ $\qquad$ $\mathcal{G}(\mathcal{G}^E)$ is the (extended) gauge group.

$$\exp\left(-\beta F_{q\bar{q}}\right) = \langle L(\vec{x}) L^{\dagger}(\vec{y})\rangle = \frac{1}{4}\left(e^{-\beta V_1(r)} + 3e^{-\beta V_3(r)}\right)$$

Assuming clustering, we conclude that, as $r = |\vec{y} - \vec{x}| \to \infty$,

$$\langle L(\vec{x}) L^{\dagger}(\vec{y})\rangle \ \to |\langle L\rangle|^2.$$

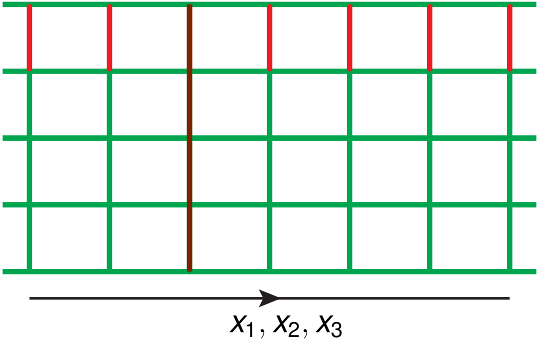
$|\langle L\rangle| = 0 \Longrightarrow$ confinement

$|\langle L\rangle| \neq 0 \Longrightarrow$ deconfinement IN PURE GAUGE THEORY

$\langle L\rangle$ furnishes the average in a given sector; i.e., average over configurations that can be approached at a finite time.
Average over time differs from the ensemble average
$\Longrightarrow$ Spontaneous breaking of the center symmetry at $T = T_c$

Center symmetry:

$\mathbb{Z}_2 : U_{x,\mu} \rightarrow - U_{x,\mu}$



$$L(x_1, x_2, x_3) \rightarrow -L(x_1, x_2, x_3)$$

$T > T_c$
spontaneous
breaking of $\mathbb{Z}_2$
signals transition
to deconfinement:

Free energy
of a free quark
becomes finite

Simulations on $N_s^3 \times N_t$ lattices

$\beta = 4/g^2$	2.478	2.488	2.495	2.501	2.506	2.508	2.509	2.510
$1/a$ (GeV)	2.143	2.213	2.263	2.307	2.344	2.359	2.367	2.374
T/T_c	0.901	0.931	0.952	0.970	0.986	0.992	0.996	0.999

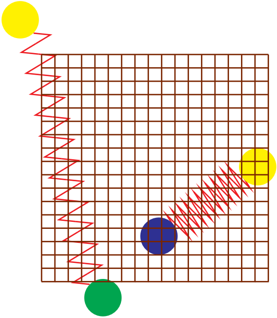
Table : $N_t = 8$

Simulations are performed at $N_s = 32, 40, 48, 56, 64, 78$;
for $\beta = 2.506, 2.510$ we also consider $N_s = 88$.

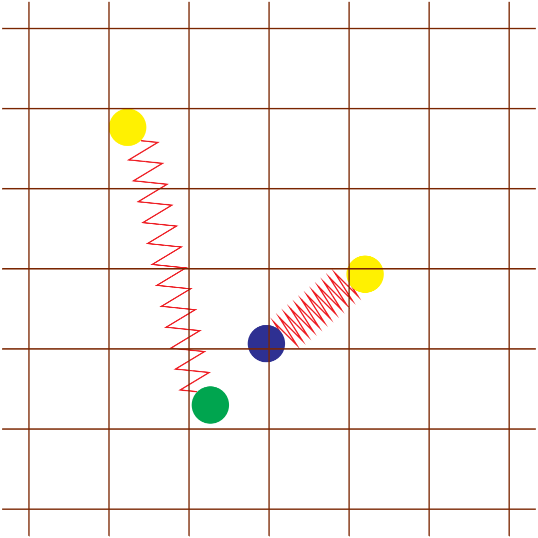
Scale fixing

$\sqrt{\sigma} = 440$ MeV

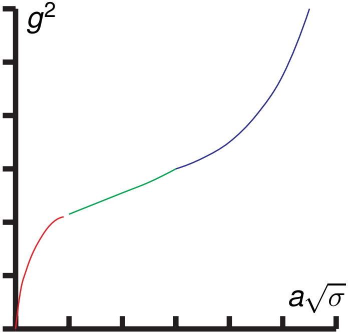
continuum limit $a \to 0$



Weak coupling



Strong coupling (TD limit, $b \to \infty$)



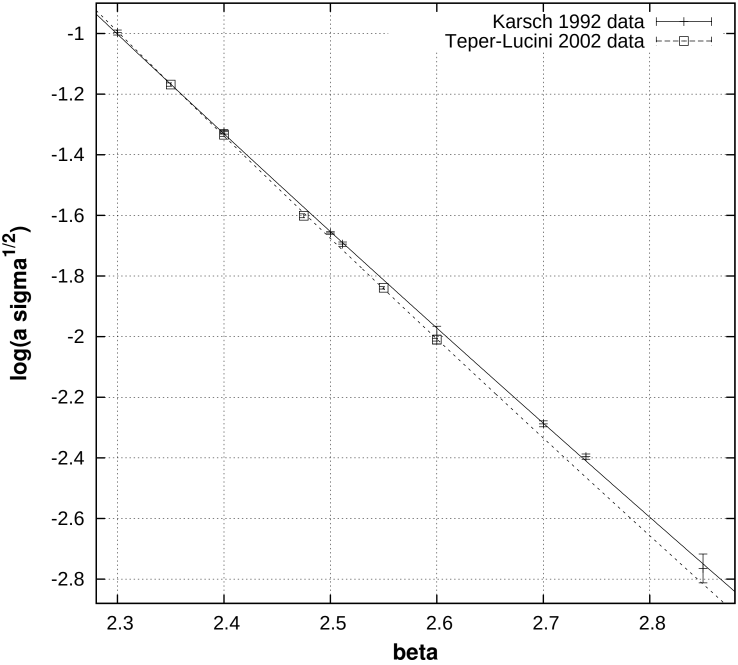
$$T = \frac{1}{aN_t}$$

$$a\Lambda = \left(\frac{b_0 g^2}{16\pi^2}\right)^{-b_1/b_0^2} \exp\left(\frac{-8\pi^2}{b_0 g^2}\right),$$

$$b_0 = \frac{22}{3}, \qquad b_1 = \frac{68}{3}.$$

$$\ln(\sigma a^2) = -\frac{4\pi^2}{b_0}\beta + \frac{2b_1}{b_0^2}\ln\left(\frac{4\pi^2}{b_0}\beta\right) + c' + \frac{4\pi^2}{b_0}\frac{d'}{\beta}$$

For a gauge and/or scheme dependence of Λ_{QCD} see R.Dashen, D.J.Gross, Phys.Rev.**D23** (1981) p.2340.



Karsch 1992 data
Teper-Lucini 2002 data
log(a sigma^1/2)
beta
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-2.6
-2.8
2.3
2.4
2.5
2.6
2.7
2.8

$$T = \frac{\sqrt{\sigma}}{N_t} \exp\left(- \frac{3\pi^2}{11} f(\beta, c, d) \right),$$

$$f(\beta, c, d) = -\beta + \frac{17}{11\pi^2} \ln\beta + c + \frac{d}{\beta}.$$

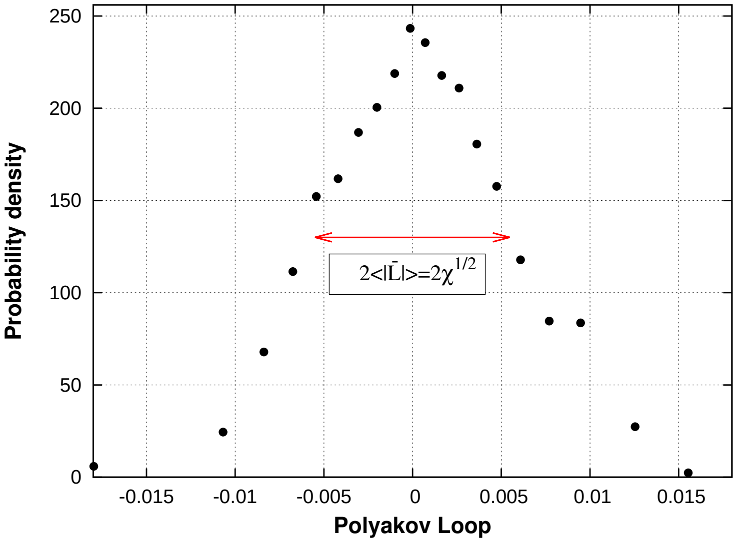
Results of the fit to the Karsch data

$$c = 0.110(21), \quad d = 1.58(5), \qquad \chi^2/N_{dof} = 1.02,\ N_{dof} = 6.$$

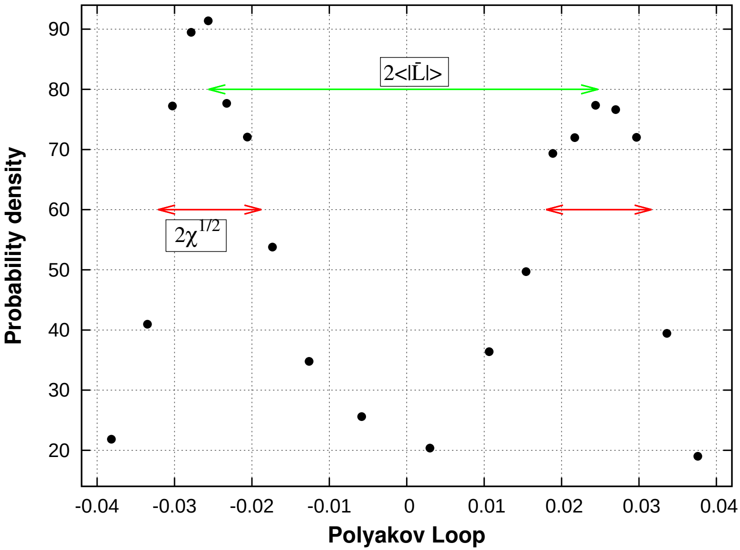
Correlation between c and $d > 99\%$

The combined fit using both Karsch and Teper-Lucini data gives

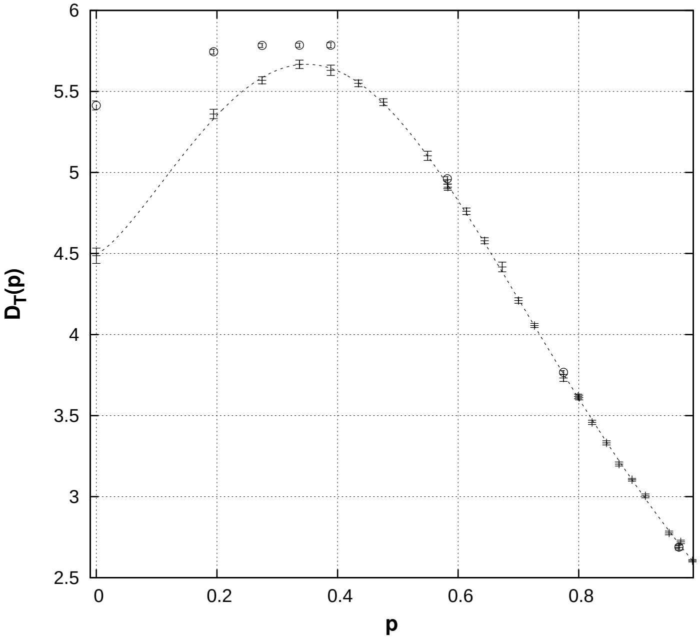
$$\chi^2/N_{dof} = 6.14,\ N_{dof} = 11, \qquad Q(\chi^2, N_{dof}) < 1.4 \times 10^{-9}$$



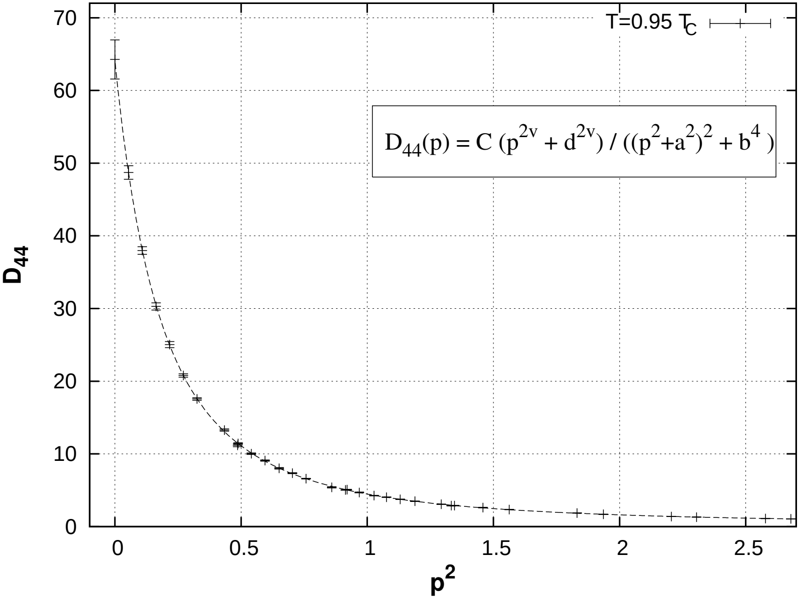
Probability density
250
200
150
100
50
0
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
Polyakov Loop
2<|L̄|>=2χ^1/2



Probability density
Polyakov Loop
2<|L̄|>
$2\chi^{1/2}$
90
80
70
60
50
40
30
20
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04



The effect of Gribov copies


T=0.95 T_C
$D_{44}(p) = C\,(p^{2v} + d^{2v}) / ((p^2+a^2)^2 + b^4)$
D_{44}
p^2
0
10
20
30
40
50
60
70
0
0.5
1
1.5
2
2.5

Definition of the electric gluon (color-screening) mass:

$$\frac{1}{D_{44}(p)} = Z\,(m_e^2 + p^2 + O(p^4))$$

At $p \leq 1 GeV$,

$$D_{44}(p) \simeq \frac{C}{(\mu^2 + p^2)^2 + rp^4} \quad \text{with } r \approx 0, \qquad (\mu = m_e\sqrt{2})$$

Perturbation theory for $SU(2)$: $m_e = \sqrt{\frac{2}{3}}gT \sim 310$ MeV

— however, in the Feynman and $A_0 = 0$ gauges; $g \simeq 1.25$.

Normalization conditions for the longitudinal propagator

- (MOM): $$D^{MOM}(p)|_{p^2=\mu^2} = \frac{1}{\hat{p}^2}, \qquad \hat{p}_\mu = \frac{2}{a}\sin\frac{p_\mu a}{2}.$$
- (NAT): $$\frac{1}{D^{NAT}(p)} = m_e^2 + p^2 + \underline{O}(p^4).$$

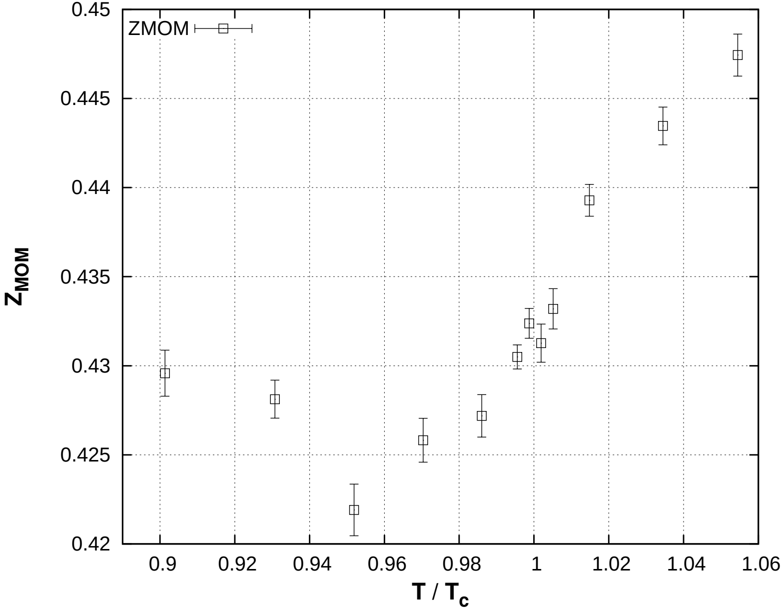
$$D^{NAT}(p) = Z_{NAT} D^{bare}(p), \qquad D^{MOM}(p) = Z_{MOM} D^{bare}(p)$$

we consider three masses,

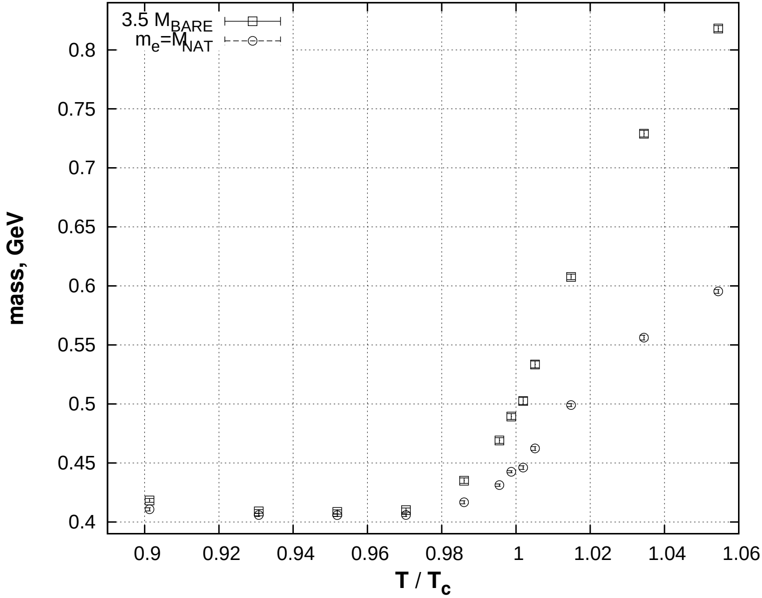
$$M_{bare} = \frac{1}{\sqrt{D^{bare}(0)}}, \tag{9}$$

$$M_{NAT} \equiv m_e = \frac{1}{\sqrt{D^{NAT}(0)}} = \frac{M_{bare}}{\sqrt{Z_{NAT}}}, \tag{10}$$

$$M_{MOM} = \frac{1}{\sqrt{D^{MOM}(0)}} = \frac{M_{bare}}{\sqrt{Z_{MOM}}}. \tag{11}$$



At $T > T_c$, $Z_{MOM} \simeq 0.423(6) + 0.064(12)\xi^{0.33(14)}$,
where $\xi = T/T_c - 1$.



Mass at $N_s = 64$; lattice size $b = 5.0 \div 5.8 fm$

Our data indicate that it is well to divide the mass into the constant (*conf*) and T-dependent (*QGP*) parts,

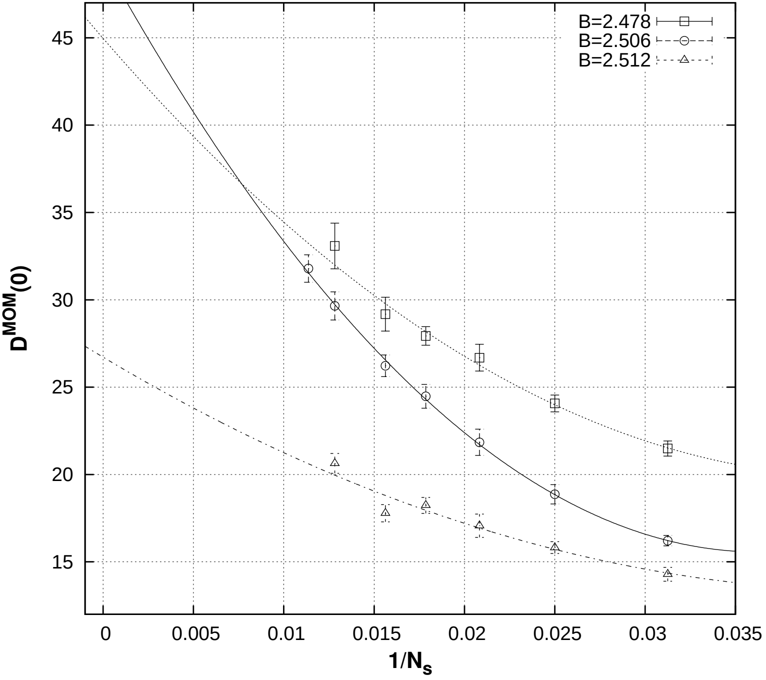
$$M_{bare} = m_{bare}^{conf} + m_{bare}^{QGP} \quad (12)$$

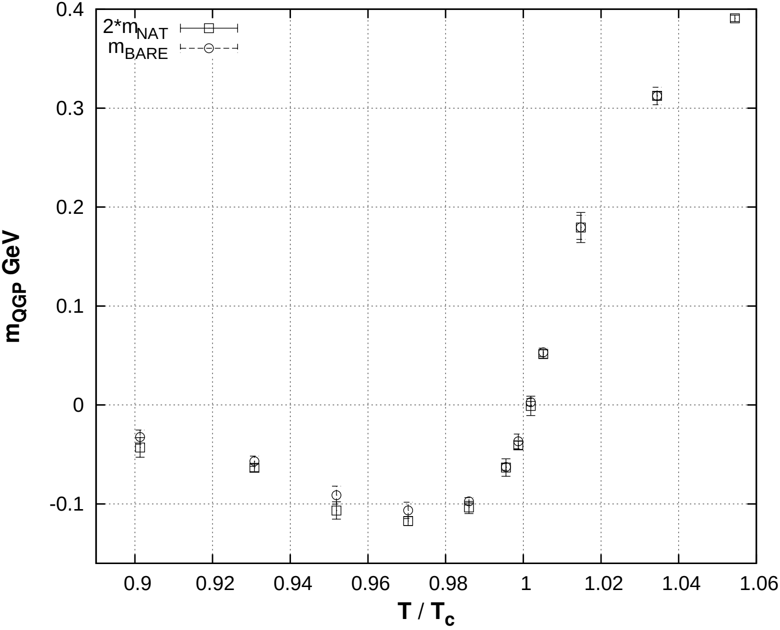
$$M_{NAT} = m_{NAT}^{conf} + m_{NAT}^{QGP} \quad (13)$$

where $m_{bare}^{conf} = 0.114$ GeV and $m_{NAT}^{conf} = 0.4$ GeV thoughout the range under consideration $0.89T_c < T < 1.06T_c$. In this case, we observe that

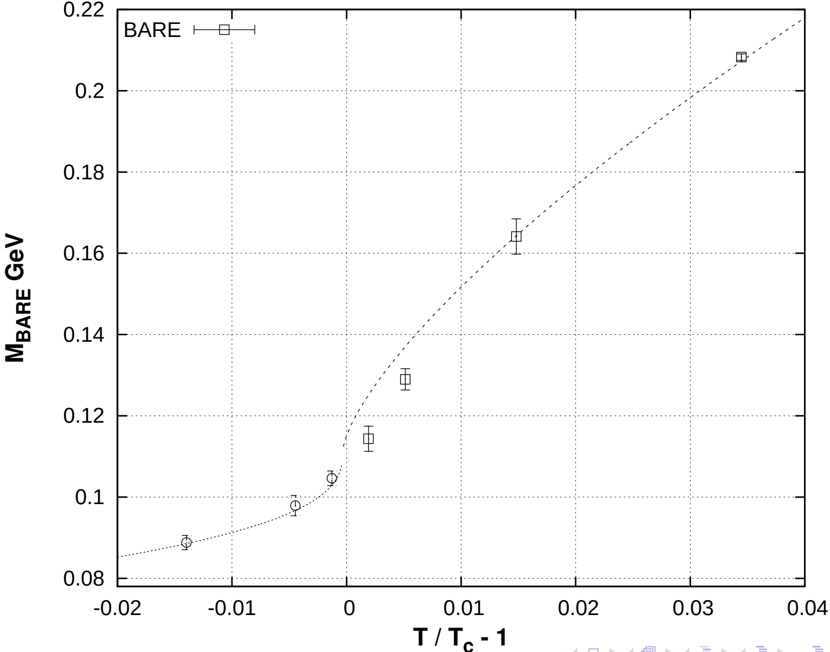
$$m_{NAT}^{conf} = \frac{m_{bare}^{conf}}{\sqrt{Z_{NAT}^{conf}}}, \qquad m_{NAT}^{QGP} \approx \frac{m_{bare}^{QGP}}{2\sqrt{Z_{NAT}^{conf}}}, \quad (14)$$

$Z_{NAT}^{conf} \approx 0.0816$ is a temperature-independent quantity (at least, over the range $0.89T_c < T < 1.06T_c$).

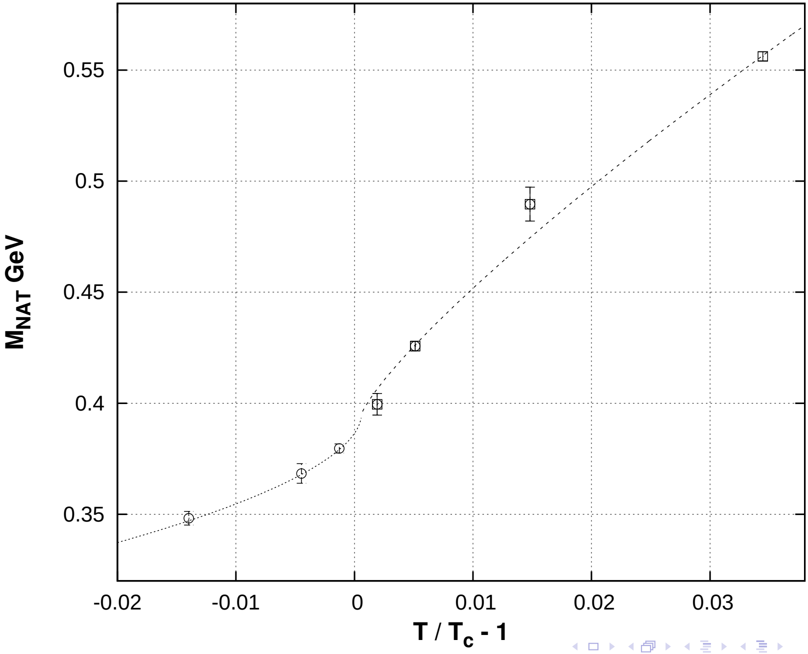

B=2.478
B=2.506
B=2.512
45
40
35
30
25
20
15
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
$D^{MOM}(0)$
$1/N_s$



0.4
0.3
0.2
0.1
0
-0.1
m_{QGP} GeV
0.9
0.92
0.94
0.96
0.98
1
1.02
1.04
1.06
T / T_c
2*m_{NAT}
m_{BARE}



BARE
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
-0.02
-0.01
0
0.01
0.02
0.03
0.04
M_{BARE} GeV
T / T_c - 1



0.55
0.5
0.45
0.4
0.35
-0.02
-0.01
0
0.01
0.02
0.03
M_{NAT} GeV
T / T_c - 1

PRELIMINARY RESULTS

We use the fit function ($\xi = \frac{T}{T_c} - 1$)

- at $\xi > d$ $\quad M_{NAT}(\xi) = m_{NAT}^{conf} + c(\xi - d)^b$
- at $\xi < d$ $\quad M_{NAT}(\xi) = m_{NAT}^{conf} - q(d - \xi)^p$

Critical exponents:

- BARE $\quad m_{BARE}^{crit} = 0.112$ GeV; $\quad b = 0.72, p = 0.36.$
- NAT $\quad m_{NAT}^{crit} = 0.395$ GeV; $\quad b = 0.63, p = 0.54.$

$$\frac{1}{D(p;N_s)} \simeq c_{00} + c_{20}p^2 + c_{40}p^4 + c_{60}p^6 + c_{01}\frac{1}{N_s} + c_{21}\frac{p^2}{N_s} + c_{41}\frac{p^4}{N_s}$$

$$\frac{1}{D(p;N_s)} \simeq c_{00} + c_{20}p^2 + c_{40}p^4 + c_{60}p^6 + c_{01}\frac{1}{N_s} + c_{21}\frac{p^2}{N_s} + c_{41}\frac{p^4}{N_s} + c_{22}\frac{p^2}{N_s^2} + c_{42}\frac{p^4}{N_s^2}$$

FISHER TEST

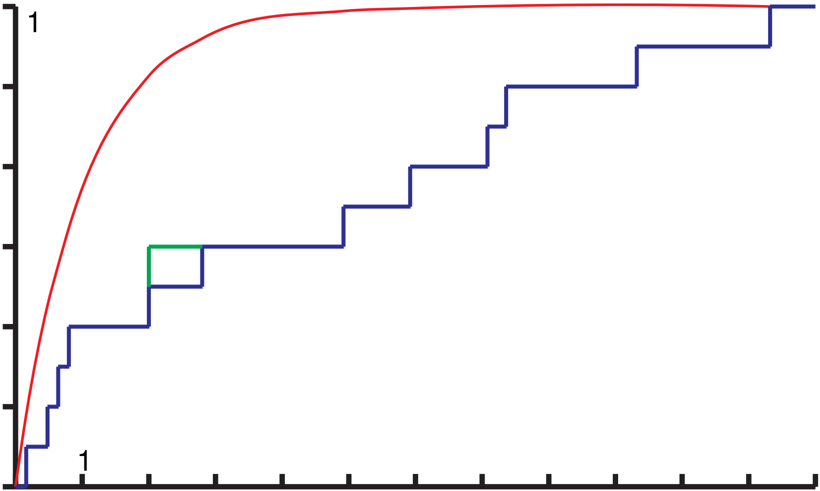
$$\chi_m = \sum_{i,j} \left(\frac{1}{D(p_i; N_s^{(j)})} - f_m(p_i, N_s^{(j)}) \right)^2 / \delta_{ij}^2$$

- residual sum of squares

$$x = \frac{\frac{\chi_n^2 - \chi_m^2}{m - n}}{\frac{\chi_m^2}{N_{dof} - m}}$$ has the Fisher CDF $P_{m-n,n}(\xi < x)$

Fisher probability density function

$$f_{2p,2q}(x) = \frac{dP_{2p,2q}(\xi < x)}{dx} = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \frac{x^{p-1}}{(1-x)^{p+q}}$$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Kolmogorov-Smirnov test

$$\sqrt{N} \sup_x |F_N(x) - F_{theor}(x)|$$

is Kolmogorov distributed.
Kolmogorov function has the form

$$K(x) = \sum_{j=-\infty}^{\infty} (-1)^j e^{-2j^2x^2}$$

In our case, $x = 1.813$ and $K(x) = 0.997$;
A minimal change in the empirical CDF gives $K(x) = 0.9787$