LPM effect in QED and QCD and jet quenching in AA-collisions

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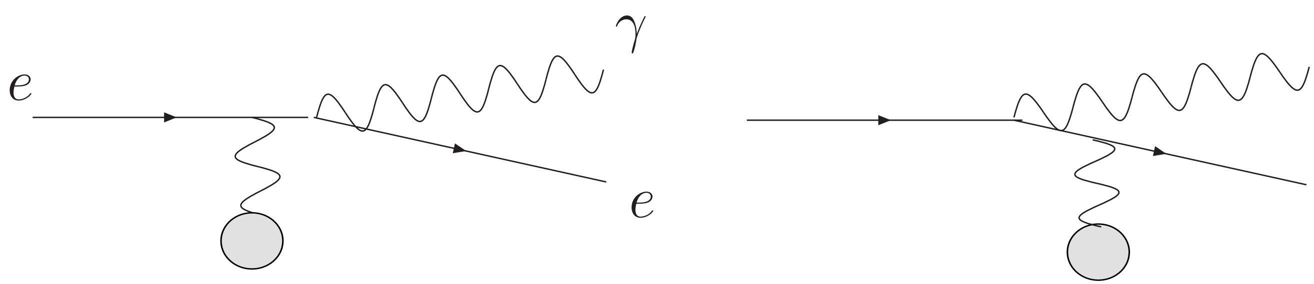
OUTLINE

- Introduction
- Basic concepts of the light-cone path integral approach
- Comparison with the data on the LPM effect for $e \to \gamma e$
- Radiative energy loss in a brick of QGP
- Jet quenching in AA-collisions
- Summary

Formation length for Bethe-Heitler process



$$L_f \sim 1/q_z \sim \frac{2E(1-x)}{m_e^2 x}$$

In 1953 Ter-Mikaelian noted that the photon formation length becomes very large at $E \to \infty$.

For classical electron trajectories in the matrix element $e \to e + \gamma$

$$M \propto \int dt \exp[i(\omega t - \vec{k}\vec{r}(t)]\vec{\epsilon}\vec{v}_e(t)$$

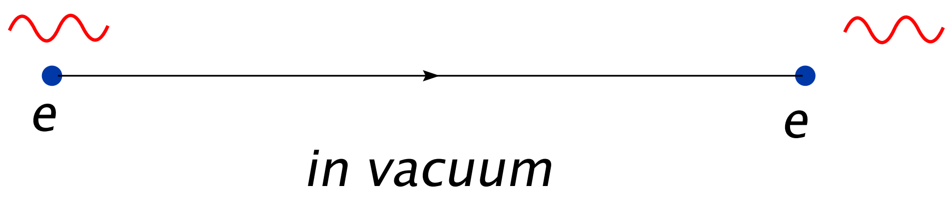
the coherence length is simply the time scale dominating the t-integral $L_f \sim \frac{2E_e^2}{m_e^2\omega}$.

It agrees with quantum estimate $L_f \sim 1/q_z$ from the uncertainty relation $\Delta p \Delta z \gtrsim 1$

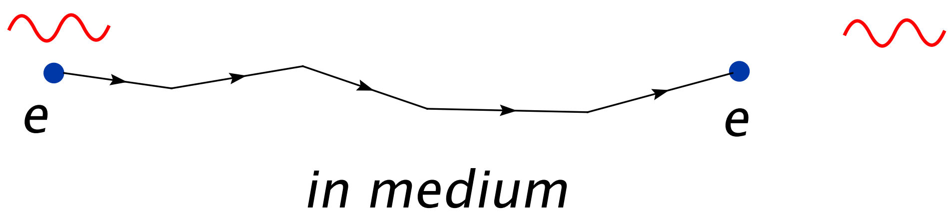
which gives $L_f \sim \frac{2E_e(1-x)}{m_e^2 x^2}$ with $x = \omega/E_e$.

The photon formation length in medium

L_f can be viewed as a longitudinal scale at which the radiated photon and electron become separated by a distance $\Delta z \gtrsim \lambda_\gamma$



$$\Delta z \sim (1 - v_e) L_f \sim \lambda_\gamma$$



In a medium due to multiple scattering $v_e \Rightarrow v_e^{eff} = v_e \langle\langle cos\theta \rangle\rangle < v_e$

$$\Delta z_{med} \sim (1 - v_e^{eff}) L_f^{eff} \sim \lambda_\gamma$$

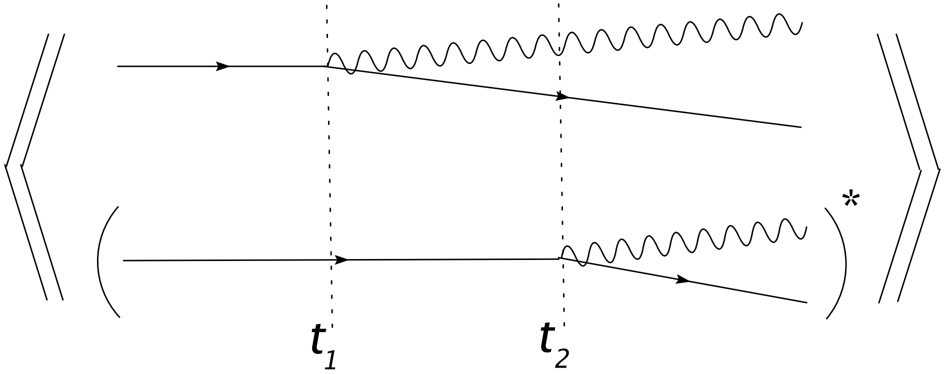
The radiation is suppressed (the LPM effect) when $L_f^{eff} \lesssim L_f$.

The history of the LPM effect

In 1953 Landau and Pomeranchuk within classical approach obtained for the photon spectrum in the regime $L_f^{eff} \ll L_f$

$$\frac{dP}{d\omega} \propto \frac{1}{\sqrt{\omega}}$$

A quantum theory, valid beyond the soft photon approximation, was developed by Migdal in 1956. The old-fashioned PT was used. Migdal's method is based on the evaluation of the electron density matrix in the Fokker-Planck approximation



The LPM effect for $e \to \gamma e$ was qualitatively confirmed in experiment in Protvino at $E_e = 40$ GeV [A.A.Varfolomeev et al. JETP 42, 218 (1975)].
The first accurate measurement (for $\omega \ll E_e$) was performed at SLAC [P.L. Anthony *et al.* Phys. Rev. Lett. 75, 1949 (1995)]. For $\omega \sim E_e$ the effect was studied in 2003 at CERN SPS [[H.D. Hansen *et al.* Phys. Rev. Lett. 91, 014801 (2003)]

Studies in the LPM effect in QCD

- M. Gyulassy and X. N. Wang, Nucl. Phys. B420, 583 (1994); X. N. Wang, M. Gyulassy and M. Plumer, Phys. Rev. D51,3436 (1995).
- R. Baier, Y.L. Dokshitzer, S. Peigné and D. Schiff, Phys. Lett. B345 (1995) 277.
- R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff (BDMPS), Nucl. Phys. B483, 291 (1997); B484, 265 (1997).
- B.G. Zakharov, JETP Lett. 63, 952 (1996); *ibid* 65, 615 (1997); 70, 176 (1999); Phys. Atom. Nucl. 61, 838 (1998).
- R. Baier, Y.L. Dokshitzer, A.H. Mueller, and D. Schiff, Nucl. Phys. B531, 403 (1998).
- P. B. Arnold, G. D. Moore and L. G. Yaffe, JHEP 0206, 030 (2002)

Now there are hundreds of papers on parton energy loss in the hot QCD matter produced

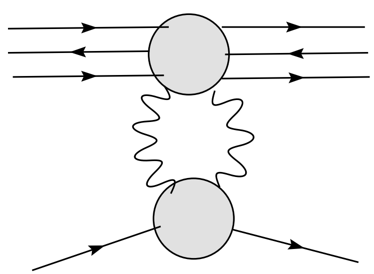
in AA-collisions and its role in the jet quenching phenomenon

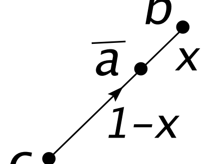
$a \to bc$ induced splitting and LCWF

In general for $a \to bc$ $L_f \sim 2Ex_bx_c/\epsilon^2$, where $\epsilon^2 = m_b^2 x_c + m_c^2 x_b - m_a^2 x_b x_c$.
At $L_f \gg a$ (a is the screening radius, in QED $a \sim r_B/Z^{1/3}$) the transverse coordinates are frozen at the longitudinal scale $\sim a$. It allows to write the cross section for $a \to bc$ in QED and QCD in terms of LCWF which is formed at the scale $\sim L_f$ [Nikolaev, Piller, BGZ (1995)]

$$|a_{phys}\rangle = |a\rangle\sqrt{1-n} + \Psi_a^{bc}(\vec\rho, x)|bc\rangle \ \Rightarrow\ \hat S|a_{phys}\rangle = S_a\{|a\rangle\sqrt{1-n} + S_{bc\bar a}\Psi_a^{bc}(\vec\rho, x)|bc\rangle\}$$

$n = \int d\vec\rho dx|\Psi_a^{bc}(\vec\rho, x)|^2$, $S_{bc\bar a} = S_b S_c S_a^*$,
$P(a \to a) = |\langle a_{phys}|\hat S|a_{phys}\rangle|^2$,
$P(a \to bc) = 1 - P(a \to a)$





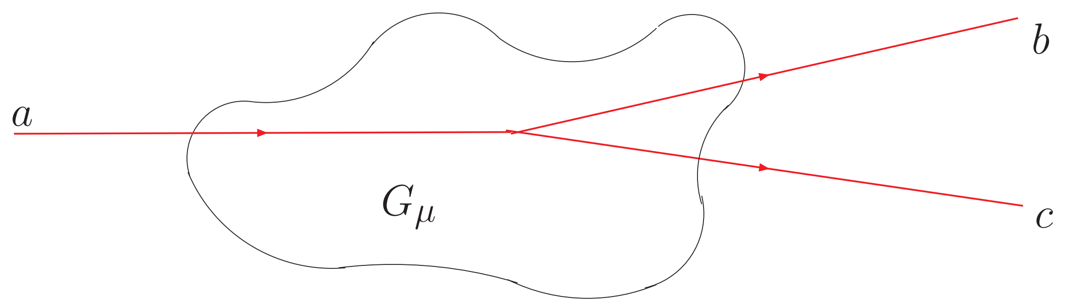
$$\Rightarrow \frac{d\sigma_{a\to bc}^{BH}(x)}{dx} = \int d\vec\rho\,|\Psi_a^{bc*}(\vec\rho, x)|^2 \sigma_{\bar a bc}(\rho, x)$$

$$|\Psi_a^{bc*}(\vec\rho, x)|^2 \propto \epsilon^2 K_1^2(\rho\epsilon) \quad \text{and} \quad \sigma_{\bar a bc}(\rho, x) \propto \rho^2 \log(a/\rho)$$

$$\Rightarrow \quad d\sigma/dx \propto P_{ba}(x)/\epsilon^2$$

for spin non-flip part of $\sigma(e \to \gamma e) \Rightarrow \left(\frac{d\sigma}{dx}\right)_{nf}^{BH} \approx \frac{4\alpha^3 Z^2[4 - 4x + 2x^2]}{3m_e^2 x} \log(2am_e)$

Basics concepts of the LCPI approach



We consider processes like $a \to bc$ in an external vector field (in QCD or QED). The S-matrix element is written in standard form in terms of incoming and outgoing WFs. For $q \to gq'$ it reads

$$\langle gq'|\hat{S}|q\rangle = -ig\int dy\bar{\psi}_{q'}(y)\gamma^{\mu}A^{*}_{\mu}(y)\psi_q(y)\,.$$

The quark wave function is written in the form

$$\psi_i(y) = \exp[-iE_i(t-z)]\hat{u}_{\lambda}\phi_i(z,\vec{\rho})/\sqrt{2E_i},$$

λ is quark helicity, $\hat{u}_\lambda$ is the Dirac spinor operator. The gluon wave function is written in a similar way. The z-dependence of the transverse wave functions ϕ_i is governed by the two-dimensional Schrödinger equation

$$i\frac{\partial\phi_i(z,\vec{\rho})}{\partial z} = \Big\{\frac{(\vec{p}-g\vec{G})^2+m_q^2}{2\mu_i} + g(G^0-G^3)\Big\}\phi_i(z,\vec{\rho})\,,$$

G is the external vector potential (the color indexes are omitted), $\mu_i = E_i$.

The transverse part $\vec{G}$ can be ignored for gauges with potential vanishing at large distances. Thus we have a Schrödinger equation with “time”-dependent potential $U = g(G^0 - G^3)$.
The z-evolution of ϕ_i can be written in terms of the Green's function as

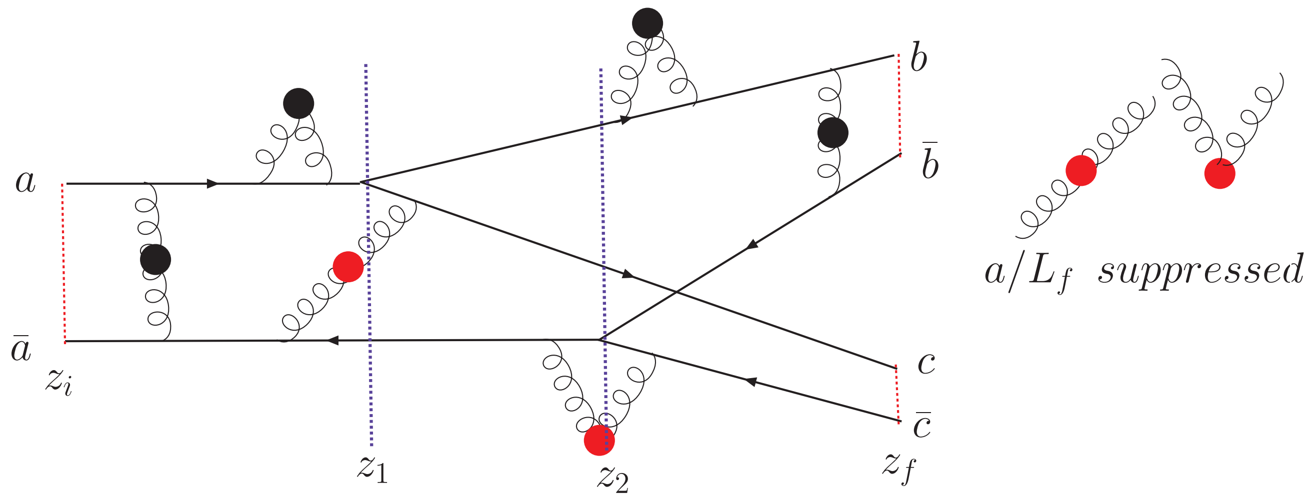
$$\phi_i(z_2, \vec{\rho}_2) = \int d\vec{\rho}_1 K_i(\vec{\rho}_2, z_2|\vec{\rho}_1, z_1)\phi_i(z_1, \vec{\rho}_1)$$

$\Rightarrow$ One can write the cross section in terms of the initial ($z = z_i$) and final ($z = z_f$) transverse density matrices and the Green's functions K and K^*.
The Green's functions are written in the path integral form

$$K_i(\vec{\rho}_2, z_2|\vec{\rho}_1, z_1) = \int D\vec{\rho}\,\exp\left\{i\int dz\left[\frac{\mu_i(d\vec{\rho}/dz)^2}{2} - U(\vec{\rho}, z)\right] - \frac{im_i^2(z_2 - z_1)}{2\mu_i}\right\}$$

In calculation of the gluon spectrum $\propto \langle\langle|\langle q'g|M|q\rangle|^2\rangle\rangle$ the averaging over the medium states $\langle\langle\ \rangle\rangle$ is performed before the path integration. In vacuum

$$K_i^{vac}(\vec{\rho}_2, z_2|\vec{\rho}_1, z_1) = \frac{\mu_i}{2\pi i(z_2 - z_1)}\exp\left[\frac{i\mu_i(\vec{\rho}_2 - \vec{\rho}_1)^2}{2(z_2 - z_1)} - \frac{im_i^2(z_2 - z_1)}{2\mu_i}\right]$$

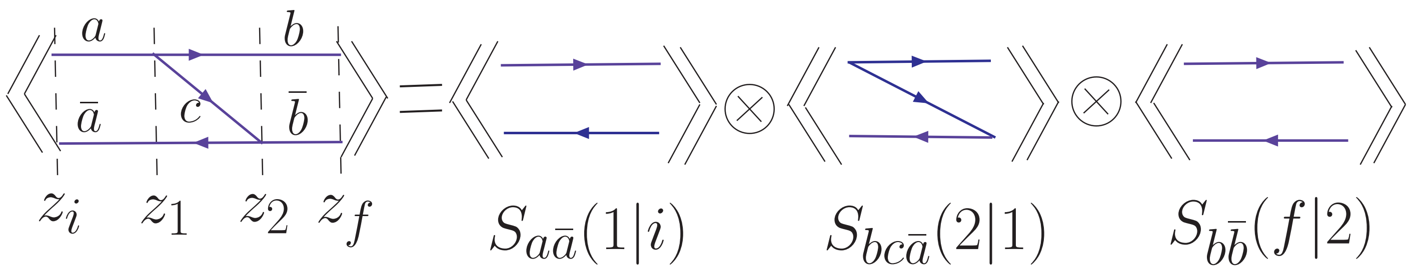


$\rightarrow$ lines (K) interact with the medium as "particle" and $\leftarrow$ (K^*) as "antiparticle". $\langle\langle\rangle\rangle$ generates an interaction between trajectories. This interaction is local in z at $L_f \gg$ the correlation radius in the medium. The effective Lagrangian for the path integral reads

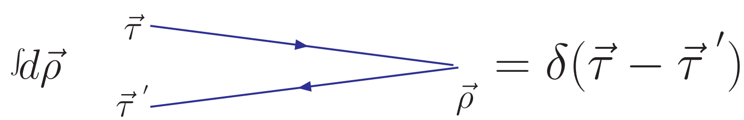
$$L_{eff} = L_0^p(\dot{\vec{\tau}}_p) - L_0^{\bar{p}}(\dot{\vec{\tau}}_{\bar{p}}) + L_{int}(\vec{\tau}_p, \vec{\tau}_{\bar{p}}), \quad L_0^p(\dot{\vec{\tau}}_p) = \sum_i \frac{\mu_i \dot{\vec{\rho}}_i^2}{2}, \quad L_{int}(\vec{\tau}_p, \vec{\tau}_{\bar{p}}) = \frac{in(z)\sigma_X(\vec{\tau}_p, \vec{\tau}_{\bar{p}})}{2}$$

$\vec{\tau}_p$, $\vec{\tau}_{\bar{p}}$ are the sets of the transverse coordinates for "particles" and "antiparticles", σ_X is the diffractive operator for $X =$ "particles"+"antiparticles" system scattering off a particle in medium, $n(z)$ is the number density of the medium. $L_{int}(\vec{\tau}_p, \vec{\tau}_{\bar{p}}) = L_{int}(\vec{\tau}_p - \vec{\tau}_{\bar{p}})$

In QED σ_X is a scalar, but in QCD for the 4-body part ($z_2 < z < z_f$) σ_X is an operator in color space. The spectrum integrated over $\vec{q}_c$ does not contain the 4-body part at all.



For this transformation we have used the relation (valid in vacuum and medium)



The spectrum reads [BGZ (1999)]

$$\frac{dP}{dx\,d\vec{q}_b} = 2\mathrm{Re}\int_{z_i}^{z_1} dz_1 \int_{z_1}^{z_f} dz_2 \hat{g}\langle\rho_f|\hat{S}_{b\bar{b}} \otimes \hat{S}_{bc\bar{a}} \otimes \hat{S}_{a\bar{a}}|\rho_i\rangle,$$

$\hat{g}$ is the vertex factor, $\hat{S}_{a\bar{a}}$, $\hat{S}_{b\bar{b}}$, $\hat{S}_{bc\bar{c}}$ are the evolution operators for the corresponding L_{eff}. The two-body parts can be evaluated analytically [BGZ (1987)]

$$\boxed{\langle\vec{\rho}\,'_a, \vec{\rho}\,'_{\bar{a}}, z'|\hat{S}_{a\bar{a}}|\vec{\rho}_a, \vec{\rho}_{\bar{a}}, z\rangle = K_a(\vec{\rho}\,'_a, z'|\vec{\rho}_a, z)K^*_{\bar{a}}(\vec{\rho}\,'_{\bar{a}}, z'|\vec{\rho}_{\bar{a}}, z)\Phi_{a\bar{a}}(\vec{\rho}\,'_a - \vec{\rho}\,'_{\bar{a}}, z'|\vec{\rho}_a - \vec{\rho}_{\bar{a}}, z)}$$

The interaction phase factor $\Phi_{a\bar{a}}{=}\exp\left[-\frac{1}{2}\int_z^{z'} dz n(z)\sigma_{a\bar{a}}(\vec{\rho}_a(z)-\vec{\rho}_{\bar{a}}(z))\right]$ should be evaluated for the straight trajectories of a and $\bar{a}$.
For the $bc\bar{a}$-part $\int D\vec{\rho}_b D\vec{\rho}_c D\vec{\rho}_{\bar{a}} = \int D\vec{\rho} D\vec{\rho}_a D\vec{\rho}_{\bar{a}}$, where $\vec{\rho} = \vec{\rho}_b - \vec{\rho}_c$, $\vec{\rho}_a = x_b\vec{\rho}_b + x_c\vec{\rho}_c$ is the center-of-mass coordinate of the bc system. The $\int D\vec{\rho}_a D\vec{\rho}_{\bar{a}}$ can be taken analytically. In the new variables $S_{bc\bar{a}}$ reads

$$\boxed{\langle\vec{\rho}_a^{\,\prime},\vec{\rho}_{\bar{a}}^{\,\prime},\vec{\rho}^{\,\prime},z'|\hat{S}|\vec{\rho}_a,\vec{\rho}_{\bar{a}},\vec{\rho},z\rangle = K_a(\vec{\rho}_a^{\,\prime},z'|\vec{\rho}_a,z)K^*_{\bar{a}}(\vec{\rho}_{\bar{a}}^{\,\prime},z'|\vec{\rho}_{\bar{a}},z)\mathcal{K}(\vec{\rho}^{\,\prime},z'|\vec{\rho},z)}$$

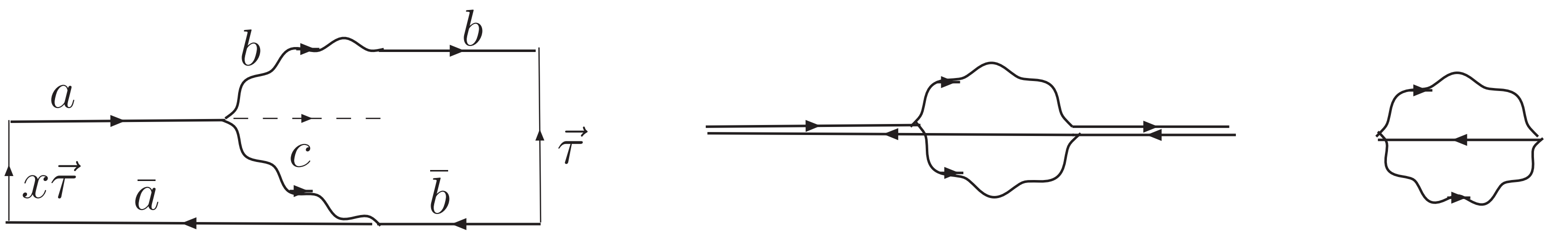
$\mathcal{K}$ is the Green's function for the Hamiltonian

$$\hat{H} = -\frac{1}{2\mu(x)}\left(\frac{\partial}{\partial\vec{\rho}}\right)^2 + v(z,\vec{\rho}) + \frac{1}{L_f}\,, \quad v(z,\vec{\rho}) = -in(z)\sigma_{bc\bar{a}}(\vec{\rho},\vec{\rho}_a-\vec{\rho}_{\bar{a}})/2$$

In this Hamiltonian the trajectories $\vec{\rho}_a$, $\vec{\rho}_{\bar{a}}$ are straight, $\mu(x) = E_a x(1-x)$,

$L_f = 2\mu(x)/\epsilon^2$ is the formation length

$\epsilon^2 = [m_b^2 x_c + m_c^2 x_b - m_a^2 x_b x_c]$.
The potential is not central. The integrals over the center of mass of the end points can be taken analytically. It makes the trajectories a and $\bar{a}$, b and $\bar{b}$ parallel

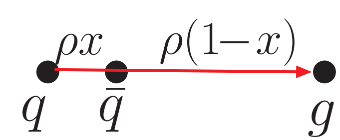


$$\frac{dP}{dx\,d\vec{q}_b} = \frac{2}{(2\pi)^2}\mathrm{Re}\int d\vec{\tau}\exp(-i\vec{q}_b\cdot\vec{\tau})\int_{z_i}^{z_f}dz_1\int_{z_1}^{z_f}dz_2\,\hat{g}\,\Phi_f(\vec{\tau},z_2)\mathcal{K}(\vec{\tau},z_2|0,z_1)\Phi_i(x\vec{\tau},z_1),$$

$$\Phi_i(\vec{\tau},z_1)=\exp\left[-\frac{\sigma_{a\bar{a}}(\vec{\tau})}{2}\int_{z_i}^{z_1}dz\,n(z)\right],\quad \Phi_f(\vec{\tau},z_2)=\exp\left[-\frac{\sigma_{b\bar{b}}(\vec{\tau})}{2}\int_{z_2}^{z_f}dz\,n(z)\right].$$

For x-spectrum the potential v is central (since $\bar{a}$ is at the center of mass of bc. The spectrum reads [BGZ (1996)]

$$\boxed{\frac{dP}{dx}=2\mathrm{Re}\int_{z_i}^{z_f}dz_1\int_{z_1}^{z_f}dz_2\,\hat{g}\left[\mathcal{K}(\vec{\rho}_2,z_2|\vec{\rho}_1,z_1)-\mathcal{K}_v(\vec{\rho}_2,z_2|\vec{\rho}_1,z_1)\right]\Big|_{\vec{\rho}_1=\vec{\rho}_2=0}.}$$

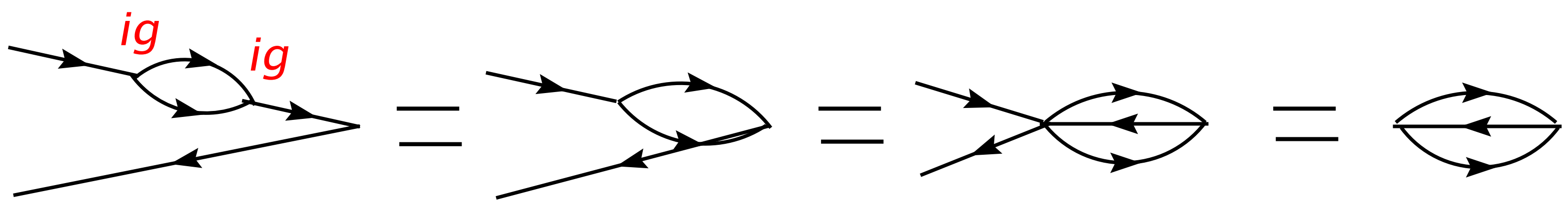
Here $\hat{g}=\frac{\alpha_s P_{ba}(x)}{2\mu^2(x)}\frac{\partial}{\partial\vec{\rho}_2}\cdot\frac{\partial}{\partial\vec{\rho}_1}$. For $q\to gq$ we have configuration  $\Rightarrow$

$q\bar{q}g\approx|88\rangle$ at $x\to 0$, $q\bar{q}g\approx|\bar{3}3\rangle$ at $x\to 1$.

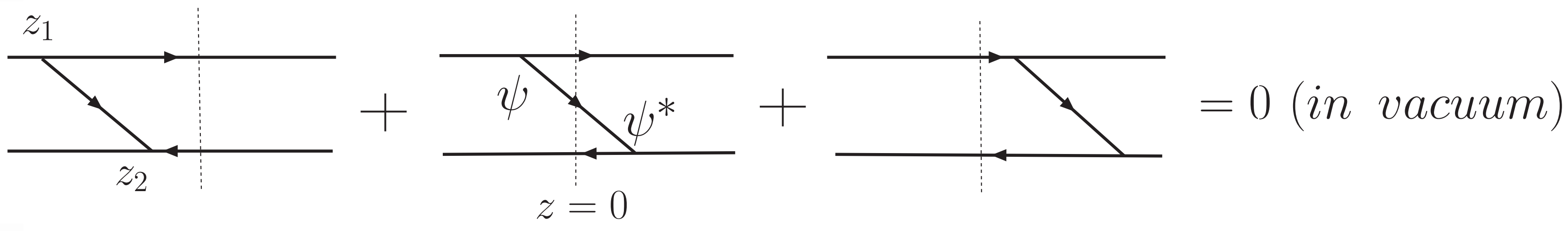
In QED and QCD the three-body cross section can be written in terms of the dipole cross sections $\sigma_2 = \sigma_{e\bar{e}}$ and $\sigma_2 = \sigma_{q\bar{q}}$. $\sigma_{\gamma e\bar{e}}(\rho) = \sigma_{e\bar{e}}(x\rho)$ and

$$\sigma_{gq\bar{q}}(\rho, x, z) = \frac{9}{8}[\sigma_2(\rho, z) + \sigma_2((1-x)\rho, z)] - \frac{1}{8}\sigma_2(x\rho, z)\,.$$ [Nikolaev, BGZ (1994)]

For $a \to a\ (ig)(ig)^* \to (ig)^2$. $\Rightarrow$ the unitarity $P(a \to a) + P(a \to bc) = 1$ is satisfied



For the $x, \vec{q}_b$ spectrum the contribution from $|z_{1,2}| \to \infty$ can be expressed via the LCWF of the $a \to bc$ transition in vacuum, Ψ_a^{bc} (we use the adiabatically switched off coupling)



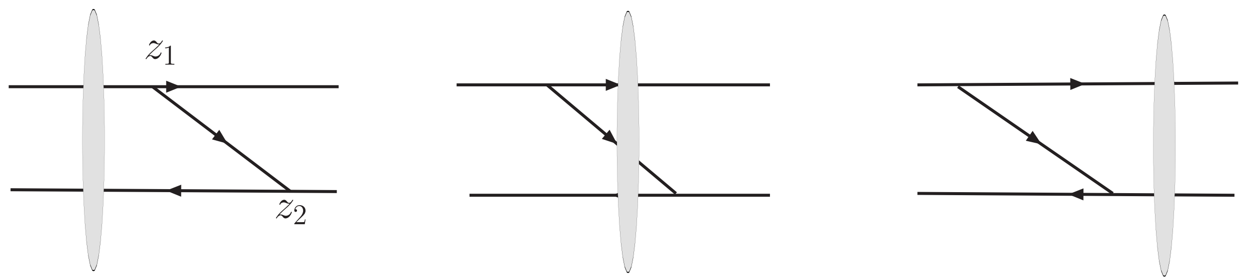
Formulas for $x, \vec{q}_b$ spectrum

$$\frac{dP}{dx d\vec{q}_b} = \frac{2}{(2\pi)^2} \mathrm{Re} \int d\vec{\tau} \exp(-i\vec{q}_b\vec{\tau}) \int_{z_i}^{z_f} dz_1 \int_{z_1}^{z_f} dz_2 \hat{g} \Big\{ \Phi_f(\vec{\tau}, z_2)[\mathcal{K}(\vec{\tau}, z_2|0, z_1)$$

$$-\mathcal{K}_v(\vec{\tau}, z_2|0, z_1)]\Phi_i(x\vec{\tau}, z_1) + [\Phi_f(\vec{\tau}, z_2) - 1]\mathcal{K}_v(\vec{\tau}, z_2|0, z_1)[\Phi_i(x\vec{\tau}, z_1) - 1] \Big\}$$

$$+ \frac{1}{(2\pi)^2} \int d\vec{\tau} d\vec{\tau}' \exp(-i\vec{q}_b\vec{\tau}) \Psi_a^{bc*}(x, \vec{\tau}' - \vec{\tau}) \Psi_a^{bc}(x, \vec{\tau}') \left[\Phi_f(\vec{\tau}, z_i) + \Phi_i(x\vec{\tau}, z_f) - 2 \right],$$

Here $z_{1,2}$ integrations comes only in the matter. For $L_f \gg L$ we have the picture



scattering at $L_f \gg L$

$$\frac{dP}{dx d\vec{q}_b} = \frac{1}{(2\pi)^2} \mathrm{Re} \int d\vec{\tau} d\vec{\tau}' \exp(-i\vec{q}_b\vec{\tau}) \Psi_a^{bc*}(x, \vec{\tau}' - \vec{\tau}) \Psi_a^{bc}(x, \vec{\tau}') \left[2\Gamma_{bc\bar{a}}(\vec{\tau}', x\vec{\tau}) - \Gamma_{b\bar{b}}(\vec{\tau}) - \Gamma_{a\bar{a}}(x\vec{\tau}) \right],$$

$\Gamma_h = 1 - \exp[-\frac{\sigma_h}{2} \int_{-\infty}^{\infty} dz n(z)]$. For $q \to gq$ at $x \ll 1$ it gives the Kovchegov-Mueller spectrum in pA-collisions.

The $x, \vec{q}_b$ spectrum, parton produced in QGP

In the case of a $a \to bc$ for a incoming from $-\infty$ the radiation rate vanishes for a zero external potential. But for a fast parton produced in a hard reaction $a \to bc$ splitting is possible even without external potential. It is the well known LO DGLAP parton showering. In this case the spectrum can be written as (now z_i is the production point)

$$\frac{dP}{dx d\vec{q}_b} = \frac{2}{(2\pi)^2} \mathrm{Re} \int d\vec{\tau} \exp(-i\vec{q}_b\vec{\tau}) \int_{z_i}^{z_f} dz_1 \int_{z_1}^{z_f} dz_2 \hat{g} \Big\{ \Phi_f(\vec{\tau}, z_2)[\mathcal{K}(\vec{\tau}, z_2|0, z_1)$$

$$-\mathcal{K}_v(\vec{\tau}, z_2|0, z_1)]\Phi_i(x\vec{\tau}, z_1) + [\Phi_f(\vec{\tau}, z_2) - 1]\mathcal{K}_v(\vec{\tau}, z_2|0, z_1)\Phi_i(x\vec{\tau}, z_1)$$

$$+\mathcal{K}_v(\vec{\tau}, z_2|0, z_1)\Phi_i(x\vec{\tau}, z_1) \Big\}$$

The $\mathcal{K}_v\Phi_i$ which is $0 \times \infty$ can be written via the LC wave function, then

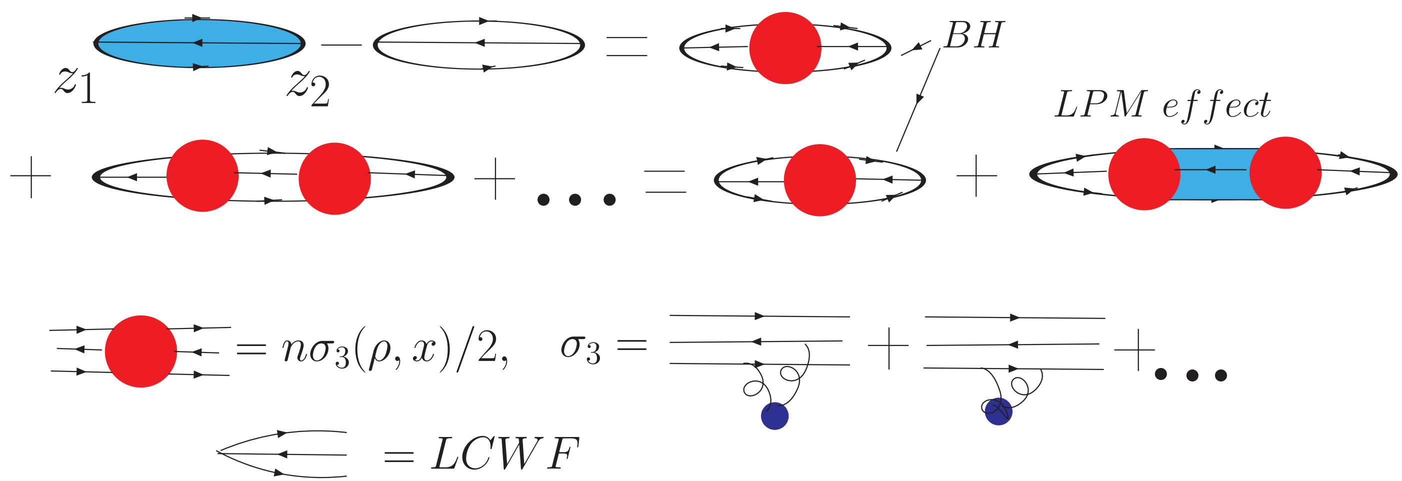
$$\frac{dP}{dx d\vec{q}_b} = \frac{2}{(2\pi)^2} \mathrm{Re} \int d\vec{\tau} \exp(-i\vec{q}_b\vec{\tau}) \int_{z_i}^{z_f} dz_1 \int_{z_1}^{z_f} dz_2 \hat{g} \Big\{ \Phi_f(\vec{\tau}, z_2)[\mathcal{K}(\vec{\tau}, z_2|0, z_1)$$

$$-\mathcal{K}_v(\vec{\tau}, z_2|0, z_1)]\Phi_i(x\vec{\tau}, z_1) + [\Phi_f(\vec{\tau}, z_2) - 1]\mathcal{K}_v(\vec{\tau}, z_2|0, z_1)\Phi_i(x\vec{\tau}, z_1) \Big\}$$

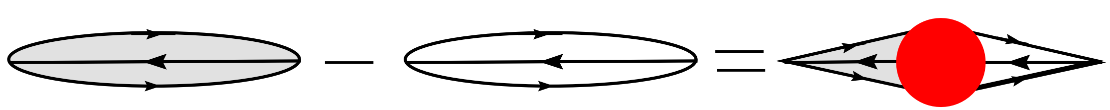
$$+ \boxed{\frac{1}{(2\pi)^2} \int d\vec{\tau} d\vec{\tau}' \exp(-i\vec{q}_b\vec{\tau}) \Psi_a^{bc*}(x, \vec{\tau}' - \vec{\tau}) \Psi_a^{bc}(x, \vec{\tau}') \Phi_i(x\vec{\tau}, z_f)},$$

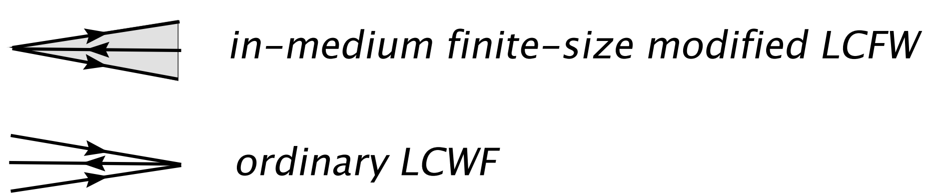
It describes DGLAP (red) and induced (blue) splitting with p_T broadening

Other representations for the x-spectrum



The LPM effects is equivalent to absorptive correction for scattering of $bc\bar{a}$ system. This form has been used for successful description of the SLAC and SPS data on the LPM effect in photon bremsstrahlung from high energy electrons [BGZ (1996,1998,2003)].
dP/dx can also be written in terms of the $bc\bar{a}$ medium and finite-size modified LCWF





This form is convenient for parton produced in a medium.

Dynamical effects in HTL method

The LCPI method applies also to the dynamical pQCD weakly coupled QGP.

$$\frac{n\sigma_{q\bar{q}}(\rho)}{2} \Rightarrow P(\vec{\rho})\,,$$

$$P(\vec{\rho}) = g^2 C_F \int\limits_{-\infty}^{\infty} dz [G(z, 0_\perp z) - G(z, \vec{\rho}, z)]\,, \quad G(x-y) = u_\mu u_\nu \langle\langle A^\mu(x) A^\nu(y) \rangle\rangle$$ [QED BGZ (1987)]

$$P(\vec{\rho}) = \int \frac{d\vec{q}}{(2\pi)^2} [1 - \exp(i\vec{\rho}\vec{q})] P(\vec{q})\,.$$

$P(\vec{q})$ can be written in terms of the HTL polarization operator $\Pi\mu\nu$.

$$P(\vec{q}) \approx g^2 C_F T C(\vec{q})\,, \quad C(\vec{q}) = \frac{m_D^2}{\vec{q}^2(\vec{q}^2 + m_D^2)}$$ [Aurenche, Gelis, Zaraket (2000)]

In LCPI approach we reproduce all AMY [Arnold, Moore, Yaffe (2001,2002)] results on photon emission [Aurenche, BGZ (2007)]. the pole $1/q^2$ (due to zero magnetic mass) changes the effective dipole cross section at $\rho \gtrsim 1/m_D$, at small ρ it is the same as in static model.

x-spectrum in OA for L=∞

In QED and QCD $\sigma_2(\rho) = C_2(\rho)\rho^2$, $C_2(\rho)$ is a smooth function. For $q\bar{q}$

$$C_2(\rho) = \frac{C_T C_F \alpha_s^2}{\rho^2} \int d\vec{q} \frac{[1-\exp(i\vec{q}\,\vec{\rho})]}{(\vec{q}^{\,2}+m_D^2)^2}, \quad C_2(\rho) \approx \frac{C_F C_T \alpha_s \pi^2}{2} \log\left(\frac{1}{\rho m_D}\right) \quad \text{at} \rho \ll 1/m_D$$

The Hamiltonian takes the oscillator form if one neglects ρ-dependence of $C_2(\rho)$ and replaces it by $C_2(\rho_{eff})$. The oscillator frequency reads

$$\Omega(z) = \frac{(1-i)}{\sqrt{2}} \left(\frac{n(z)C_3(x,z)}{E_q x(1-x)}\right)^{1/2},$$

$C_3(x,z) = \frac{1}{8}\left\{9[1+(1-x)^2]-x^2\right\}C_2$. The OA is used in BDMPS calculations (for massless partons), $C_2 = \hat{q}C_F/2nC_A$ [Baier, Dokshitzer, Mueller, Peigne, Schiff (1997)]. In BDMPS $\langle p_T^2\rangle = \hat{q}L$ for gluon. In QED for $\gamma e\bar{e}$ $C_3 = x^2 C_2$.

The OA corresponds to the Fokker-Planck approximation in Migdal's approach. In OA $\langle p_T^2\rangle = 2C_2 nL$. The oscillator approximation is clearly not good for $q \to gq$ in the BH regime when $\rho_{eff} \sim 1/\epsilon$. One could expect that the OA should be applicable when $\rho_{eff} \ll 1/\epsilon$. In reality this is not the case when $L \lesssim L_f$.

$$\frac{dP}{dx dL} = n \frac{d\sigma_{OA}^{BH}}{dx} S_{LPM}(\eta) \,, \quad \frac{d\sigma_{OA}^{BH}}{dx} = \frac{2\alpha_s P_{Gq}(x) C_3(x)}{3\pi\epsilon^2} \,, \quad P_{Gq} = \frac{C_F[1+(1-x)^2]}{x}$$

$$\eta = L_f|\Omega| = \frac{2\sqrt{nEx(1-x)C_3(x)}}{\epsilon^2} \,,$$ $\eta \ll 1$ BH regime, $\eta \gg 1$ strong LPM effect.

$$S_{LPM}(\eta) = \frac{3}{\eta\sqrt{2}} \int_0^\infty dy \left(\frac{1}{y^2} - \frac{1}{\mathrm{sh}^2 y}\right) \exp\left(-\frac{y}{\eta\sqrt{2}}\right) \left[\cos\left(\frac{y}{\eta\sqrt{2}}\right) + \sin\left(\frac{y}{\eta\sqrt{2}}\right)\right] .$$

For $e \to \gamma e$ $C_F \to 1$. At $\eta \ll 1$ $S_{LPM}(\eta) \simeq 1 - 16\eta^4/21$. For $\eta \gg 1$ $S_{LPM}(\eta) \approx \frac{3}{\eta\sqrt{2}}\left(1 - \frac{\pi}{\eta 2\sqrt{2}}\right)$. The leading term gives

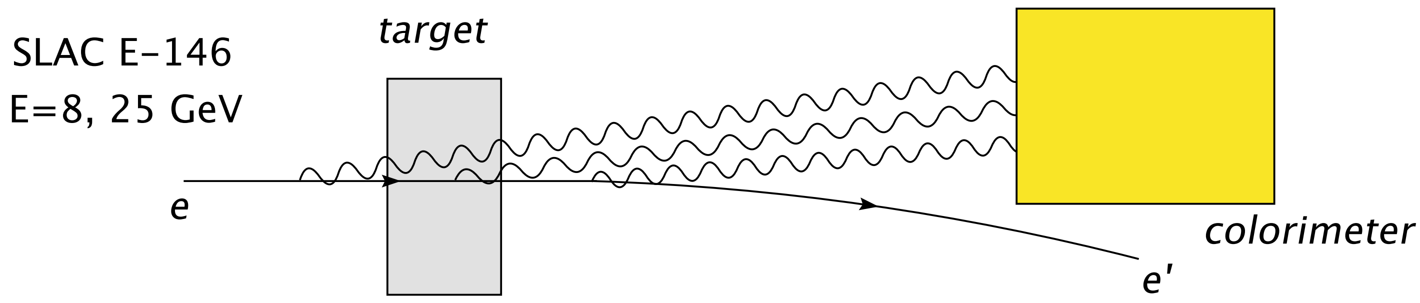
$$\frac{dP}{dx dL} \approx \frac{\alpha_s P_{Gq}(x)}{\pi} \sqrt{\frac{nC_3(x)}{2Ex(1-x)}} \propto x^{-3/2}$$ at $x \ll 1$ [BGZ (1996)].

The factor $\left(1 - \frac{\pi}{\eta 2\sqrt{2}}\right)$ gives the heavy-to-light K-factor (strong LPM regime)

$$K \approx 1 - \frac{\pi}{2\sqrt{2}} \frac{(M_Q^2 - m_q^2)x^{3/2}}{\sqrt{2E(1-x)nC_3(x)}}$$ $\left(K_{DK} = \left[1 + \frac{M_Q^2 x^{3/2}}{\sqrt{18EnC_2/4}}\right]^{-2} \right.$ Dokshitzer, Kharzeev (2001) $\Big)$

Comparison with SLAC and SPS data on $e \to \gamma e$

Experimentally the total electron energy loss $\Delta E_e = \omega_1 + \omega_2 + \ldots$ is measured.



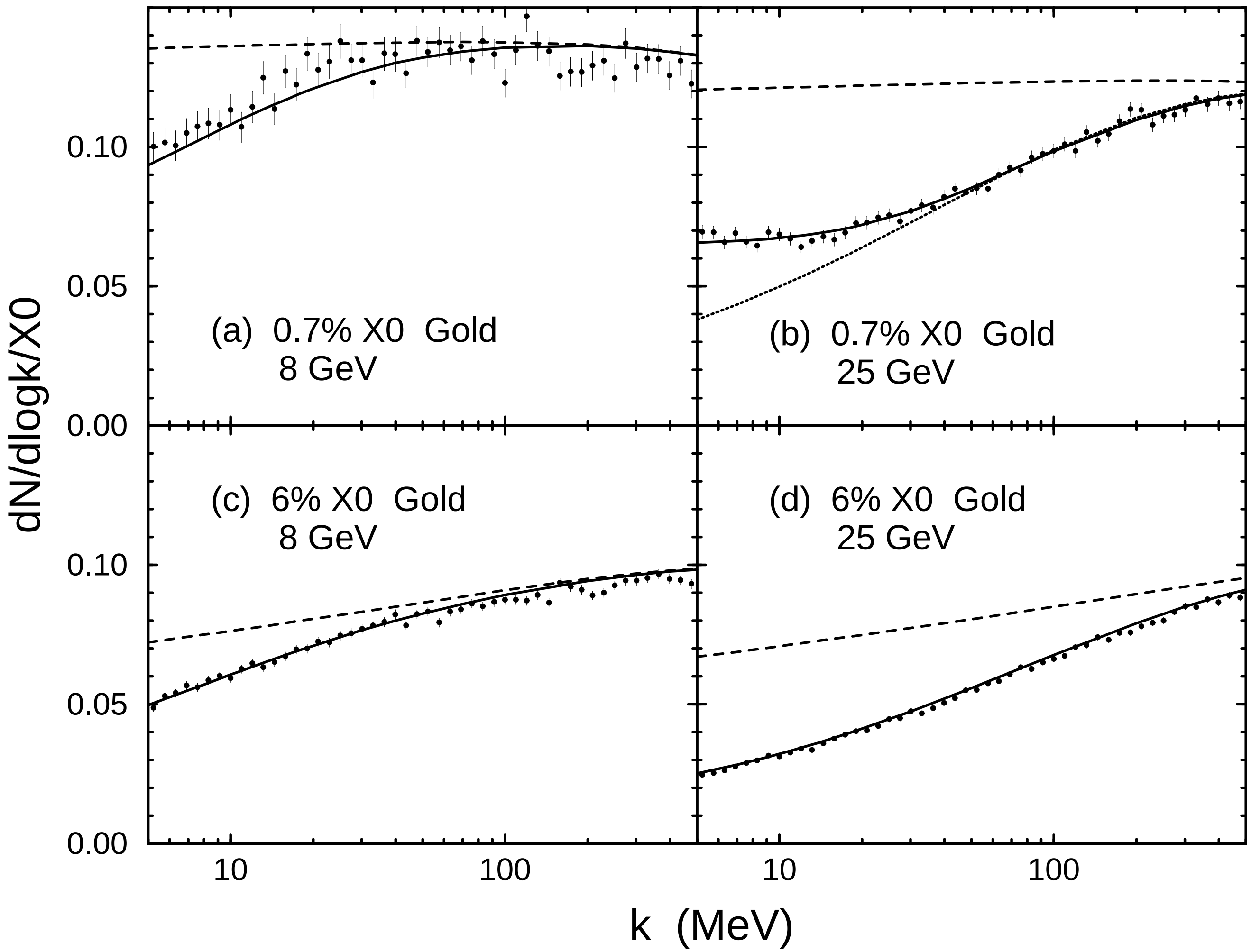
For SLAC and SPS experiments $L_f^{eff} \ll L$. The finite-size effects are important for the gold target $L = 0.7\% X_0$ at $E_e = 25$ GeV, where the multi-photon effects are negligible.
We evaluated the multi-photon K-factor in the probabilistic approach (dilute loop gas approximation)

$$K(x) \approx \exp\left[-\int_x^1 dx_1 \frac{dP_\gamma}{dx_1}\right]\left\{1 - \frac{1}{2}\int_0^x dx_1 \left[\frac{dP_\gamma}{dx_1} + \frac{dP_\gamma}{dx_2} - \frac{dP_\gamma}{dx_1}\frac{dP_\gamma}{dx_2}\left(\frac{dP_\gamma(x)}{dx}\right)^{-1}\right]\right\}, \; x_2 = x - x_1$$

Here $x \ll 1$. For SPS we use $K(x)$ valid at any x. The inaccuracy $\lesssim 0.5\%$.

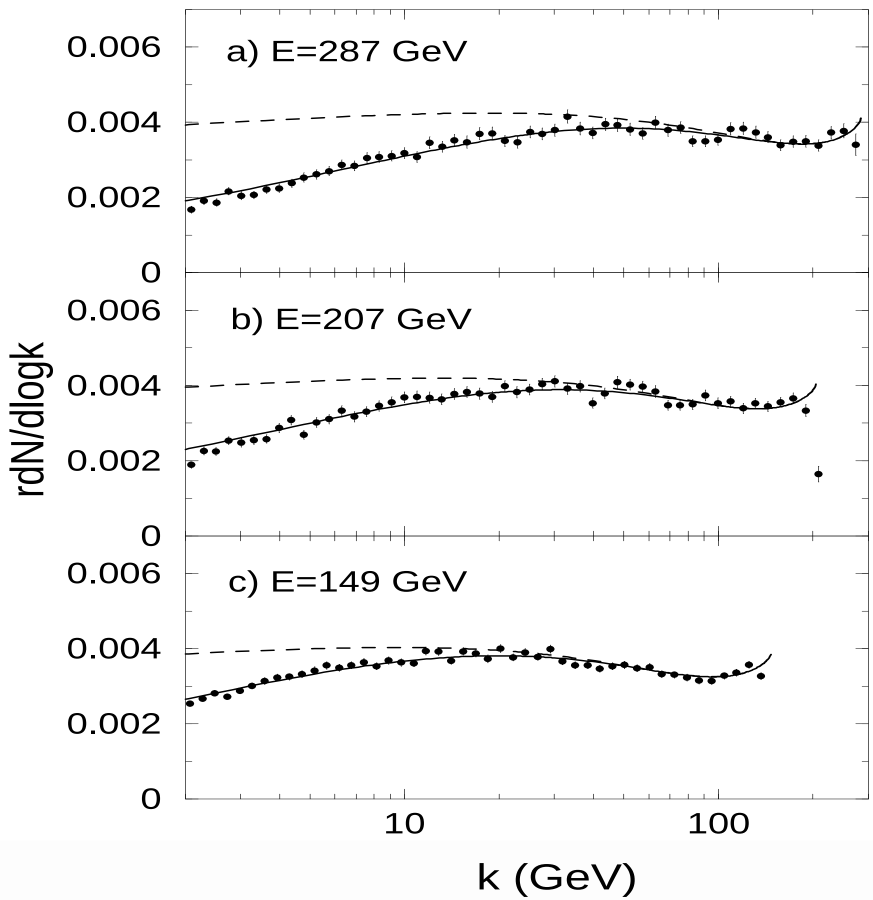
Comparison with SLAC E-146 data

The spectra in the radiated energy. The dashed line shows the Bethe-Heitler spectrum, the dotted line our calculations without finite-size effects.



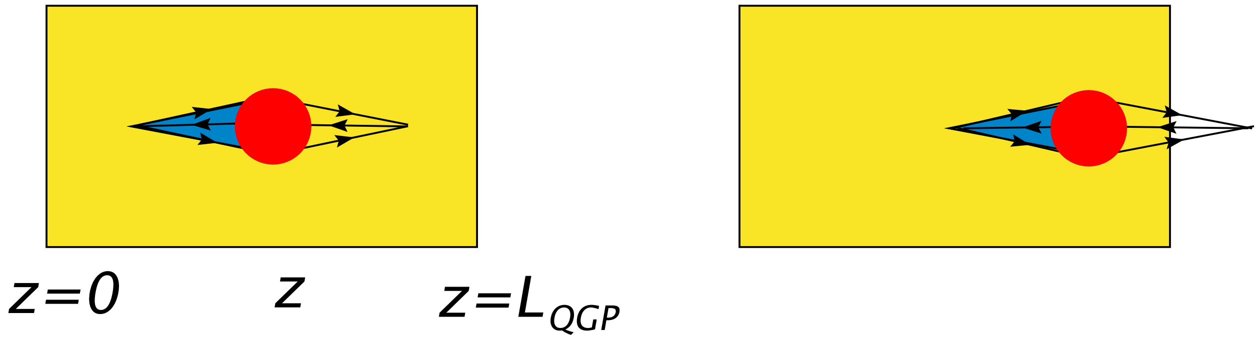
Comparison with the CERN SPS data

The spectra in the radiated energy on $4.36\%\,X_0$ Ir target. The dashed line shows the Bethe-Heitler spectrum. $r = 2\tanh(\Delta/2)$, $\Delta = \log(10)/25$.



Parton produced in a brick of QGP

For a fast parton produced in a medium $z_2 > z_1 > z_i = 0$. Expanding the spectrum over density for the induced part we have



The spectrum can be written via an effective BH cross section

$$\frac{dP}{dx} = \int_0^L dz\, n(z) \frac{d\sigma^{BH}_{eff}(x,z)}{dx}\,, \qquad \frac{d\sigma^{BH}_{eff}(x,z)}{dx} = \text{Re} \int d\vec{\rho}\, \psi^*(\vec{\rho},x)\sigma_3(\rho,x,z)\psi_m(\vec{\rho},x,z)\,,$$

where $\psi(\vec{\rho},x)$ is the LCWF for the $q \to qg$ transition in vacuum, and $\psi_m(\vec{\rho},x,z)$ is the in-medium finite-size modified LCWF for $q \to qg$ transition in medium at the longitudinal coordinate z. At $n \to 0$ and $z \to \infty$ $\psi_m(\vec{\rho},x,z) = \psi(\vec{\rho},x)$, and $\sigma^{BH}_{eff} = \sigma^{BH}$. At $z \to 0$ $\frac{d\sigma^{BH}_{eff}(x,z)}{dx} \Big/ \frac{d\sigma^{BH}(x)}{dx} \propto z$. This is a direct consequence of the Schrödinger diffusion relation $\rho^2 \sim z/\mu$ for the transverse size of the qg Fock component of the quark produced at $z = 0$. This effect is responsible for the L^2-dependence of ΔE_q at $E_q \to \infty$. In QED the situations is the same.

Qualitative pattern of induced gluon emission

The Schrödinger diffusion relation $\rho^2 \sim l/\mu$ relates the typical transverse and longitudinal scales. In an infinite medium

$$\rho_{eff}^{\infty} \sim \min(\rho_{BH}, \rho_{LPM}) \quad l_{eff}^{\infty} \sim (\rho_{eff}^{\infty})^2 \mu$$

$\rho_{BH} = 1/\epsilon$, $l_{BH} = 2Ex(1-x)/\epsilon^2$ (BH regime, weak LPM effect)
$\rho_{LPM} = [Ex(1-x)nC_3]^{-1/4}$, $l_{LPM} = \sqrt{Ex(1-x)/nC_3}$ (strong LPM effect).

For a finite-size matter two situations are possible.

- Infinite medium regime: $L \gtrsim l_f^{\infty}$, and $\rho_{eff} \sim \rho_{eff}^{\infty}$. The spectrum can roughly be calculated using the effective BH cross section for an infinite medium. There can exist a region with strong LPM effect (number of rescatterings $N \gg 1$) with $dP/dx \propto x^{-3/2}$. But at $x \to 0, 1$ we always have the BH regime ($N = 1$ dominates).
- Diffusion regime: $L \lesssim l_f^{\infty}$ $\rho_{eff} \sim \rho_d(L)$ ($\rho_d(L) = \sqrt{L/2\mu}$ is the diffusion radius). The effective BH cross section is chiefly controlled by the finite-size effects. The $N = 1$ term dominates, the LPM suppression is small.

Failure of OA in diffusion regime

For massless partons the OA for finite medium gives [BDMPS (1997)]

$$\frac{dP_{BDMPS}}{dx} = \frac{\alpha_s P_{Gq}(x)}{\pi} \log|\cos \Omega L|$$

$$\Rightarrow \quad \frac{dP_{BDMPS}}{dx} \approx \frac{\alpha_s P_{Gq}(x)}{16\pi} \frac{L^4 C_3^2 n^2}{[x(1-x)E]^2}$$ $\Rightarrow$ It is $N=2$ rescattering! [BGZ, (2001)]

The mass effects gives nonzero $N=1$ term, but for massless partons it vanishes. It is a consequence of neglecting the Coulomb effects

$$\left.\frac{d\sigma_{eff}^{BH}(x,z)}{dx}\right|_{N=1} = \frac{\alpha_s z P_{Gq}(x)}{8\pi\rho_d^2(z)} \mathrm{Im} \int\limits_0^z \frac{d\xi}{\xi^2} \int\limits_0^\infty d\rho^2 \rho^2 C_3(\rho) \exp\left(\frac{i\rho^2}{4\rho_d^2(\xi)}\right).$$

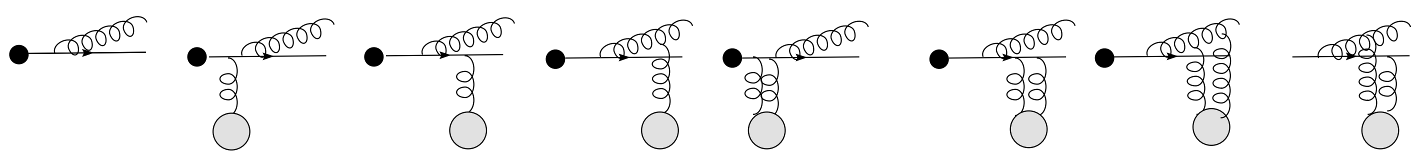
For $C_2(\rho) = $ const Im $\int d\rho^2 = 0$ With the Coulomb effects
$C_2(i\rho) = C_2(\rho) - \text{const} \cdot i\pi/2$. Only $i\pi/2$ contributes! It gives (no Coulomb $\log(1/\rho_d m_D)$!).

$$\left.\frac{d\sigma_{eff}^{BH}(x,z)}{dx}\right|_{N=1} = \frac{\alpha_s^3 \pi C_T C_A \rho_d^2(z) P_{Gq}(x)[1+(1-x)^2 - x^2/9]}{8} \propto z\,.$$

Naively one could expect for $N=1$ $\sigma^{BH} \propto \rho_d^2 \log(1/\rho_d m_D)$. But there is no the Coulomb $\log$ in the $N=1$ term! For photon emission at $L_f \gg L$ $\sigma^{BH} \propto 1/x$ is replaced dependence $\sigma^{BH} \propto 1/(1-x)$

$$\left.\frac{d\sigma_{eff}^{BH}(x,z)}{dx}\right|_{N=1} = \frac{z\alpha_{em}\alpha_s^2 \pi C_T C_F [1+(1-x)^2]}{4E(1-x)} \propto z\,, \quad \text{[BGZ (2004)]}$$

The $N=1$ can also be calculated using the momentum representation [Gyulassy, Lévai, Vitev (2001); BGZ (2001)]



Each diagram can be calculated as $\langle bc|M|a\rangle \propto \int dz d\vec{\rho} \phi_b^*(z,\vec{\rho})\phi_c^*(z,\vec{\rho})\phi_a(z,\vec{\rho})$ with the plane wave functions with sharp $\vec{p}_T$ change at the rescattering point [BGZ (2004)]

$N = 1$ term in momentum representation

$$\frac{d\sigma^{BH}_{eff}(x,z)}{dx} = \frac{\alpha_s^3 C_T P_{Gq}(x)}{\pi^2 C_F}\left[F(1,z) + F(1-x,z) - F(x,z)/9\right] ,$$

$$F(y,z) = \int \frac{d\vec{p}d\vec{k}}{(\vec{k}^{\,2}+m_D^2)^2} H(y\vec{k},\vec{p})\cdot\left[1-\cos\left((\vec{p}^{\,2}+\epsilon^2)\rho_d^2(z)\right)\right] , \quad \rho_d^2(z) = \frac{z}{2Ex(1-x)} ,$$

$$H(\vec{k},\vec{p}) = \frac{\vec{p}^{\,2}}{(\vec{p}^{\,2}+\epsilon^2)^2} - \frac{(\vec{p}-\vec{k})\vec{p}}{(\vec{p}^{\,2}+\epsilon^2)((\vec{p}-\vec{k})^2+\epsilon^2)} , \quad \langle H(\vec{k},\vec{p})\rangle_{\epsilon=0} = \frac{\theta(k-p)}{p^2}$$

$F = F_0 + \delta F$, $F_0 = F(\epsilon = 0)$, δF is mass correction.
The momentum integration gives for $\epsilon = 0$ $F_0(y,z) = \pi^3 y^2 \rho_d^2(z)/2$. There is no any $\log$ terms! LLA fails since for $\epsilon = 0$ $\nabla_k^2 H(\vec{k},\vec{p}) = 0$. Integration over the position of the rescattering gives $dP/dx \propto L^2$

$$\left.\frac{dP_{N=1}}{dx}\right|_{\epsilon=0} = \frac{\pi n L^2 \alpha_s^3 C_T P_{Gq}(x)[1+(1-x)^2 - x^2/9]}{8 C_F E x(1-x)}$$

The Debye mass does not appear in the spectrum! In massless limit the mass scale is given by $1/\rho_d(z)$. The photon spectrum has the same L^2-dependence! [BGZ (2001,2004)].

Mass correction

$$\delta F \approx \frac{\pi^2\epsilon^2\rho_d^4 y^2}{2}\left\{2\log^2\left(\frac{1}{\epsilon^2\rho_d^2}\right)+\log\left(\frac{1}{\epsilon^2\rho_d^2}\right)\log\left(\frac{\epsilon^2}{y^4 m_D^4\rho_d^2}\right)-3\log\left(\frac{1}{\epsilon^2\rho_d^2}\right)\right.$$

$$\left.-\frac{y^2 m_D^2}{\epsilon^2}\log\left(\frac{1}{\epsilon^2\rho_d^2}\right)\right\}\approx\frac{3\pi^2\epsilon^2\rho_d^4 y^2}{2}\log^2\left(\frac{1}{\epsilon^2\rho_d^2}\right)\quad \text{for}\quad \log\left(\frac{1}{\epsilon^2\rho_d^2}\right)\gg 1\,.$$

$\Rightarrow$ The mass correction to $\delta(dP/dx)$ is $\propto L^3$ and positive [Aurenche, BGZ (2009)]

$$\delta\frac{dP_{N=1}}{dx}=\frac{\alpha_s^3 P_{Gq}(x)[1+(1-x)^2-x^2/9]Ln\epsilon^2\rho_d^4(L)}{2C_F}\log^2\left(\frac{1}{\epsilon^2\rho_d^2(L)}\right)\,.$$

In the OA mass correction also has an anomalous mass dependence. To obtain $N=1$ term in the OA one should make replacement

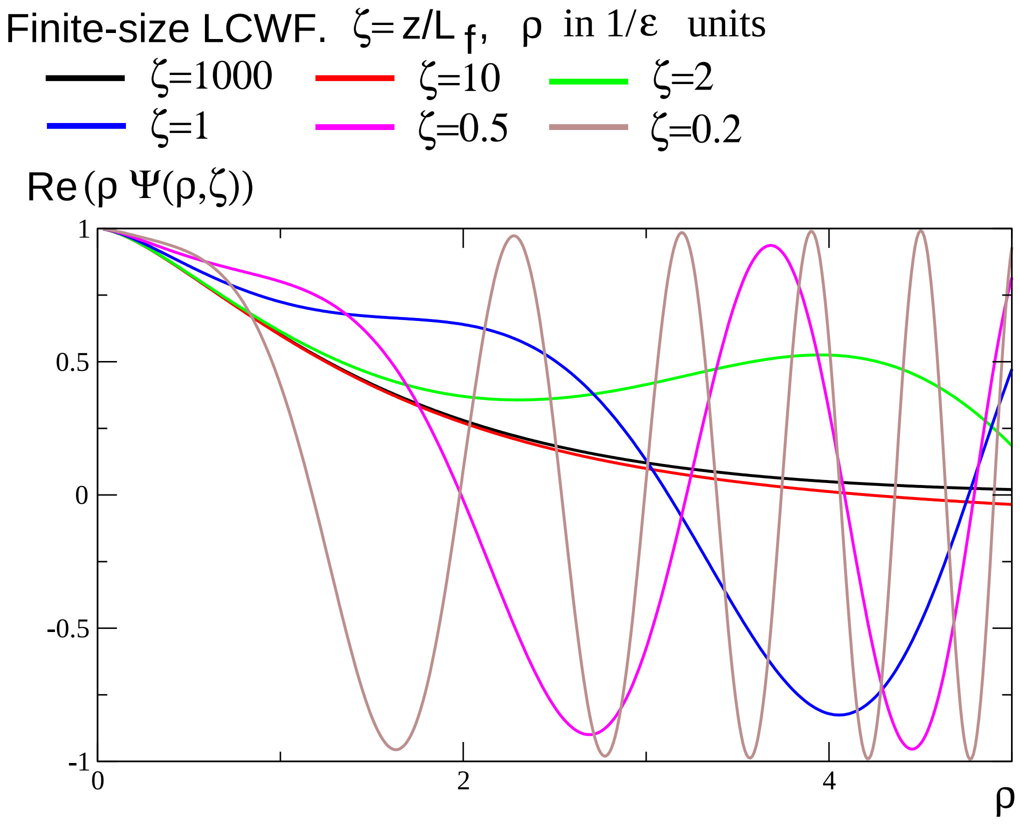
$$\frac{1}{(\vec{k}^2+m_D^2)^2}\Rightarrow\frac{2\hat{q}\delta(\vec{k})}{n\alpha_s^2 C_A C_T\vec{k}^2}\quad \text{which gives}\quad \sigma_{q\bar{q}}(\rho)=\frac{\hat{q}C_F}{2nC_A}\rho^2$$

$$\frac{dP_{N=1}^{OA}}{dx}=\frac{4\hat{q}L\alpha_s P_{Gq}(x)[1+(1-x)^2-x^2/9]\epsilon^2\rho_d^4(L)}{6\pi C_A C_F}\log\left(\frac{1}{\epsilon^2\rho_d^2(L)}\right)$$

Mass dependence and LCWF in ρ space

$$\frac{d\sigma_{eff}(x,z)}{dx} = \mathrm{Re} \int d\vec{\rho}\, \rho^2 \psi^*(\vec{\rho},x) \left(\frac{\sigma_3(\rho,x)}{\rho^2}\right) \psi_m(\vec{\rho},x,z)\,,$$

$\rho|\psi(\vec{\rho},x)| \propto \exp(-\epsilon\rho)$, $\rho|\psi_m(\vec{\rho},x,z=\infty)| \propto \exp(-\epsilon\sqrt{\eta}\rho)$ and the integrand is smooth like that for the BH cross section. At $z \lesssim L_f$ $\psi_m(\rho,x,z)$ oscillates. This can give anomalous mass dependence of the spectrum.

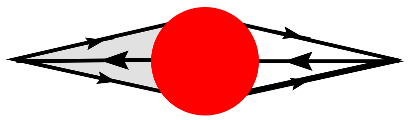


Accurate numerical method for all rescatterings

$$\frac{d\sigma_{eff}^{BH}(x,z)}{dx} = \text{Re}\int_0^z dz_1 \int_z^\infty dz_2 \int d\vec{\rho}\, g(x)\mathcal{K}_v(z_2,\vec{\rho}_2|z,\vec{\rho})\sigma_3(\rho)\mathcal{K}(z,\vec{\rho}|z_1,\vec{\rho}_1)\Big|_{\vec{\rho}_1=\vec{\rho}_2=0}.$$

For the vacuum Green's function z_2-integration comes up to infinity. The integral equals the LCWF with the azimuthal quantum number $m=\pm 1$ $\psi(\vec{\rho},x)\propto K_1(\epsilon\rho)\exp(im\phi)$. It allows one to represent the effective Bethe-Heitler cross section in the form [BGZ (2004)]

$$\frac{d\sigma_{eff}^{BH}(x,z)}{dx} = -\frac{\alpha_s P_{Gq}(x)}{\pi\mu(x)}\text{Im}\int_0^z d\xi\, \frac{\partial}{\partial\rho}\left(\frac{F(\xi,\rho)}{\sqrt{\rho}}\right)\Big|_{\rho=0},$$

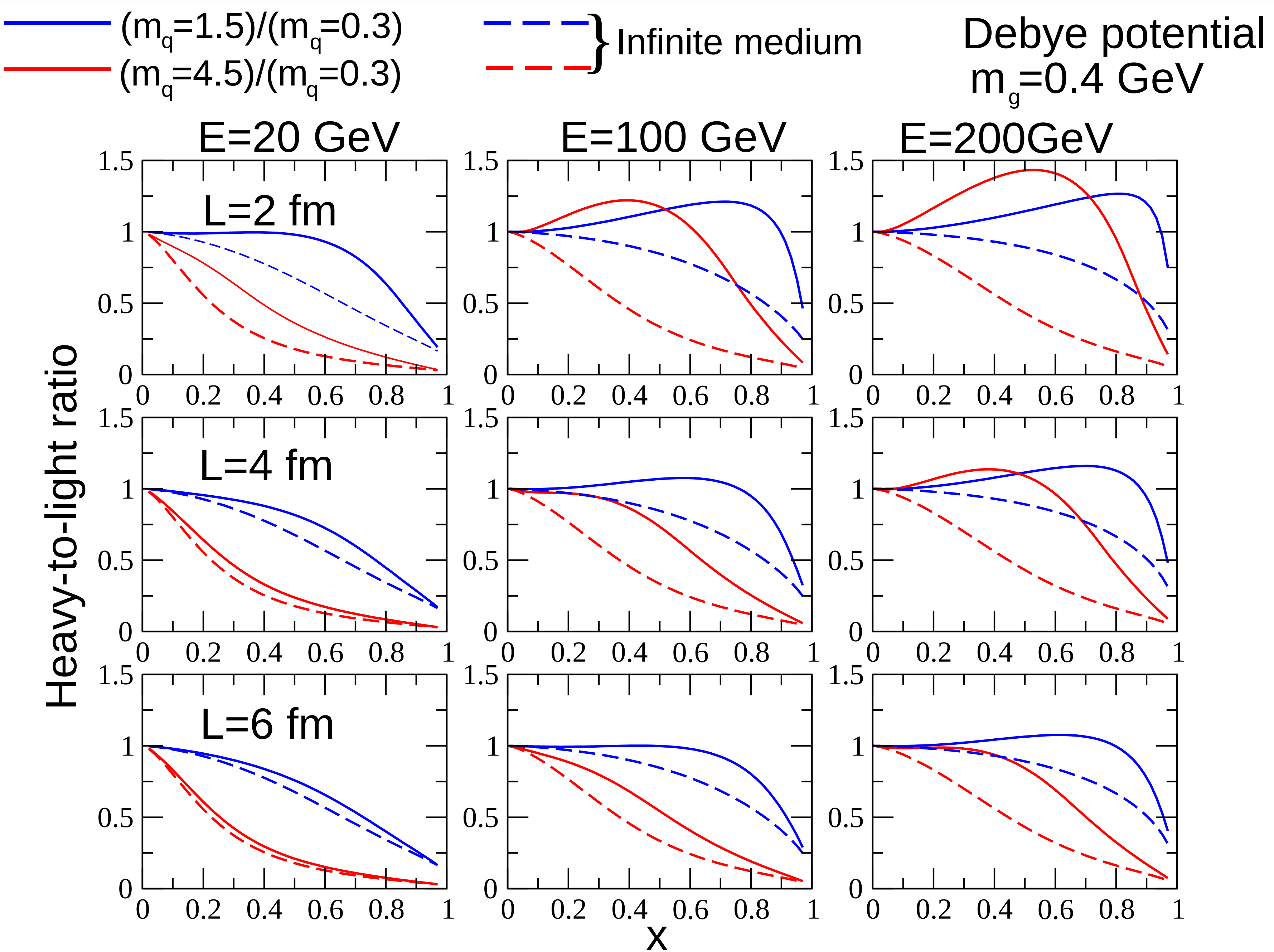


F is the solution to the radial Schrödinger equation for $m=1$

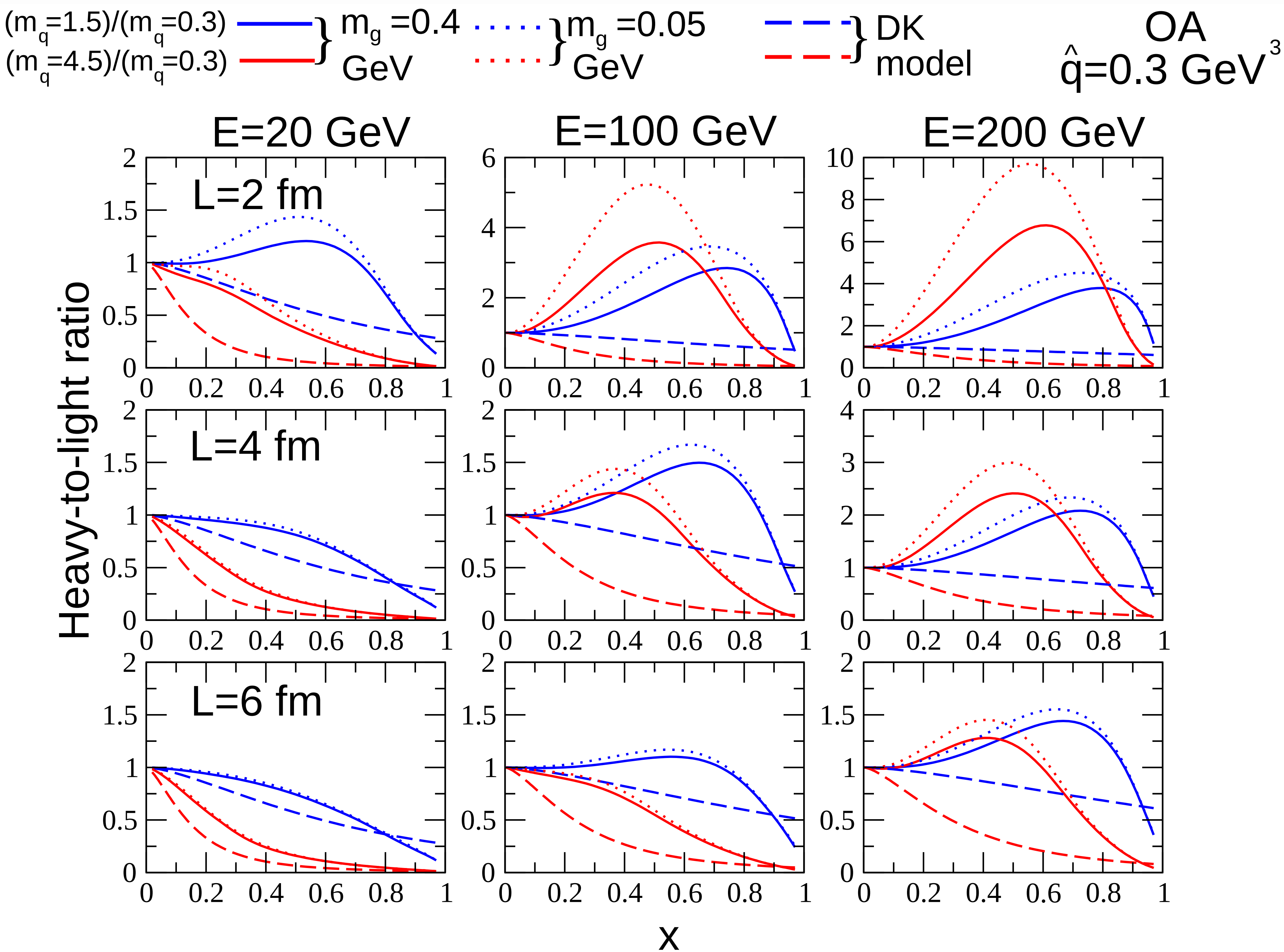
$$i\frac{\partial F(\xi,\rho)}{\partial\xi} = \left[-\frac{1}{2\mu(x)}\left(\frac{\partial}{\partial\rho}\right)^2 - i\frac{n(z-\xi)\sigma_3(\rho)}{2} + \frac{4m^2-1}{8\mu(x)\rho^2} + \frac{1}{L_f}\right]F(\xi,\rho).$$

with the boundary condition $F(\xi=0,\rho)=\sqrt{\rho}\sigma_3(\rho)\epsilon K_1(\epsilon\rho)$. We solve the Schrödinger equation back in time. It allows one to have a smooth boundary condition.

We take $\alpha_s=0.4$, QGP temperature $T=250$ MeV, $m_u=0.3$, $m_c=1.5$, $m_b=4.5$, $m_g=0.4$ GeV, $m_D=\sqrt{2}m_g$. In the OA we use $\hat{q}=0.3$ GeV^3



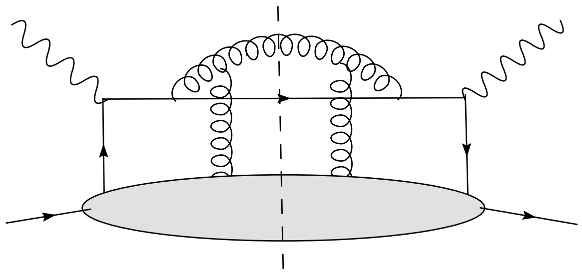
$(m_q=1.5)/(m_q=0.3)$
$(m_q=4.5)/(m_q=0.3)$
Infinite medium
Debye potential
$m_g=0.4$ GeV
E=20 GeV
E=100 GeV
E=200GeV
L=2 fm
L=4 fm
L=6 fm
Heavy-to-light ratio
x

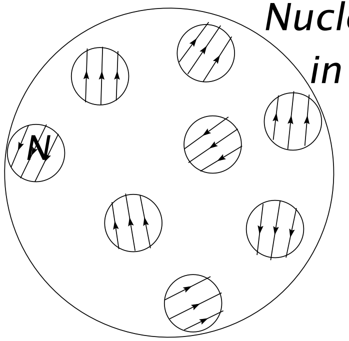


$(m_q=1.5)/(m_q=0.3)$
$(m_q=4.5)/(m_q=0.3)$
m_g =0.4 GeV
m_g =0.05 GeV
DK model
OA
$\hat{q}$=0.3 GeV^3
E=20 GeV
E=100 GeV
E=200 GeV
L=2 fm
L=4 fm
L=6 fm
Heavy-to-light ratio
x

Oscillator approximation and collinear expansion

The $N = 1$ term in DIS in the higher twist method [Wang, Guo (2001); Zhang, Wang (2003)] comes from the diagrams like





Collinear expansions is equivalent to the linear approximation of vector potential

$$A^+(y^-,\vec{y}_T+\vec{b}_T) \approx A^+(y^-,\vec{b}_T)+\vec{y}_T\frac{\partial}{\partial\vec{y}_T}A^+(y^-,\vec{b}_T)\,, \qquad \Rightarrow \qquad \sigma_{q\bar{q}}(y_T) \propto \vec{y}_T^{\,2}\langle F_T^{+\,2}\rangle\,.$$

$$\sigma_{q\bar{q}}(y_T) \propto \int d\vec{b}_T\langle N|[W(\vec{y}_T+\vec{b}_T)-W(\vec{b}_T)]^2|\rangle\,, \qquad W(\vec{y}_T) = \int dy^-A^+(y^-,\vec{y}_T)\,.$$

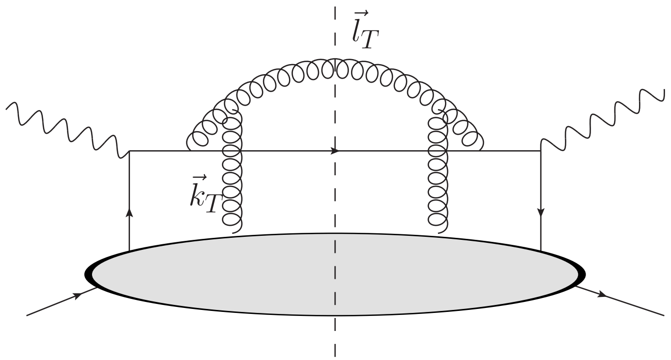
$\Rightarrow$ HT Collinear expansion=BDMPS in the OA, $\Rightarrow$ $N = 1$ term (and any $N = 2k+1$)

in HT is absent for massless partons. The calculations in momentum space confirm this

[BGZ (2001); Aurenche, BGZ, Zaraket (2008); Arnold (2009)].

N=1 term in higher twist GWZ's model

In Guo, Wang, Zhang (2001,2003) calculations (citation=430!) the non-zero result in the collinear expansion comes from this diagram (for $z \ll 1$). GWZ use for the integral variable the final gluon momentum l_T, then



$$H_{diag}^{GWZ}(\vec{l}_T, \vec{k}_T, z, \xi) \propto \frac{R(\vec{l}_T - \vec{k}_T)}{(\vec{l}_T - \vec{k}_T)^2}\,, \qquad R(\vec{l}_T - \vec{k}_T) = 1 - \cos\left(\frac{i(\vec{l}_T - \vec{k}_T)^2 \xi}{2Ez(1-z)}\right).$$

GWZ DO NOT DIFFERENTIATE the factor R, and obtain

$$\left(\frac{\partial}{\partial \vec{k}_T}\right)^2 \frac{R(\vec{l}_T - \vec{k}_T)}{(\vec{l}_T - \vec{k}_T)^2}\Bigg|_{k=0} = R(\vec{l}_T)\left(\frac{\partial}{\partial \vec{k}_T}\right)^2 \frac{1}{(\vec{l}_T - \vec{k}_T)^2}\Bigg|_{k=0} = \frac{4R(\vec{l}_T)}{l_T^4}\,.$$

However, the neglected terms are important. If one includes them one obtains

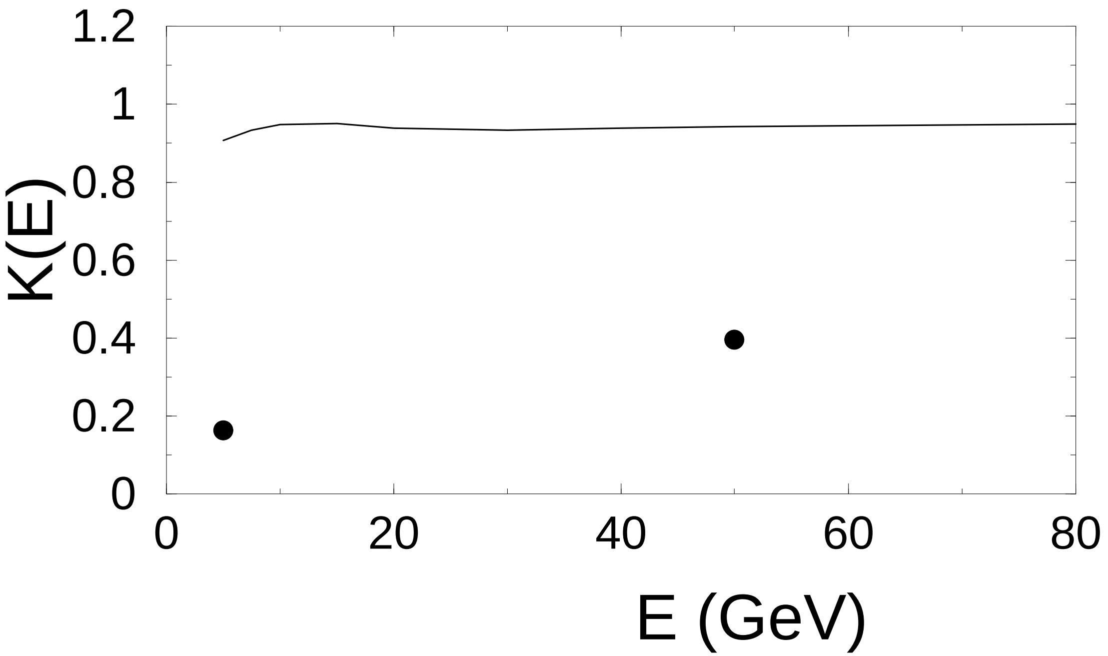
$$\left(\frac{\partial}{\partial \vec{k}_T}\right)^2 \int d\vec{l}_T\, \frac{R(\vec{l}_T - \vec{k}_T)}{(\vec{l}_T - \vec{k}_T)^2}\Bigg|_{k=0} \approx \left(\frac{\partial}{\partial \vec{k}_T}\right)^2 \int d\vec{l}_T\, \frac{R(\vec{l}_T)}{\vec{l}_T^2} = 0\,.$$

The role of kinematical bounds for N=1

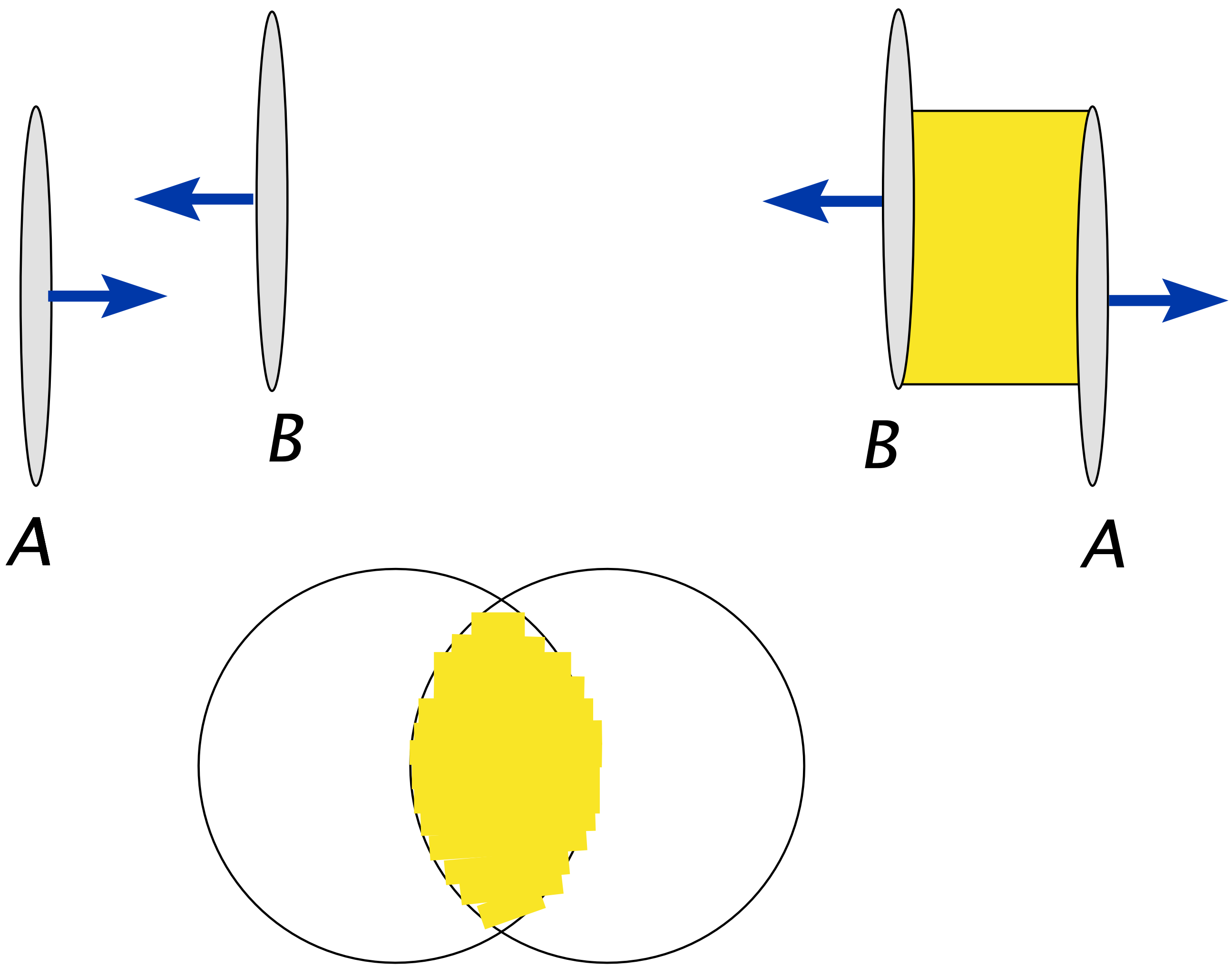
The GLV [Gyulassy, Lévai, Vitev (2001,2002,2003,2004)] group obtained even at $E \sim 50 - 500$ GeV a strong kinematical suppression of energy loss

$$\Delta E = E \int dx x \frac{dP}{dx}$$

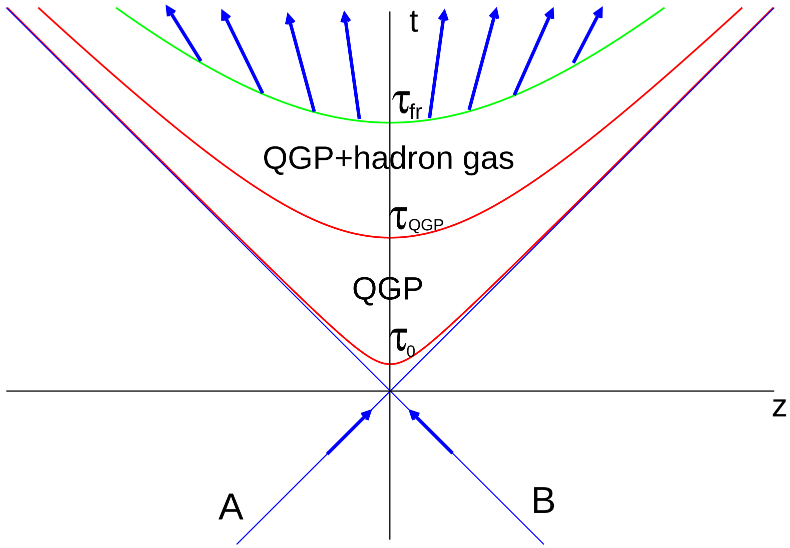
The kinematic K-factor for $N = 1$ term for QGP with n =const [BGZ (2004)] for constant α_s=const. Curves: $q_i < q_{max}(E)$ (like that in GLV) my calculations; points: GLV results



QGP in AA-collisions



1+1 Bjorken model for expanding QGP

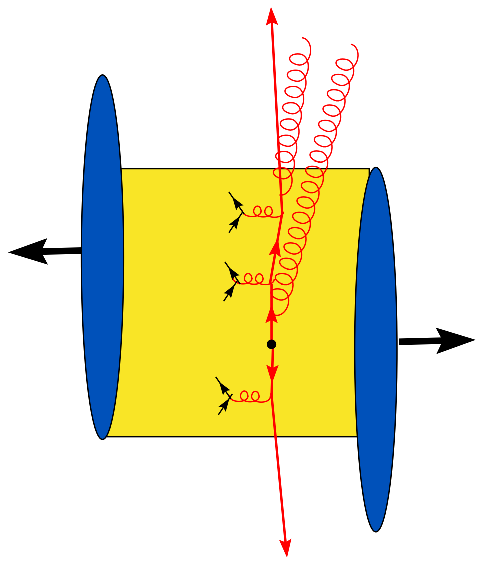


We use the Bjorken $1+1$ QGP expansion $T^3\tau = T_0^3\tau_0$. $n(\tau) \approx n_0(\tau_0/\tau)$ in the whole range of t. To simplify the numerical calculations for each value of the impact parameter b we neglect the variation of T_0 in the transverse directions. We take for the time of QGP creation $\tau_0 = 0.5$ fm, $\tau_{max} = L_{max} = 8$ fm. We fix T_0 using $dS/dy \Big/ dN_{ch}/d\eta \approx 7.67$ [B. Mueller and K. Rajagopal (2005)]

$\Rightarrow$ $\langle T_0 \rangle \approx 320$ MeV (central Au+Au, $\sqrt{s} = 200$ GeV), $\langle T_0 \rangle \approx 420$ MeV (central Pb+Pb, $\sqrt{s} = 2.76$ TeV).

Jet quenching in AA-collisions

Radiative (Bethe-Heitler) and collisional (Bjorken) energy losses modify jet evolution.



The radiative mechanism dominates, $\Delta E_{rad}^{N=1} \propto \alpha_s^3$. The theoretical uncertainties in R_{AA} are large (about a factor of 2) but the variation of R_{AA} from RHIC to LHC is more robust.

It is interesting to compare R_{AA} for RHIC ($\sqrt{s} \sim 200$ GeV) and LHC ($\sqrt{s} = 2.76$ TeV).

$S(\sqrt{s} = 2760)/S(\sqrt{s} = 200) \sim 2.2 \Rightarrow T_0(2.76\text{TeV}) \sim 1.3 T_0(0.2\text{TeV}) \Rightarrow \alpha_s$ should be suppressed at LHC. Can we see it from jet quenching?

Can we see the flavor dependence?

The nuclear modification factor for AA-collisions

$$R_{AA}(b) = \frac{dN(A+A\to h+X,\vec{b})/d\vec{p}_T dy}{T_{AA}(b)d\sigma(N+N\to h+X)/d\vec{p}_T dy}\,,$$

$T_{AA}(b) = \int d\vec{\rho}T_A(\vec{\rho})T_A(\vec{\rho}-\vec{b})$, $T_A(\vec{\rho}) = \int dz\rho_A(\vec{\rho},z)$ is the nucleus profile function.

$$\frac{dN(A+A\to h+X,\vec{b})}{d\vec{p}_T dy} = \int d\vec{\rho}T_A(\vec{\rho})T_A(\vec{\rho}-\vec{b})\frac{d\sigma_m(N+N\to h+X,\vec{\rho})}{d\vec{p}_T dy}\,,$$

$d\sigma_m(N+N\to h+X,\vec{\rho})/d\vec{p}_T dy$ is the medium-modified cross section for a hard reaction at $\vec{\rho}$. In analogy to the ordinary pQCD we write

$$\frac{d\sigma_m(N+N\to h+X)}{d\vec{p}_T dy} = \sum_i\int_0^1\frac{dz}{z^2}D^m_{h/i}(z,Q)\frac{d\sigma(N+N\to i+X)}{d\vec{p}^{\,i}_T dy}\,,\quad \vec{p}^{\,i}_T = \vec{p}_T/z$$

$D^m_{h/i}$ is the medium-modified FF for transition $i\to h$, $Q\sim p^i_T$.
$d\sigma(N+N\to i+X)/d\vec{p}^{\,i}_T dy \propto p^{-n}$ ($n\sim 8$ for RHIC and $n\sim 5$ for LHC) then qualitatively

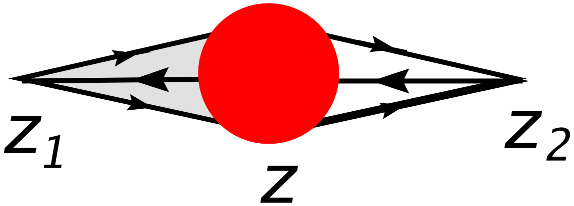
$$R_{AA}(b,p_T)\approx\int_0^1 dz z^{n-2}D^m_{h/i}(z,p_T)\Big[\int_0^1 dz z^{n-2}D_{h/i}(z,p_T)\Big]^{-1}$$

Induced one gluon emission in LCPI approach

$dP/dx = \int_0^L dz n(z) d\sigma_{eff}^{BH}(x,z)/dx$. The effective Bethe-Heitler cross section for $q \to g + q$ reads [BGZ (1997)]

$$\frac{d\sigma_{eff}^{BH}(x,z)}{dx} = \mathrm{Re} \int\limits_0^z dz_1 \int\limits_z^\infty dz_2 \int d\vec{\rho}\, \hat{g}(x) \mathcal{K}_v(z_2, \vec{\rho}_2|z,\vec{\rho}) \sigma_3(\rho) \mathcal{K}(z,\vec{\rho}|z_1,\vec{\rho}_1)\Big|_{\vec{\rho}_1=\vec{\rho}_2=0}$$

$x = \omega_g/E$, z is the position of the scattering center in QGP, $\sigma_3 = \sigma_{q\bar{q}g}$. For the vacuum Green's function $\mathcal{K}_v$ z_2-integration up to infinity gives the LCWF with the azimuthal quantum number $m = \pm 1$ $\psi(\vec{\rho}, x) \propto K_1(\epsilon\rho)\exp(im\phi)$ with $\epsilon^2 = m_q^2 x^2 + m_g^2(1-x)$. The result reads [BGZ (2004)]



$$\frac{d\sigma_{eff}^{BH}(x,z)}{dx} = -\frac{P_{Gq}(x)}{\pi\mu(x)} \mathrm{Im} \int\limits_0^z d\xi \alpha_s(Q_{eff}) \frac{\partial}{\partial\rho}\left(\frac{F(\xi,\rho)}{\sqrt{\rho}}\right)\Big|_{\rho=0},$$

$\mu = Ex(1-x)$, $Q_{eff}^2 = 1.85\mu/\xi$, F is the solution to the radial Schrödinger equation for $m = 1$

$$i\frac{\partial F(\xi,\rho)}{\partial\xi} = \left[-\frac{1}{2\mu(x)}\left(\frac{\partial}{\partial\rho}\right)^2 - i\frac{n(z-\xi)\sigma_3(\rho)}{2} + \frac{4m^2-1}{8\mu(x)\rho^2} + \frac{1}{L_f}\right] F(\xi,\rho)$$

with $L_f = 2\mu(x)/\epsilon^2$, $F(\xi=0,\rho) = \sqrt{\rho}\sigma_3(\rho)\epsilon K_1(\epsilon\rho)$. We solve the Schrödinger equation backward in time to have a smooth boundary condition.

Collisional energy loss, $2 \to 2$ processes

$$\frac{dE_{col}}{dz} = \frac{1}{2Ev} \sum_{p=q,g} g_p \int \frac{d\vec{p}'}{2E'(2\pi)^3} \int \frac{d\vec{k}\, n_p(k)}{2k(2\pi)^3}$$

$$\times \int \frac{d\vec{k}'[1+\epsilon_p n_p(k')]}{2k'(2\pi)^3} (2\pi)^4 \delta^4(P+K-P'-K') \omega \langle |M(s,t)|^2 \rangle \theta(\omega_{max}-\omega)$$

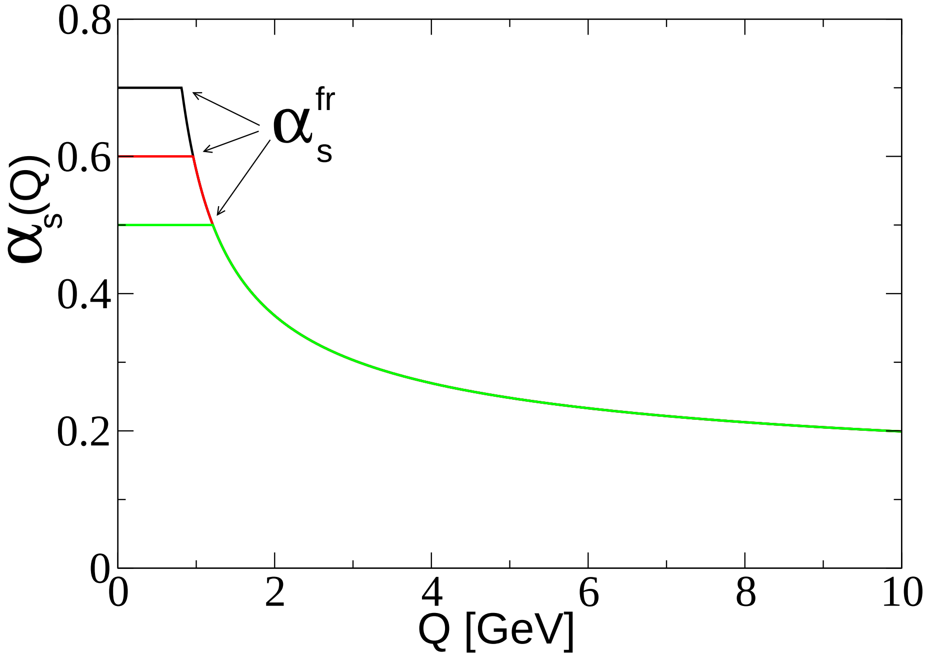
$\omega = E - E'$ is the energy transfer, $v \approx 1$ is the quark velocity, $P = (E, \vec{p})$ and $K = (k, \vec{k})$ 4-momenta for incoming partons, $P' = (E', \vec{p}')$ and $K' = (k', \vec{k}')$ 4-momenta for outgoing partons, $M(s,t)$ is matrix element for $Qp \to Qp$ scattering, $n_q(k) = (e^{k/T}+1)^{-1}$ and $n_g(k) = (e^{k/T}-1)^{-1}$, $\epsilon_q = -1$, $\epsilon_g = 1$, $g_q = 4N_c N_f$, $g_g = 2(N_c^2 - 1)$. Similarly to the radiative energy loss we take $\omega_{max} = E/2$.

$$\omega = \frac{-t - tk_z/E + 2\vec{k}_\perp \vec{q}_\perp}{2(k - k_z)} .$$

Bjorken neglected the red terms. In this case neglecting the statistical Pauli-blocking and Bose enhancement factors one can obtain

$$\frac{dE_{col}}{dz} \approx \frac{1}{2(2\pi)^3} \sum_{p=q,g} g_p \int d\vec{k} \frac{n_p(k)}{k} \int_0^{|t|_{max}} dt |t| \frac{d\sigma}{dt} , \quad |t|_{max} \approx 2(k - k_z)\omega_{max} .$$

Parametrization of $\alpha_s(Q)$

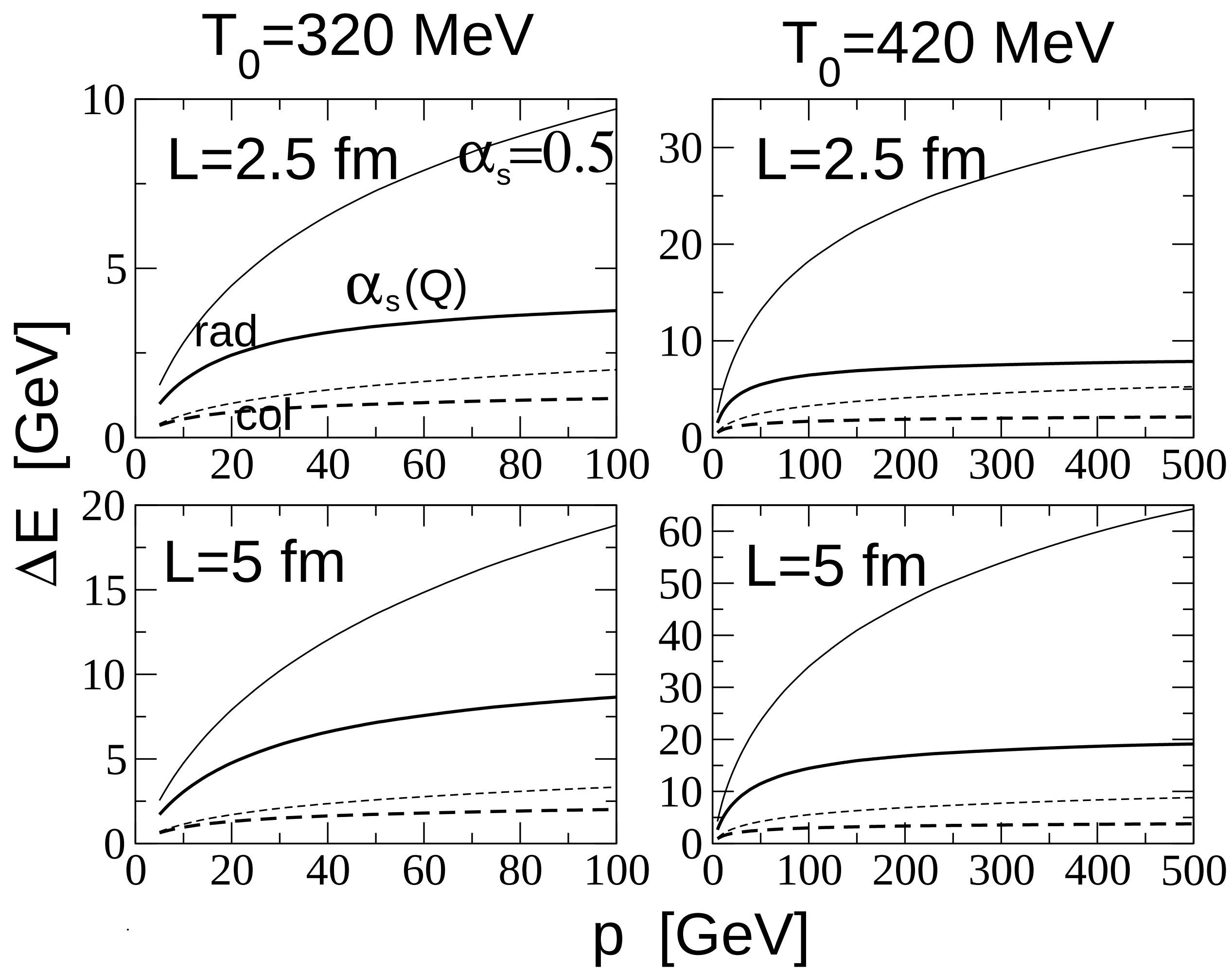


We use running α_s frozen at $\alpha_s^{fr} = 0.5,\ \ 0.4$. $\alpha_s^{fr} \approx 0.7$ was obtained from the data on F_2^p at low x [Nikolaev, BGZ (1991,1994)], it agrees with

$$\int_0^{2\,GeV} dQ \frac{\alpha_s(Q^2)}{\pi} \approx 0.36 \text{ GeV}$$

obtained from the analysis of the heavy quark energy loss in vacuum [Dokshitzer, Khoze, Troyan (1996)].

ΔE for quark: running α_s is important

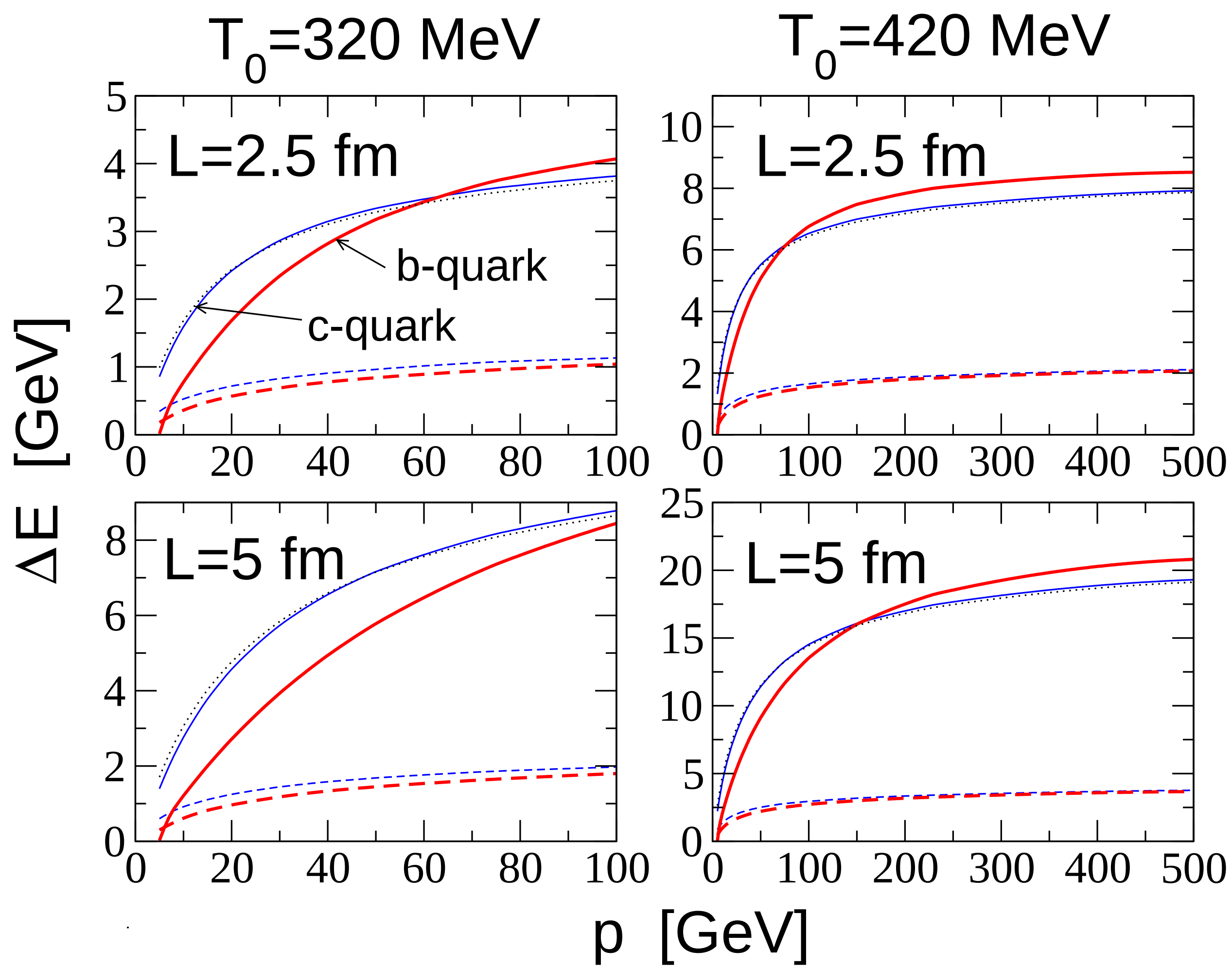


The radiative (solid) and collisional (dashed) quark energy losses in expanding QGP for $L = 2.5$ and 5 fm, $\tau_0 = 0.5$ fm, $m_q = 300$ MeV, $m_g = 400$ MeV [Lévai, Heinz (1998)].

thick: running α_s with $\alpha_s^{fr} = 0.5$, thin: $\alpha_s = 0.5$.

T-dependent Debye mass from the lattice calculations [O. Kaczmarek and F. Zantow, Phys. Rev., D71, 114510 (2005)]

ΔE for u, c, b quarks



The radiative (solid) and collisional (dashed) quark energy losses in expanding QGP for $L = 2.5$ and 5 fm, $\tau_0 = .5$ fm, $m_q = 300$ MeV, $m_g = 400$ MeV, $m_c = 1.2$, $m_b = 4.75$ GeV.

running α_s with $\alpha_s^{fr} = 0.5$

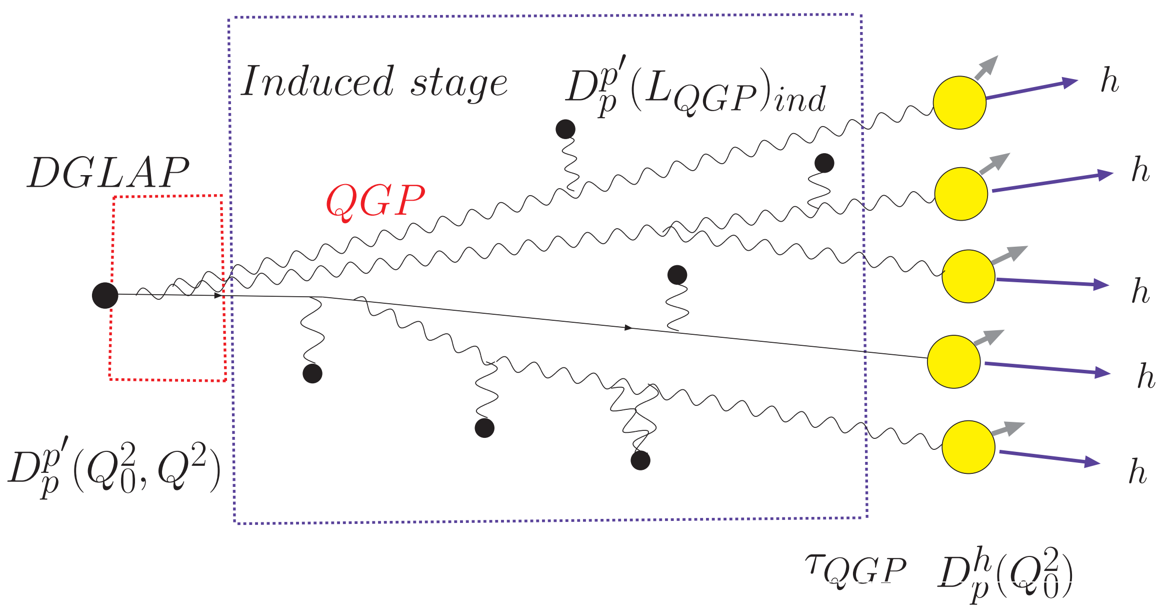
red: b-quark, blue: c-quark, dotted: u-quark.

The space-time pattern of jet distortion

The formation length for the DGLAP $\bar{l}_F \sim 0.3 - 1$ fm for $E \lesssim 100$ GeV (if $m_q \sim 0.3$ GeV and $m_g \sim 0.75$ GeV). $\Rightarrow$The DGLAP stage gives initial condition for the induced emission stage at $\tau_{DGLAP} \sim \tau_0$.

$$\Rightarrow \quad D^m_{h/i}(Q) \approx D_{h/j}(Q_0) \otimes D^{ind}_{j/l}(E_l) \otimes D^{DGLAP}_{l/i}(Q_0, Q)\,,$$

$D^{ind}_{j/l}$ is the induced radiation FF (it depends on the parton energy E, but not the virtuality), $D^{DGLAP}_{l/i}$ is calculated with the PYTHIA event generator. Our scheme of the stages of jet evolution



The FF for the induced stage

To calculate the $D^{ind}_{j/l}$ one needs to take into account the multiple gluon emission. There is no an accurate method of incorporating the multiple gluon emission.
We use Landau method developed for photon emission [BDMS (2001)]

$$P(\Delta E) = \sum_{n=1}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i \frac{dP(\omega_i)}{d\omega} \right] \delta \left(\Delta E - \sum_{i=1}^{n} \omega_i \right) \exp \left[- \int d\omega \frac{dP}{d\omega} \right] ,$$

$dP/d\omega$ is the distribution for one gluon emission. The situation is similar to multi-photon emission QED

$$P(\Delta E) = \frac{dP_\gamma}{d\omega} K(x) \, , \;\; \Delta E = \omega = xE$$

For thin targets the multi-photon K-factor can be evaluated analytically [BGZ (1998)]

$$K(x) = \exp \left[- \int_x^1 dx_1 \frac{dP_\gamma}{dx_1} \right] \left\{ 1 - \frac{1}{2} \int_0^x dx_1 \left[\frac{dP_\gamma}{dx_1} + \frac{dP_\gamma}{dx_2} - \frac{dP_\gamma}{dx_1} \frac{dP_\gamma}{dx_2} \left(\frac{dP_\gamma(x)}{dx} \right)^{-1} \right] \right\} ,$$

where $x_2 = x - x_1$. The major x-dependence of the K-factor comes from the Sudakov exponential factor.

Induced FF for $q \to q$, $q \to g$, and $g \to g$

For $q \to q$ we take (like Eskola, Honkanen, Salgado and Wiedemann (2004))

$$D^{ind}_{q/q}(z) = K_{qq} P_{Landau}(\Delta E = E(1-z))\,, \quad K_{qq} = \int_0^\infty d\Delta E P(\Delta E) / \int_0^E d\Delta E P(\Delta E)$$

K_{qq} accounts for the leakage of the probability to $\Delta E > E$ (gluons are not soft enough!).
For momentum conservation we include $q \to g$ transition. At the one gluon level

$$D^{ind}_{g/q}(z) = dP(z)/dz, \quad \text{and} \quad \int dzz[D^{ind}_{g/q}(z) + D^{ind}_{q/q}(z)] = 1$$

.
FF for multiple gluon emission we take $D^{ind}_{g/q}(z) = K_{gq} dP(z)/dz$ with K_{gq} fixed from momentum conservation $\int dzz[D^{ind}_{g/q}(z) + D^{ind}_{q/q}(z)] = 1$.
It is reasonable since R_{AA} is sensitive to FFs at $z \sim 1$ [BDMS (2001)] where $q \to g$ distribution should not be very sensitive to the multiple gluon emission.

For $g \to g$ we can use only the momentum conservation. In the first step we define $\bar{D}^{ind}_{g/g}(z) = P_{Landau}(\Delta E(1-z)) \quad z > 0.5$. At $z < 0.5$ (where the multiple gluon emission and the Sudakov suppression strongly compensate each other) we use the one gluon formula $\bar{D}^{ind}_{g/g}(z) = dP/dz$. Finally we define $D^{ind}_{g/g}(z) = K_{gg}\bar{D}^{ind}_{g/g}(z)$. K_{gg} is fixed from the momentum sum rule $\int dzz D^{ind}_{g/g}(z) = 1$.

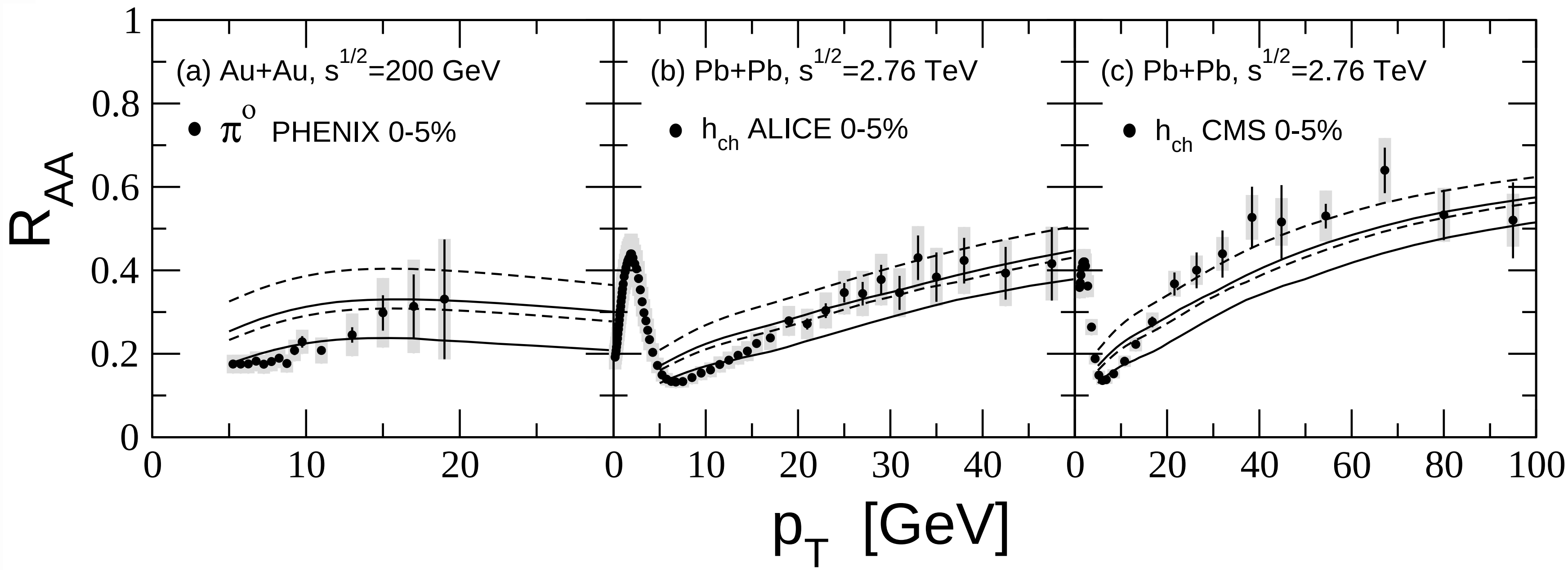
We treat the collisional loss as a perturbation and incorporate it by a small renormalization of T_{QGP} according to the change in the ΔE due to the collisional energy loss

$$\Delta E_{rad}(T') = \Delta E_{rad}(T) + \Delta E_{col}(T)$$

The collisional loss decreases R_{AA} by 15-25 %.

We calculate the cross sections $d\sigma(N+N \to i+X)/d\vec{p}_T^{\,i}dy$ using the LO pQCD formula with the CTEQ6 PDFs. To account for the nuclear modification of the PDFs (which leads to some small deviation of R_{AA} from unity even without energy loss) we include the EKS98 correction [K.J. Eskola, V.J. Kolhinen, and C.A. Salgado, Eur. Phys. J. C**9**, 61 (1999)]. . To simulate the higher order K-factor in the hard cross sections we use $\alpha_s(cQ)$ with $c = 0.265$ (like that in PYTHIA). For the FFs $D_{h/q(g)}(z, Q_0)$ we use the KKP parametrization [B. A. Kniehl, G. Kramer, and B. Potter, Nucl. Phys. B**582**, 514 (2000)]

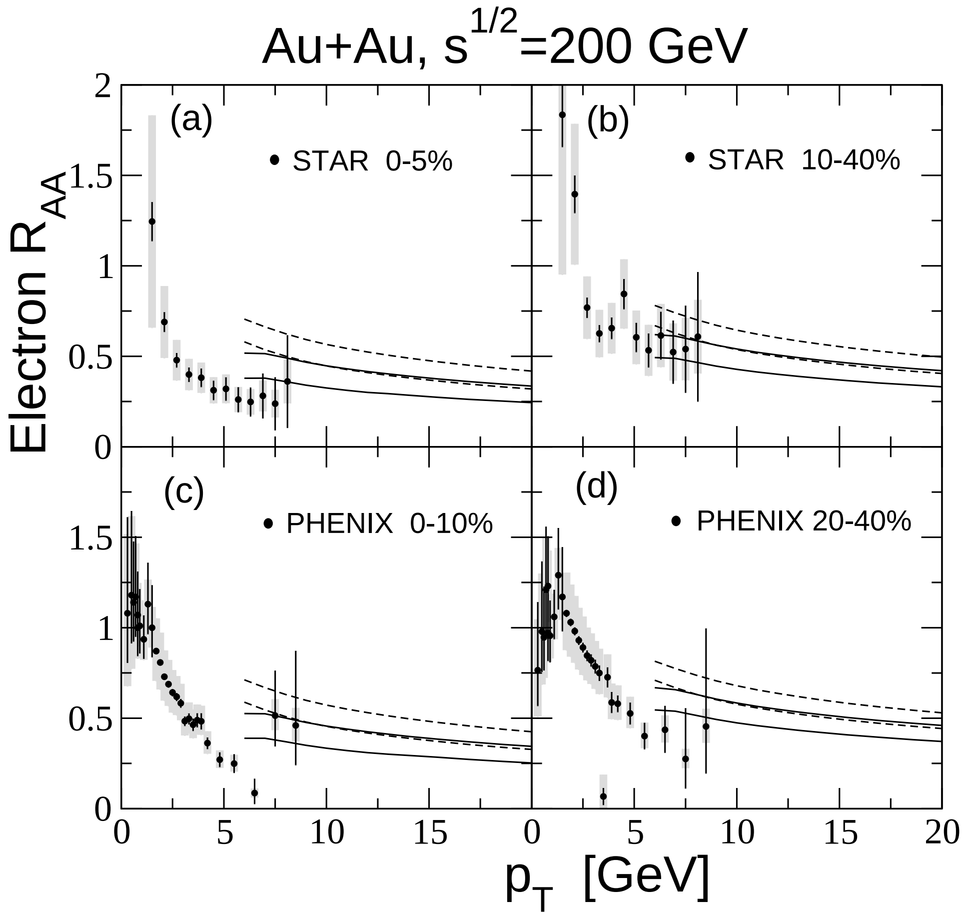
Comparison with R_{AA} for light hadrons



$\alpha_s^{fr} = 0.4$ (upper curves) and 0.5 (lower curves)

solid line: radiative+collisional; dashed line: radiative

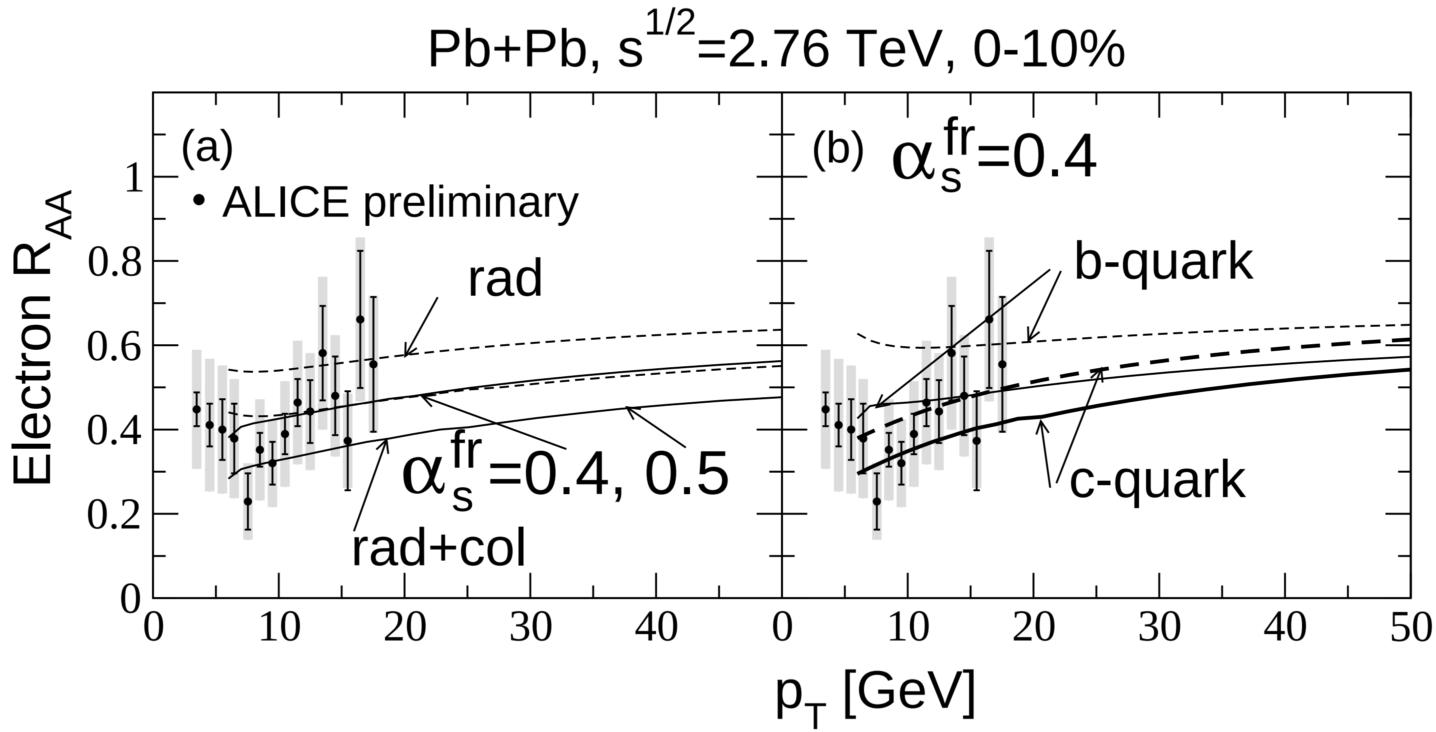
R_{AA} for non-photonic electrons $s^{1/2} = 200$ GeV



$\alpha_s^{fr} = 0.4$ (upper curves) and 0.5 (lower curves)

solid line: radiative+collisional; dashed line: radiative

R_{AA} for non-photonic electrons $s^{1/2} = 2.76$ TeV

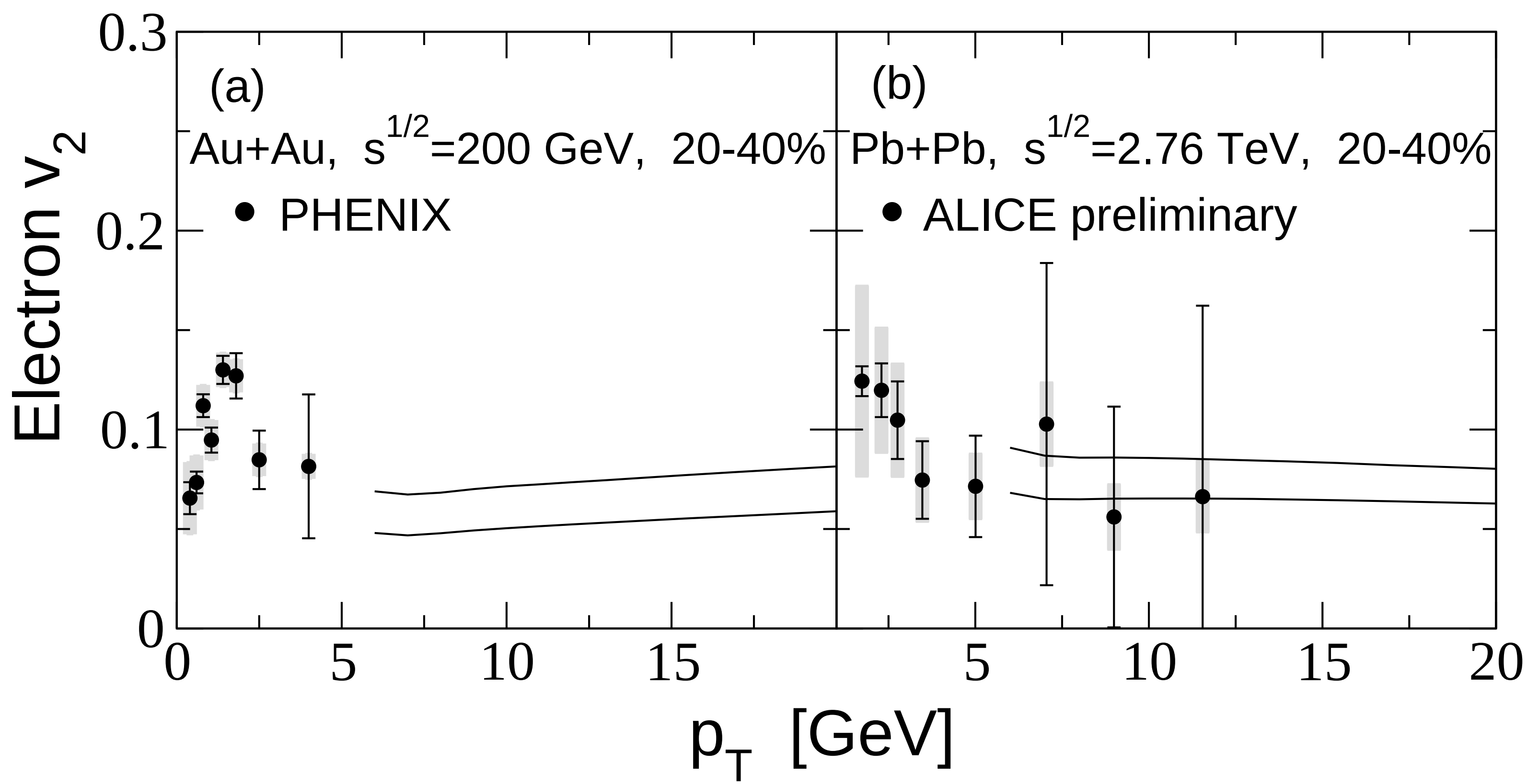


solid line: radiative+collisional; dashed line: radiative

(a) $\alpha_s^{fr} = 0.4$ (upper curves) and 0.5 (lower curves) including charm and bottom.

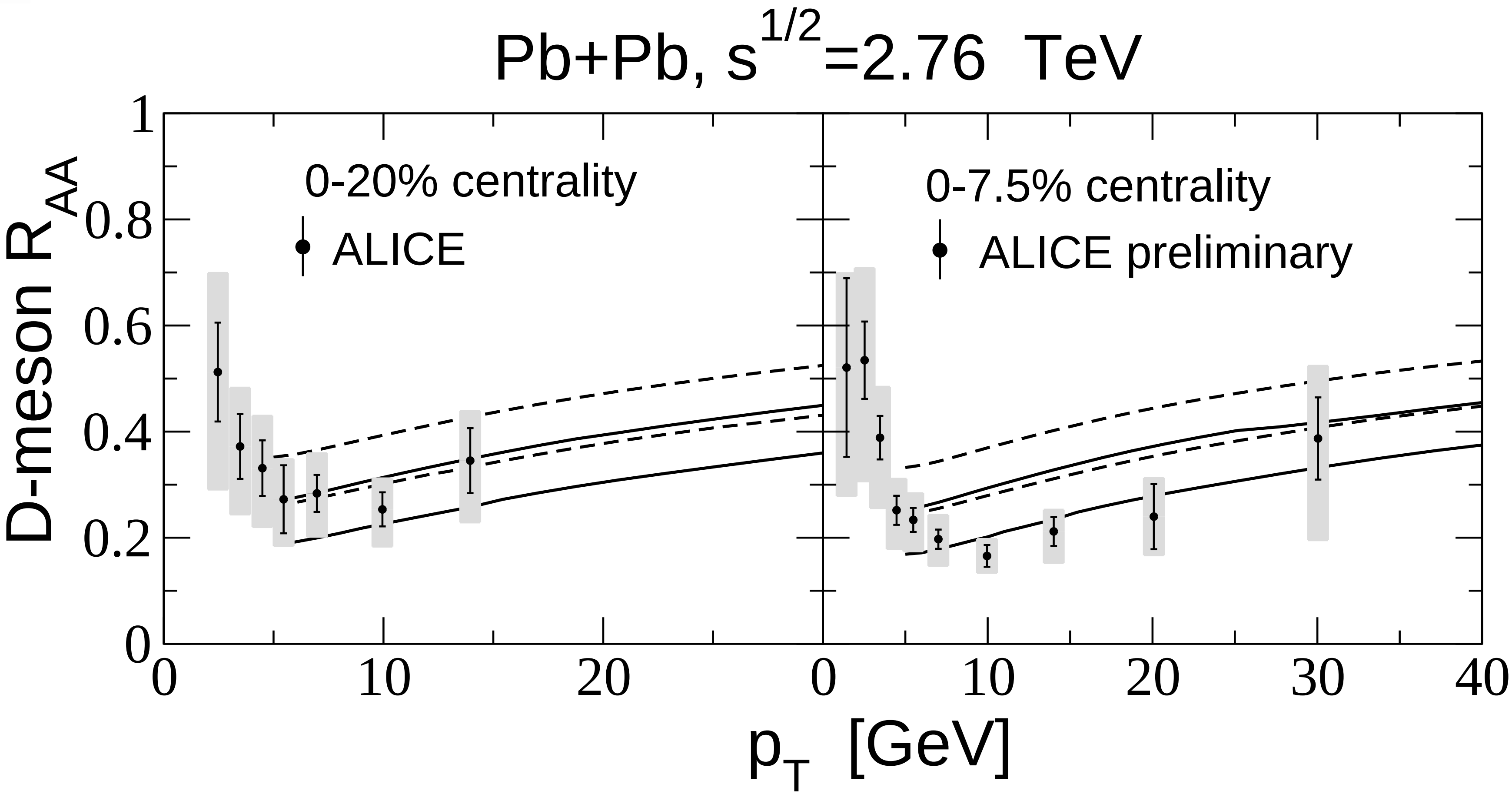
(b) $\alpha_s^{fr} = 0.4$ charm (thick) and bottom (thin).

v_2 for non-photonic electrons



$\alpha_s^{fr} = 0.5$ (upper curves) and 0.4 (lower curves)
The theoretical curves are for both the charm and bottom contributions, radiative+collisional

R_{AA} for D-mesons $s^{1/2} = 2.76$ TeV



solid line: radiative+collisional; dashed line: radiative

$\alpha_s^{fr} = 0.4$ (upper curves) and 0.5 (lower curves)

Conclusions:

- The LCPI approach describes very well the data on the LPM effect for $e \to \gamma e$ obtained at SLAC and CERN SPS
- The finite-size effects can lead to an enhancement of the gluon emission from heavy quarks as compared to that from the light quarks at $L \lesssim L_f$. The results of our calculations in the OA disagree strongly with Dokshitzer-Kharzeev estimates obtained neglecting the finite-size effects.
- In expanding QGP for RHIC-LHC conditions the gluon emission from the c-quark is very similar to that from the light quarks. At the $E \gtrsim 100$ GeV b-quark radiates stronger than c and light quarks at $x \lesssim 0.5$.
- In the higher-twist model the $N = 1$ term vanishes in the massless limit. In the calculations by Wang, Guo and Zhang the collinear expansion is made incorrectly, and for this reason they obtained nonzero result.
- The nuclear modification factor R_{AA} for the RHIC and LHC is calculated accounting for both the radiative and collisional energy losses with the running α_s and fluctuations of parton path length in QGP. The effect of the collisional energy loss is relatively small.
- Comparison with the RHIC PHENIX and LHC CMS-ALICE data on R_{AA} gives evidence in favor of somewhat stronger thermal suppression of α_s at LHC. We have $\alpha_s^{fr} \approx 0.5$ at $\sqrt{s} = 200$ GeV and $\alpha_s^{fr} \approx 0.4$ at $\sqrt{s} = 2.76$ TeV. It seems that the QGP really becomes more perturbative at LHC. The pQCD gives a reasonable description of the flavor dependence of R_{AA}.

BACK UP SLIDES

Formulas for $e \to \gamma e$

$$\frac{dP_\gamma}{dx} = \frac{dP_\gamma^{BH}}{dx} + \frac{dP_\gamma^{abs}}{dx} \,,$$

$$\frac{dP_\gamma^{BH}}{dx} = T\frac{d\sigma^{BH}}{dx} \,, \qquad T = \int_0^L dz n(z) \,,$$

$$\frac{d\sigma^{BH}}{dx} = \int d\vec{\rho}\, W_e^{e\gamma}(x,\vec{\rho})\sigma(\rho x) \;, \quad W_e^{e\gamma}(x,\vec{\rho}) = \frac{1}{2}\sum_{\{\lambda_i\}} |\Psi(x,\vec{\rho},\{\lambda_i\})|^2 \,,$$

$$\frac{dP_\gamma^{abs}}{dx} = -\frac{1}{4}\mathrm{Re}\sum_{\{\lambda_i\}} \int\limits_0^L dz_1 n(z_1) \int\limits_{z_1}^L dz_2 n(z_2) \int d\vec{\rho}\, \Psi^*(x,\vec{\rho},\{\lambda_i\})$$

$$\times \sigma(\rho x)\Phi(x,\vec{\rho},\{\lambda_i\},z_1,z_2)\exp\left[-\frac{i(z_2-z_1)}{L_f}\right] \,.$$

$$i\frac{\partial\Phi(x,\vec{\rho},\{\lambda_i\},z_1,z_2)}{\partial z_2} = \left[-\frac{1}{2\mu(x)}\left(\frac{\partial}{\partial\vec{\rho}}\right)^2 - i\frac{n(z)\sigma_{e\bar{e}}(x|\vec{\rho}|)}{2}\right]\Phi(x,\vec{\rho},\{\lambda_i\},z_1,z_2)$$

The boundary condition reads

$$\Phi(x,\vec{\rho},\{\lambda_i\},z_1,z_1) = \Psi(x,\vec{\rho},\{\lambda_i\})\sigma(\rho x)\,.$$

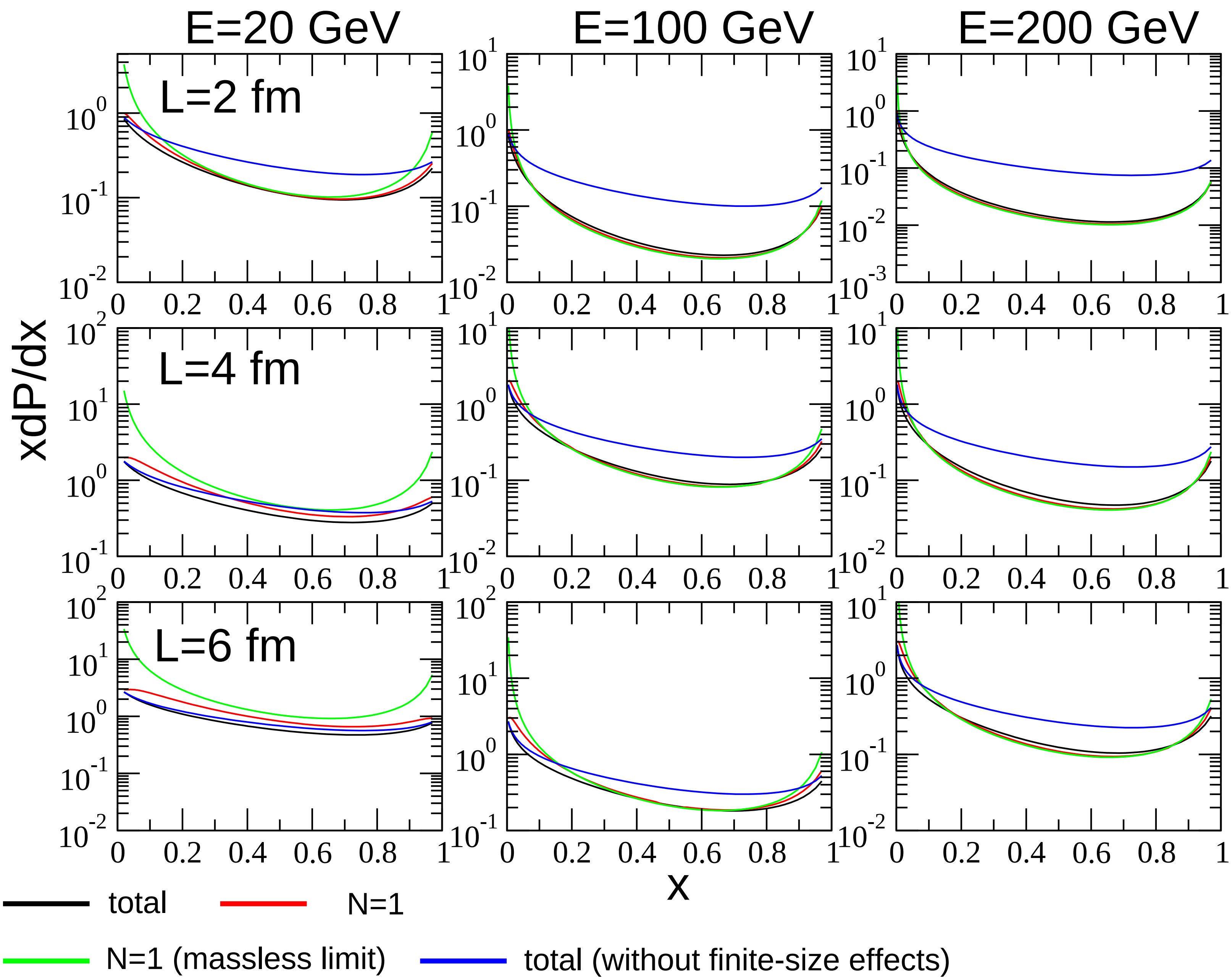
$$\sigma(\rho) = \rho^2 C(\rho)\,,\quad C(\rho) = Z^2 C_{el}(\rho) + Z C_{in}(\rho)\,.$$

$$C_{el}(\rho) = 4\pi\alpha^2\left[\log\left(\frac{2a_{el}}{\rho}\right) + \frac{(1-2\gamma)}{2} - f(Z\alpha)\right]\,,\ \ a_{el} = 0.81 r_B Z^{-1/3}\,,$$

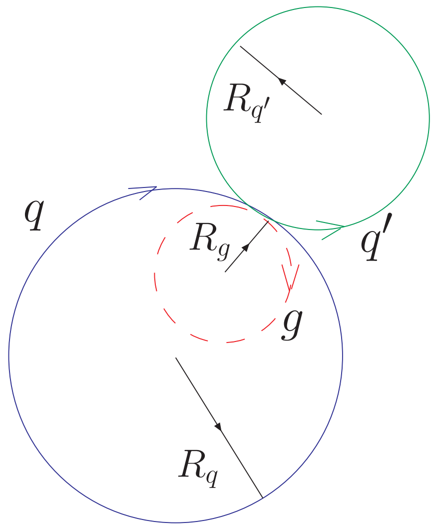
$$f(y) = y^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+y^2)}\ ,$$

$$C_{in}(\rho) = 4\pi\alpha^2\left[\log\left(\frac{2a_{in}}{\rho}\right) + \frac{(1-2\gamma)}{2}\right]\,,\ \ a_{in} = 5.3 r_B Z^{-2/3}$$

Gluon spectrum for m_q=0.3, m_g=0.4 GeV (Debye potential)



Gluon synchrotron radiation



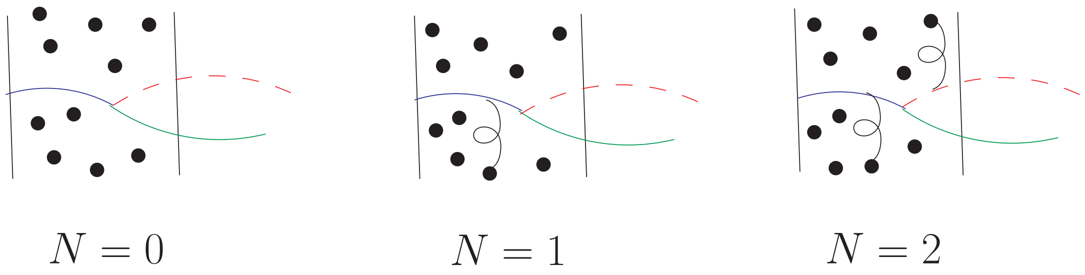
$\omega \gg m_g$

$E, E' \gg m_q$

$L_f \ll min(R_q, R_{q'}, R_g)$

The small angle approximation is applicable at the scale $L \sim L_f \Rightarrow$ One can define $dP/dxdL$, and calculate it for a slab with thickness $R_{g,q} \gg L \gg L_f$.

Gluon emission due to multiple scattering and synchrotron radiation should be treated on even footing, but we neglect the interference of the two mechanisms



- For $SU(3)$ it is enough to consider chromomagnetic field with color components $a = 3$ and $a = 8$. For radiated gluons we use the color states $Q = (Q_A, Q_B)$ with definite color isospin, Q_A, and color hypercharge, Q_B.
  There are 2 neutral gluons $A = G_3$ and $B = G_8$, and 6 charged gluons $X, Y, Z, \bar{X}, \bar{Y}, \bar{Z}$ given by

$$
\begin{aligned}
X &= (G_1 + iG_2)/\sqrt{2}, \quad Q = (-1, 0), \\
Y &= (G_4 + iG_5)/\sqrt{2}, \quad Q = (-1/2, -\sqrt{3}/2), \\
Z &= (G_6 + iG_7)/\sqrt{2}, \quad Q = (1/2, -\sqrt{3}/2).
\end{aligned}
$$

- The S-matrix element of the $q \to gq'$ synchrotron transition can be written as

$$\langle gq'|\hat{S}|q\rangle = -ig \int dy \bar{\psi}_{q'}(y) \gamma^{\mu} G^{*}_{\mu}(y) \psi_q(y)\,.$$

  We write each quark wave function in the form
  $\psi_i(y) = \exp[-iE_i(t-z)] \hat{u}_{\lambda} \phi_i(z, \vec{\rho})/\sqrt{2E_i}$, where λ is quark helicity, $\hat{u}_{\lambda}$ is the Dirac spinor operator.

The z-dependence of the transverse wave functions ϕ_i for a parton with color vector $Q = (Q_A, Q_B)$ is governed by the two-dimensional Schrödinger equation

$$i\frac{\partial \phi_i(z,\vec{\rho})}{\partial z} = \left\{\frac{(\vec{p} - gQ_n\vec{G}_n)^2 + m_q^2}{2E_i} + gQ_n(G_n^0 - G_n^3)\right\}\phi_i(z,\vec{\rho})\,,$$

where G is the external vector potential (the superscripts are the Lorentz indexes and $n = A, B$). The gluon wave function can be represented in a similar way.

We take the external potential in the form $G_n^3 = [\vec{H}_n \times \vec{\rho}]^3$, $\vec{G}_n = 0$, $G_n^0 = 0$ (the electric field can be included as well). $-gQ_nG_n^3$ can be viewed as the potential energy in the impact parameter plane $U_i = -\vec{F}_i \cdot \vec{\rho}$, where $\vec{F}_i$ is the corresponding Lorentz force. The $\phi_i(z,\vec{\rho})$ can be taken in the form

$$\phi_i(z,\vec{\rho}) = \exp\left\{i\vec{p}_i(z)\vec{\rho} - \frac{i}{2E_i}\int_0^z dz'[\vec{p}_i^2(z') + m_q^2]\right\},$$

$$\frac{d\vec{p}_i}{dz} = \vec{F}_i(z)\,.$$

$$\langle gq'|\hat{S}|q\rangle = -ig(2\pi)^3\delta(E_g + E_{q'} - E_q)\int_{-\infty}^{\infty} dz V(z,\{\lambda\})\delta(\vec{p}_g(z) + \vec{p}_{q'}(z) - \vec{p}_q(z))$$
$$\times \exp\left\{-i\int_0^z dz'\left[\frac{\vec{p}_q^{\,2}(z') + m_q^2}{2E_q} - \frac{\vec{p}_g^{\,2}(z') + m_g^2}{2E_g} - \frac{\vec{p}_{q'}^{\,2}(z') + m_q^2}{2E_{q'}}\right]\right\},$$

where V is the spin vertex factor.

Using $\vec{p}_g(z) + \vec{p}_{q'}(z) = \vec{p}_q(z)$ (since $\vec{F}_q = \vec{F}_g + \vec{F}_{q'}$) we obtain the gluon spectrum

$$\frac{dP}{dx} = \frac{1}{(2\pi)^2}\int d\vec{p}_g(\infty)\int dz_1 dz_2 g(z_1, z_2)$$
$$\times \exp\left\{i\int_{z_1}^{z_2} dz\left[\frac{\vec{p}_q^{\,2}(z) + m_q^2}{2E_q} - \frac{\vec{p}_g^{\,2}(z) + m_g^2}{2E_g} - \frac{\vec{p}_{q'}^{\,2}(z) + m_q^2}{2E_{q'}}\right]\right\},$$

$$g(z_1, z_2) = \frac{C\alpha_s}{8E_q^2 x(1-x)}\sum_{\{\lambda\}} V^*(z_2,\{\lambda\})V(z_1,\{\lambda\}) = g_1\vec{q}(z_2)\vec{q}(z_1)/\mu^2 + g_2 \quad (1)$$

with $g_1 = C\alpha_s(1 - x + x^2/2)/x$, $g_2 = C\alpha_s m_q^2 x^3/2\mu^2$, $C = |\lambda^a_{fi}\chi^*_a/2|^2$.

$\vec{q}(z) = \vec{p}_g(z)(1-x) - \vec{p}_{q'}(z)x$, $\mu = E_q x(1-x)$.

Gluon emission in a uniform field

For a uniform field $\vec{q}(z_2)\vec{q}(z_1) = [\vec{Q}^{\,2} - \vec{f}^{\,2}\tau^2/4]$, $\vec{Q} = \vec{q}((z_1 + z_2)/2)$, $\tau = z_2 - z_1$, and $\vec{f} = d\vec{q}/dz = \vec{F}_g(1-x) - \vec{F}_{q'}x$. After replacing the integration over $\vec{p}_g(\infty)$ by the integration over $\vec{Q}$ we obtain

$$\frac{dP}{dLdx} = \frac{1}{(2\pi)^2}\int d\vec{Q}\int_{-\infty}^{\infty} d\tau \left[\frac{g_1}{\mu^2}\left(\vec{Q}^{\,2} - \frac{\vec{f}^{\,2}\tau^2}{4}\right) + g_2\right] \exp\left[-i\Phi(\tau,\vec{Q})\right],$$

with $\Phi(\tau,\vec{Q}) = \frac{(\epsilon^2+\vec{Q}^2)\tau}{2\mu} + \frac{\vec{f}^{\,2}\tau^3}{24\mu}$ $\epsilon^2 = m_q^2x^2 + m_g^2(1-x)$. Integrating over τ by parts one can kill $\vec{Q}^2$

$$\frac{dP}{dLdx} = -\frac{1}{(2\pi)^2}\int d\vec{Q}\int_{-\infty}^{\infty} d\tau \left[\frac{g_1}{\mu^2}\left(\epsilon^2 + \frac{\vec{f}^{\,2}\tau^2}{2}\right) - g_2\right] \exp\left[-i\Phi(\tau,\vec{Q})\right].$$

$$\Rightarrow \quad \frac{dP}{dLdx} = \frac{i\mu}{2\pi}\int_{-\infty}^{\infty} \frac{d\tau}{\tau}\left[\frac{g_1}{\mu^2}\left(\epsilon^2 + \frac{\vec{f}^{\,2}\tau^2}{2}\right) - g_2\right] \exp\left\{-i\left[\frac{\epsilon^2\tau}{2\mu} + \frac{\vec{f}^{\,2}\tau^3}{24\mu}\right]\right\}.$$

Here it is assumed that τ has a small negative imaginary part. The integral around the lower semicircle near the pole at $\tau = 0$ plays the role of the $\vec{f} = 0$ subtraction term.

In terms of the Airy function $\mathrm{Ai}(z)=\frac{1}{\pi}\sqrt{\frac{z}{3}}K_{1/3}(2z^{3/2}/3)$ ($K_{1/3}$ is the Bessel function) the spectrum reads

$$\frac{dP}{dLdx}=\frac{a}{\kappa}\mathrm{Ai}'(\kappa)+b\int_{\kappa}^{\infty}dy\mathrm{Ai}(y)\,,$$

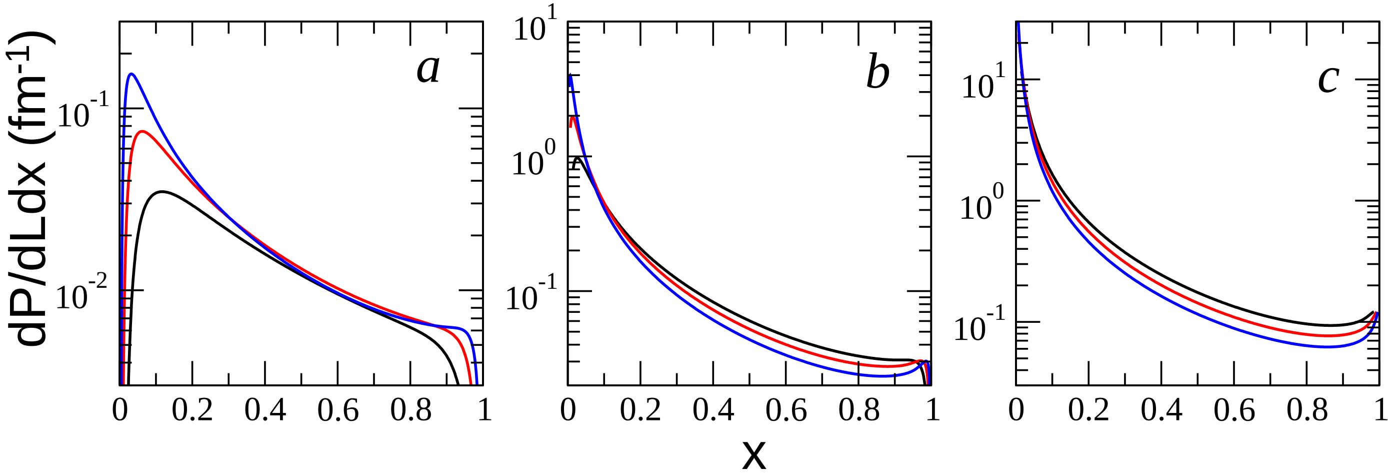
where $a=-2\epsilon^2 g_1/\mu$, $b=\mu g_2-\epsilon^2 g_1/\mu$, $\kappa=\epsilon^2/(\mu^2\vec{f}^{\,2})^{1/3}$.

Thus the effect of field is only accumulated in $\vec{f}^{\,2}=\vec{F}_{q'}^{\,2}x_g^2-2\vec{F}_{q'}\vec{F}_g x_{q'}x_g+\vec{F}_g^2 x_q^{\,2}$.

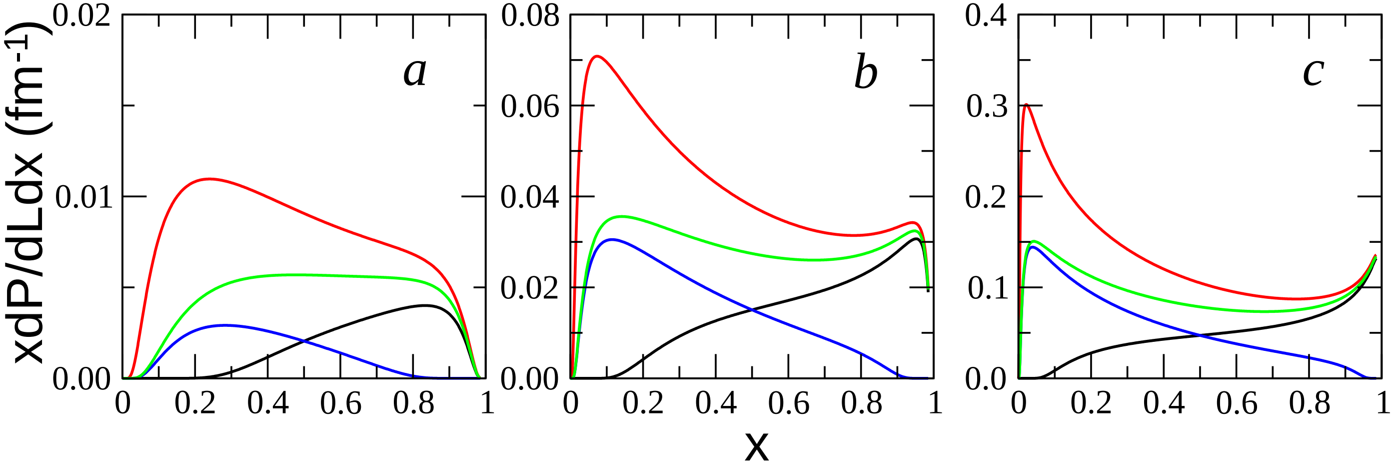
Our spectrum disagrees with that obtained by Shuryak and Zahed [Phys. Rev. D**67**, 054025 (2003)] in the soft gluon limit within the Schwinger's proper time method.

- In the SZ formula the argument of the exponential contains $\vec{F}_{q'}^{\,2}x_g^2+\vec{F}_g^{\,2}$.
- In the pre-exponential factor instead of $\vec{f}^2$ SZ have $\vec{F}_{q'}^{\,2}x_g^2$.

The SZ predictions are physically absurd: The spectrum is insensitive to the relation between the signs of the color charges of the final partons. The $q_1\to g_Z q_3$ transition for the chromomagnetic field in the color state A in the massless limit vanishes (since $\vec{F}_{q'}=0$). This process is analogous to the synchrotron radiation in QED, and there is no physical reason why it should vanish. Since the pre-exponential factor contains the Lorentz forces in non-symmetric form it is clear that the $g\to gg$ spectrum will have incorrect permutation properties.



The $q \rightarrow gq'$ spectrum for magnetic field in the color state A for $gH_A/m_D^2 = 0.05$ (a), 0.25 (b) and 1 (c), $\alpha_s = 0.3$, $m_q = 0.3$ GeV, $m_g = 0.4$ GeV, the initial quark energies: $E_q = 20$ GeV, $E_q = 40$ GeV, $E_q = 80$ GeV.



The $q \to gq'$ spectrum for specific color states. $\alpha_s = 0.3$, $E_q = 20$ GeV, magnetic field is in the color state A with $gH_A/m_D^2 = 0.05$ (a), 0.25 (b) and 1 (c).

$q_1 \to g_A q_1$, $Q_A^q = 1/2$, $Q_A^{q'} = 1/2$, $Q_A^g = 0$ (analogous to $e \to \gamma e$ in QED)

$q_1 \to g_Z q_3$, $Q_A^q = 1/2$, $Q_A^{q'} = 0$, $Q_A^g = 1/2$ (vanishes in SZ)

$q_1 \to g_{\bar{X}} q_2$, $Q_A^q = 1/2$, $Q_A^{q'} = -1/2$, $Q_A^g = 1$

$q_3 \to g_Y q_1$, $Q_A^q = 0$, $Q_A^{q'} = 1/2$, $Q_A^g = -1/2$

Energy loss due to synchrotron radiation

Without magnetic field jet quenching is dominated by the induced gluon emission due to multiple scattering on thermal partons. The collisional energy loss gives small effect $\Delta E_{col}/\Delta E_{rad} \sim 0.2 - 03$, and $\Delta E_{col}/E \sim 0.03 - 0.05$ at $E \lesssim 40$ GeV

Can the synchrotron radiation modify strongly the jet quenching?

$$\frac{\epsilon_{mag}}{\epsilon_{thermal}} \sim \alpha_s \left(\frac{gH}{m_D^2}\right)^2$$

This ratio is ~ 0.3 if $gH \sim m_D^2$. Such a value of magnetic field is required by the scenario with turbulent viscosity [Asakawa, Bass, Müller (2007)] for explaining small η/s.

$gH \sim m_D^2$ gives $\Delta E/E \sim 0.1 - 0.2$ at $E \sim 10 - 20$ GeV for $L \sim 2 - 4$ fm.

The finite-size effects become important if $L_c \sim L$. We have $L_c \sim 1 - 2$ fm. The finite-size effects may suppress the energy loss by a factor ~ 0.5.

The finite coherence length of the turbulent magnetic field, L_m, suppresses the radiation as well. For the unstable magnetic field modes the wave vector $k^2 \lesssim \xi m_D^2$ [Asakawa, Bass, Müller (2007)], we have $L_m/L_c \gtrsim 1$. The turbulent suppression should not be strong, and as a plausible estimate one can take the turbulent suppression factor ~ 0.5.

With these suppression factors we have

$$\Delta E_{synch} \sim \Delta E_{coll}$$

Conclusions:

- We have developed a quasiclassical theory of the synchrotron-like gluon radiation.
- In the QGP the gluon spectrum is dominated by the processes with emission of the charged gluons, the effect of the neutral gluons is relatively small.
- The parton energy loss due to the synchrotron radiation may be important in the jet quenching if the QGP instabilities generate magnetic field $H \sim m_D^2/g$.
- Our gluon spectrum disagrees with that obtained by Shuryak and Zahed. Simple physical arguments are given that the SZ spectrum is incorrect.