On the possibility of charged pion condensation phenomenon in dense quark matter

K. G. Klimenko

Institute for High Energy Physics, 142281 Protvino, Russia

Protvino, 2015

Motivation and objective

QCD is a theory of dense baryonic matter (neutron stars, heavy-ion collisions). At physically acceptable densities and/or energies the QCD coupling constant is large $\Rightarrow$ the perturb. theory does not work. To describe the low density baryonic matter one can use effective models, e. g., The Nambu–Jona-Lasinio model (NJL):

$$L = \bar{q}\left[\gamma^\nu \mathrm{i}\partial_\nu + m_0 + \mu\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0\right]q + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\right],$$

where the quark field $q(x)$ is a flavor doublet and color N_c-plet; τ_k $(k = 1,2,3)$ are Pauli matrices; the quark number chemical potential μ is responsible for the nonzero baryonic density of quark matter, whereas the isospin chemical potential μ_I is taken into account in order to study properties of quark matter at nonzero isospin densities (in this case the densities of u and d quarks are different). At present time the model is a basis for the consideration of color supercond. as well as pion condensation (PC) effects, etc.

- (3+1)-dimensional NJL model :

$$\begin{aligned}\tilde{L} &= \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + m_0 + \mu\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q \\ &- \frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big], \end{aligned} \tag{1}$$

where $\sigma(x) = -2\frac{G}{N_c}(\bar{q}q)$; $\pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q)$. At $m_0 = 0$, $\mu_I \neq 0$ Lagrangian is symm with respect to $U_{I_3}(1) \times U_{AI_3}(1)$, where $I_3 = \tau_3/2$ is the third component of the isospin operator. $U_{I_3}(1)$ is the isospin subgroup ($q \to \exp(\mathrm{i}\alpha\tau_3)q$) and $U_{AI_3}(1)$ is the axial isospin subgroup ($q \to \exp(\mathrm{i}\alpha\gamma^5\tau_3)q$).

$$\mathcal{S}_{\mathrm{eff}}(\sigma, \pi_a) = -N_c \int \frac{\sigma^2 + \pi_a^2}{4G} d^4x + \tilde{\mathcal{S}}_{\mathrm{eff}}, \tag{2}$$

$$\begin{aligned}\exp(\mathrm{i}\tilde{\mathcal{S}}_{\mathrm{eff}}) = N' \int [d\bar{q}][dq] \exp\Big(\mathrm{i}\int \bar{q}[\mathrm{i}\gamma^\rho\partial_\rho + m_0 + \mu\gamma^0 \\ + \frac{\mu_I}{2}\tau_3\gamma^0 - \sigma(x) - \mathrm{i}\gamma^5\pi_a(x)\tau_a]q\, d^4x\Big). \end{aligned} \tag{3}$$

The ground state expectation values $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ of the composite bosonic fields are determined by the saddle point equations (S. Coleman in "Secret symmetry", formula (3.18)),

$$\frac{\delta\mathcal{S}_{\rm eff}}{\delta\sigma(x)}=0, \qquad \frac{\delta\mathcal{S}_{\rm eff}}{\delta\pi_a(x)}=0. \tag{4}$$

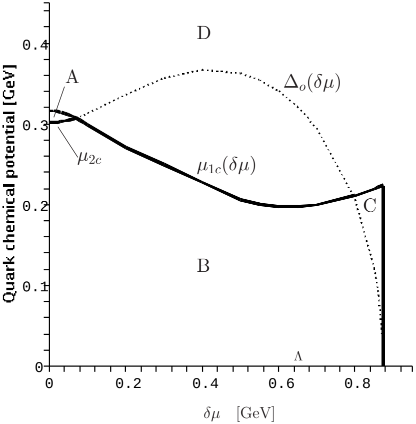
It is assumed usually that order parameters $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinates. In this case

$$\int d^4x\Omega(\sigma,\pi_a)=-\frac{1}{N_c}\mathcal{S}_{\rm eff}\big(\sigma(x),\pi_a(x)\big)\Big|_{\sigma(x)=\sigma,\pi_a(x)=\pi_a}, \tag{5}$$

$$U_{I_3}(1): \qquad \sigma\to\sigma;\ \ \pi_3\to\pi_3;\ \ \pi_1^2+\pi_2^2={\rm inv}_1,$$
$$U_{AI_3}(1): \qquad \pi_1\to\pi_1;\ \ \pi_2\to\pi_2;\ \ \sigma^2+\pi_3^2={\rm inv}_2.$$

$\Rightarrow$ the effective potential depend on $U_{I_3}(1)\times U_{AI_3}(1)$- invariants ${\rm inv}_1$, ${\rm inv}_2 \Rightarrow$ without loss of generality one can put $\langle\pi_3\rangle=0$ and $\langle\pi_2\rangle=0$. $\langle\sigma\rangle$ and $\langle\pi_1\rangle$ are order parameters: If $\langle\sigma\rangle\neq0$ then $U_{AI_3}(1)$ is spontaneously broken.

If $\langle\pi_1\rangle \sim \langle\bar{q}i\gamma^5\tau_1 q\rangle \neq 0$ then isotopic symmetry $U_{I_3}(1)$ is spont broken and we have charged pion condensed phase. Phase structure of the model at $m_0 = 0$ in 3+1 dim case for $\Lambda = 0.65$ GeV, $G = 5.01$ GeV^{-1}, where $\delta\mu = \mu_I/2$:



(1+1)-dim NJL model with homogeneous condensates, $\langle\sigma\rangle$ and $\langle\pi_1\rangle$ do not depend on spatial coordinate x.

In the leading order of $1/N_c$ expansion the TDP looks like (here $M - m_0 \equiv \sigma$ and $\Delta \equiv \pi_1$):

$$\Omega^{un}(M,\Delta) = \frac{(M-m_0)^2+\Delta^2}{4G} - \int_{-\infty}^{\infty}\frac{dp_1}{2\pi}\Big\{E_\Delta^+ + E_\Delta^- + (\mu - E_\Delta^+)\theta(\mu - E_\Delta^+) + (\mu - E_\Delta^-)\theta(\mu - E_\Delta^-)\Big\}, \qquad (6)$$

where $E_\Delta^\pm = \sqrt{(E^\pm)^2+\Delta^2}$, $E^\pm = E\pm\nu$, $\nu = \mu_I/2$ and $E = \sqrt{p_1^2+M^2}$. Coordinates of the global min point (GMP),or gaps, of TDP give us the ground state expect values of σ and π_1 fields, i.e. $M_{min} - m_0 = \langle\sigma(x)\rangle$, $\Delta_{min} = \langle\pi_1(x)\rangle$. $\Rightarrow$ if $\Delta_{min} \neq 0$, the charged pion condensation occurs. Moreover, M_{min} is the dynamical quark mass. The TDP is an ultraviolet divergent quantity, so in order to get any physical information one should renormalize it, using a special dependence of such quantities as the bare coupling constant G and the bare quark mass m_0 on the cutoff parameter Λ (Λ restricts the integration region in the divergent integral in (6), $|p_1| < \Lambda$). It is easy to see that, cutting of the divergent integral in (6) and then using the substitution $G \equiv G(\Lambda)$ and $m_0 \equiv mG(\Lambda)$, where

$$\frac{1}{2G(\Lambda)} = \frac{2}{\pi}\ln\left(\frac{2\Lambda}{M_0}\right) \qquad (7)$$

and m, M_0 are new free massive parameters of the model. Then in the limit $\Lambda\to\infty$ it is possible to obtain a finite expression for the TDP. Namely,

$$\Omega^{ren}(M,\Delta) = V_0(M,\Delta) - \frac{mM}{2} - \int_{-\infty}^{\infty}\frac{dp_1}{2\pi}\Big\{E_\Delta^+ + E_\Delta^- - 2\sqrt{p_1^2+M^2+\Delta^2} + (\mu - E_\Delta^+)\theta(\mu - E_\Delta^+) + (\mu - E_\Delta^-)\theta(\mu - E_\Delta^-)\Big\}$$

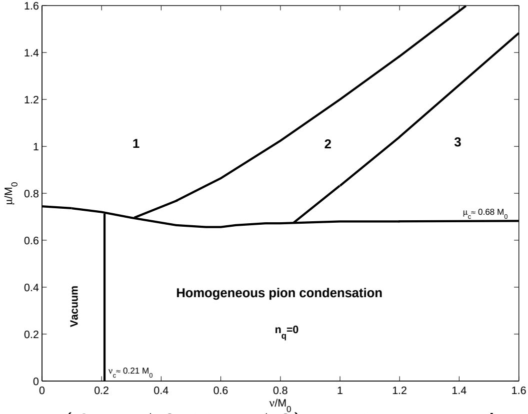
where

$$V_0(M,\Delta) = \frac{M^2+\Delta^2}{2\pi}\left[\ln\left(\frac{M^2+\Delta^2}{M_0^2}\right) - 1\right] \qquad (9)$$

is the TDP in vacuum, i.e. at $\mu = 0$ and $\mu_I = 0$, taken in the chiral limit, i.e. at $m = 0$. M_0 is the dynamical quark mass in vacuum, i.e. at $\mu, \nu = 0$.

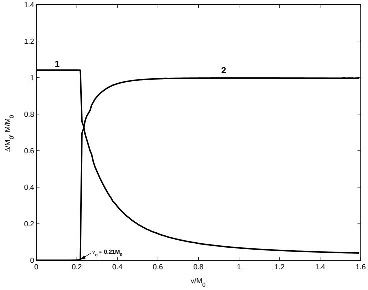
Phase structure

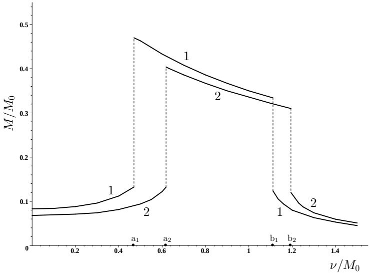
We suppose that $m = 0.05M_0$ (in this case the relation between the dynamical quark mass $M_{dyn} \equiv M_{min}$ and π-meson mass M_π is the same as in the framework of NJL$_4$ model, where $M_{dyn} = 350$ MeV, $M_\pi = 140$ MeV), i.e. $M_{dyn}/M_\pi = 5/2$.



In the PC phase: $(\Delta_{min} \neq 0, M_{min} \neq 0)$, In Vacuum and 1-, 2-, 3-phases: $(\Delta = 0, M_{min} \neq 0)$.

The behavior of M_{min} and Δ_{min} vs $\nu = \mu_I/2$



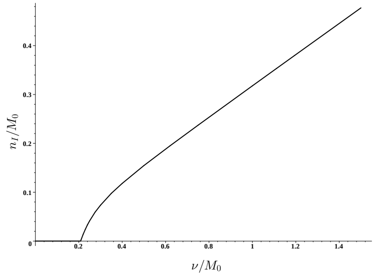


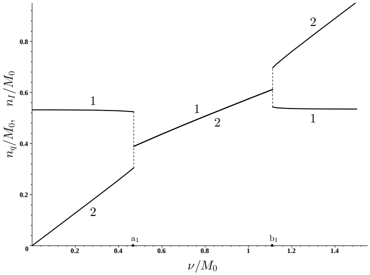
Left figure: The behavior of M_{min} (1) and Δ_{min} (2) in Vacuum and PC phases at small fixed values of μ.

Right figure: The behavior of M_{min} vs ν at $\mu/M_0 = 0.84$ (1) and $\mu/M_0 = 0.94$ (2). In this case $\Delta_{min} = 0$.

Particle number n_q and isospin n_I densities vs ν

$$n_q = \partial\Omega(M,\Delta)/\partial\mu\big|_{min}, \qquad n_I = \partial\Omega(M,\Delta)/\partial\mu_I\big|_{min}$$





Left figure: The behavior of n_I in Vacuum and PC phases at small fixed values of μ. In this case $n_q \equiv 0$.
Right figure: The behavior of n_q (1) and n_I (2) at $\mu/M_0 = 0.84$.

Quasiparticles

Each quasiparticle is a one-quark excitation of a ground state. In the GN model (1) there are 4 quasiparticles, i.e. one u, d, $\bar{u}, \bar{d}$-excitations, in each phase. Their energies vs p_1 are

$$E_u(p_1) = E_\Delta^- - \mu, \quad E_d(p_1) = E_\Delta^+ - \mu,$$
$$E_{\bar{u}}(p_1) = -(E_\Delta^+ + \mu), \quad E_{\bar{d}}(p_1) = -(E_\Delta^- + \mu).$$

- In Vacuum and PC phase all quarks are gapped particles.
- In the phase 1 u, d-quarks are gapless.
- In the phases 2, 3 u-quarks are gapless, but d-quarks are gapped.

Namely, we suppose the following structure of the condensates
$\langle\sigma(x)\rangle = M - m_0$, $\langle\pi_3(x)\rangle = 0$, $\langle\pi_1(x)\rangle = \Delta\cos(2bx)$, $\langle\pi_2(x)\rangle = \Delta\sin(2bx)$

$$\Omega^{un}(M,b,\Delta) = \frac{(M-m_0)^2+\Delta^2}{4G} + \mathrm{i}\int\frac{d^2p}{(2\pi)^2}\ln\left(\eta^4+A\eta^2+B\eta+C\right), \tag{10}$$

where the notations $\eta = p_0+\mu$ and

$$\begin{aligned} A &= -2(M^2+b^2+p_1^2+\nu^2+\Delta^2), \quad B=-8p_1b\nu, \\ C &= (M^2+b^2+p_1^2+\nu^2+\Delta^2)^2 - 4(p_1^2\nu^2+b^2\nu^2+\Delta^2b^2+M^2\nu^2+p_1^2b^2) \end{aligned} \tag{11}$$

are used. The argument of the ln-function in (10) can be expanded into a product of four linear multipliers,

$$\Omega^{un}(M,b,\Delta) = \frac{(M-m_0)^2+\Delta^2}{4G} + \mathrm{i}\int\frac{d^2p}{(2\pi)^2}\ln\left[(\eta-\eta_1^+)(\eta-\eta_1^-)(\eta-\eta_2^+)(\eta-\eta_2^-)\right], \tag{12}$$

$$p_{0u} = \eta_1^- - \mu,\quad p_{0d} = \eta_1^+ - \mu,\quad p_{0\bar u} = \eta_2^- - \mu,\quad p_{0\bar d} = \eta_2^+ - \mu. \tag{13}$$

$$\int_{-\infty}^{\infty} dp_0 \ln\left(p_0 - a\right) = \mathrm{i}\pi|a|, \tag{14}$$

$$\Omega^{un}(M,b,\Delta) = \frac{(M-m_0)^2+\Delta^2}{4G} - \int_{-\infty}^{\infty}\frac{dp_1}{4\pi}\left[|p_{0u}|+|p_{0d}|+|p_{0\bar u}|+|p_{0\bar d}|\right]. \tag{15}$$

Renormalization

$$\Omega^{reg}(M,b,\Delta) = \frac{(M-m_0)^2+\Delta^2}{4G} - \int_{-\Lambda}^{\Lambda}\frac{dp_1}{4\pi}\left[|p_{0u}|+|p_{0d}|+|p_{0\bar{u}}|+|p_{0\bar{d}}|\right]. \qquad (16)$$

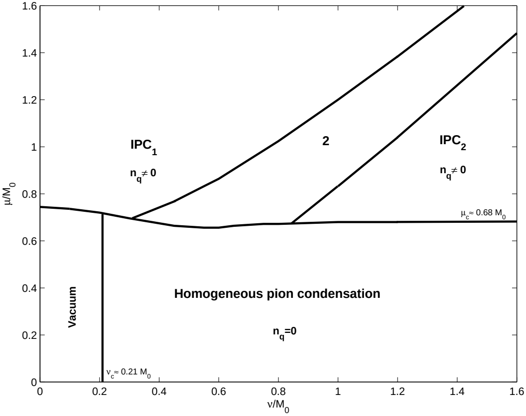
$$\Omega^{reg}(M,b,\Delta) = \left(\Omega^{reg}(M,b,\Delta) - \Omega^{reg}(M,b,\Delta)\big|_{b=0,\mu=0,\nu=0}\right) + \Omega^{reg}(M,b,\Delta)\big|_{b=0,\mu=0,\nu=0}. \qquad (17)$$

Since $|p_{0u}| = |p_{0d}| = |p_{0\bar{u}}| = |p_{0\bar{d}}| = \sqrt{p_1^2+M^2+\Delta^2}$, we have

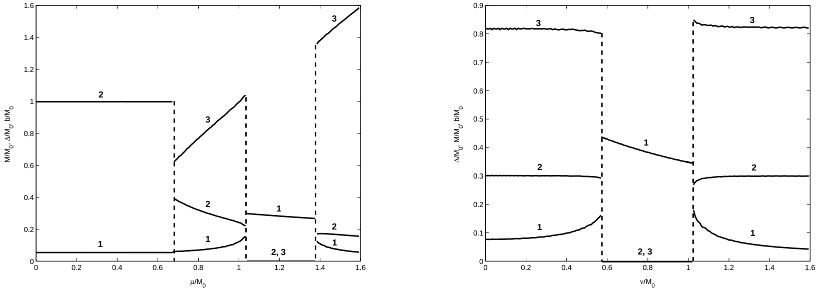
$$\Omega^{reg}(M,b,\Delta)\big|_{b=0,\mu=0,\nu=0} = \frac{(M-m_0)^2+\Delta^2}{4G} - \int_{-\Lambda}^{\Lambda}\frac{dp_1}{\pi}\sqrt{p_1^2+M^2+\Delta^2}. \qquad (18)$$

$$\Omega^{un}(M,b,\Delta) \longrightarrow \Omega^{ren}(M,b,\Delta) = V_0(M,\Delta) - \frac{mM}{2} - \lim_{\Lambda\to\infty}\left\{\int_{-\Lambda}^{\Lambda}\frac{dp_1}{4\pi}\Big[|p_{0u}|+|p_{0d}| + |p_{0\bar{u}}|+|p_{0\bar{d}}| - 4\sqrt{p_1^2+M^2+\Delta^2}\Big]\right\}. \qquad (19)$$

where $V_0(M,\Delta)$ is given in (9).

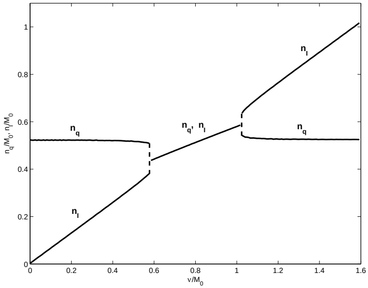


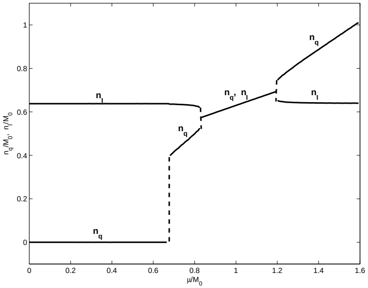
Phase portrait of the model when inhomogeneity of pion condensates is taken into account. Here $\text{IPC}_{1,2}$ denote inhomogeneous charged pion condensation phases, n_q is a quark number density, 2 is a normal quark matter phase with $\Delta = 0$, $M \neq 0$.



Left figure: Gaps M (line 1), Δ (line 2) and inhomogeneity wave vector b (line 3) vs μ at fixed $\nu = 1.2M_0$.
Right figure: Gaps M (line 1), Δ (line 2) and inhomogeneity wave vector b (line 3) vs ν at fixed $\mu = 0.85M_0$.

Behavior of the quark number n_q and isospin n_I densities vs μ and $\nu = \mu_I/2$





Left figure: Quark number, n_q, and isospin, n_I, densities vs ν at $\mu = 0.85M_0$.
Right figure: Quark number, n_q, and isospin, n_I, densities vs μ at $\nu = M_0$.

Literature

1) D. Ebert and K. G. Klimenko, "Gapless pion condensation in quark matter with finite baryon density," J. Phys. G **32**, 599 (2006) [hep-ph/0507007].

2) D. Ebert and K. G. Klimenko, "Properties of the massive Gross-Neveu model with nonzero baryon and isospin chemical potentials," Phys. Rev. D **80**, 125013 (2009) [arXiv:0911.1944 [hep-ph]].

3) N. V. Gubina, K. G. Klimenko, S. G. Kurbanov and V. C. Zhukovsky, "Inhomogeneous charged pion condensation phenomenon in the NJL_2 model with quark number and isospin chemical potentials," Phys. Rev. D **86**, 085011 (2012) [arXiv:1206.2519 [hep-ph]].