Duality correspondence between chiral symmetry breaking and charged pion condensation in (1+1)-dimensional Gross-Neveu model with baryon-, isospin- and axial (chiral) isospin chemical potentials with spatially inhomogeneous condensates.

R.N. Zhokhov

10 октября 2016 г.

Model and its Lagrangian

We consider a two-dimensional model which describes dense quark matter with two massless quark flavors (u and d quarks).

$$L = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5\Big]q +$$

$$+\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big], \qquad (1)$$

The model (1) is a generalization of the two-dimensional Gross-Neveu model with a single massless quark color N_c-plet to the case of two quark flavors and additional baryon μ_B-, isospin μ_I- and axial isospin μ_{I5} chemical potentials. These parameters are introduced in order to describe in the framework of the model (1) quark matter with nonzero baryon n_B-, isospin n_I- and axial isospin n_{I5} densities, respectively.

Symmetries of Lagrangian

Lagrangian is invariant with respect to the abelian $U_B(1)$, $U_{I_3}(1)$ and $U_{AI_3}(1)$ groups,

$$U_B(1): \;\; q \to \exp(\mathrm{i}\alpha/3)q; \tag{2}$$
$$U_{I_3}(1): \;\; q \to \exp(\mathrm{i}\alpha\tau_3/2)q; \tag{3}$$
$$U_{AI_3}(1): \;\; q \to \exp(\mathrm{i}\alpha\gamma^5\tau_3/2)q. \tag{4}$$

Lagrangian (1) is invariant with respect to the electromagnetic $U_Q(1)$ group,

$$U_Q(1): \;\; q \to \exp(\mathrm{i}Q\alpha)q, \tag{5}$$

where $Q = \mathrm{diag}(2/3, -1/3)$.

Equivalent Lagrangian

To find the thermodynamic potential of the system, we use a semi-bosonized version of the Lagrangian (1), which contains composite bosonic fields $\sigma(x)$ and $\pi_a(x)$ $(a = 1, 2, 3)$

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^1 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q$$

$$-\frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big]. \qquad (6)$$

For bosonic fields one has

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q). \qquad (7)$$

Chiral density wave and pion wave

In vacuum, i.e. in the state corresponding to an empty space with zero particle density and zero values of the chemical potentials μ, ν and ν_5, the quantities $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate x. However, in a dense medium, when $\mu \neq 0$, $\nu \neq 0$ and $\nu_5 \neq 0$, the ground state expectation values of bosonic fields might have a nontrivial dependence on x. In particular, in this paper we will use the following ansatz:

$$\langle\sigma(x)\rangle = M\cos(2bx), \ \langle\pi_3(x)\rangle = M\sin(2bx),$$

$$\langle\pi_1(x)\rangle = \Delta\cos(2b'x), \ \langle\pi_2(x)\rangle = \Delta\sin(2b'x), \tag{8}$$

where M, b, b' and Δ are constant dynamical quantities. In fact, they are coordinates of the global minimum point of the thermodynamic potential (TDP) $\Omega(M, b, b', \Delta)$.

In the leading order of the large N_c-expansion it is defined by the following expression:

$$\int d^2x\Omega(M, b, b', \Delta) = -\frac{1}{N_c}\mathcal{S}_{\text{eff}}\{\sigma(x), \pi_a(x)\}\Big|_{\sigma(x)=\langle\sigma(x)\rangle, \pi_a(x)=\langle\pi_a(x)\rangle}, \quad (9)$$

for the thermodynamic potential one can obtain

$$\Omega(M, b, b', \Delta) = \frac{M^2 + \Delta^2}{4G} + \mathrm{i} \int \frac{d^2 p}{(2\pi)^2} \ln P_4(p_0). \tag{10}$$

where

$$P_4(p_0) = \eta^4 - 2a\eta^2 - b\eta + c, \eta = p_0 + \mu$$

$$a = M^2 + \Delta^2 + p_1^2 + \tilde{\nu}^2 + \tilde{\nu}_5^2; \quad b = 8p_1\tilde{\nu}\tilde{\nu}_5;$$

$$c = a^2 - 4p_1^2(\tilde{\nu}^2 + \tilde{\nu}_5^2) - 4M^2\tilde{\nu}^2 - 4\Delta^2\tilde{\nu}_5^2 - 4\tilde{\nu}^2\tilde{\nu}_5^2.$$

$$\tilde{\nu} = \nu + b, \qquad \tilde{\nu}_5 = \nu_5 + b'.$$

$$\mu \equiv \mu_B/3, \ \nu = \mu_I/2, \ \nu_5 = \mu_{I5}/2$$

.

Duality

The thermodynamic potential is invariant with respect to the so-called duality transformation

$$\mathcal{D}: \; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5, \quad b \longleftrightarrow b'. \tag{11}$$

If we change axes $\nu \longleftrightarrow \nu_5$ then we should exchange PC $\longleftrightarrow$ CSB.

$$F_1(M) \equiv \Omega^{ren}(M, \Delta = 0)$$
$$F_2(\Delta) \equiv \Omega^{ren}(M = 0, \Delta)$$

$$F_2(\Delta) \;=\; F_1(\Delta)\Big|_{\nu \longleftrightarrow \nu_5}. \tag{12}$$

Inhomogeneous case

At $M = 0$ and $\Delta = 0$ the expression for thermodynamic potential does depend on b and b'. This is quite unphysical and somehow we need to change the expresssion for thermodynamic potential.

$$\tilde{F}_1(M, b) = \tilde{\Omega}(M, b, b', 0) = F_1(M, b) - F_1(0, b) + F_1(0, 0) \quad (13)$$

$$\tilde{F}_2(\Delta, b') = \tilde{\Omega}(0, b, b', \Delta) = F_2(\Delta, b') - F_2(0, b') + F_2(0, 0) \quad (14)$$

for thermodynamic potential

$$\tilde{\Omega}(M, b, b', \Delta) = \Omega(M, b, b', \Delta) - \Omega(M, b, b', 0) \quad (15)$$

$$+\Omega(M, b, 0, 0) - \Omega(0, b, b', \Delta) + \Omega(0, 0, b', \Delta)$$

$$-\Omega(0, b, 0, 0) - \Omega(0, 0, b', 0) + \Omega(0, b, b', 0) + \Omega(0, 0, 0, 0)$$

Phase portrait (μ, ν)

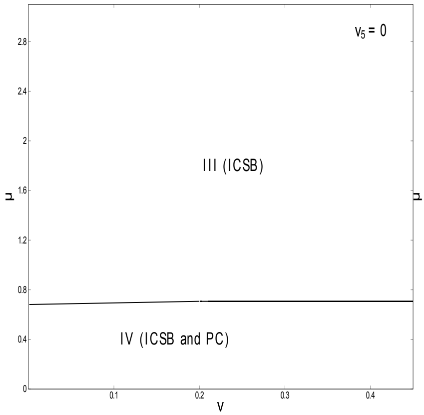

Рис.: (μ, ν) phase diagram at $\nu_5 = 0$

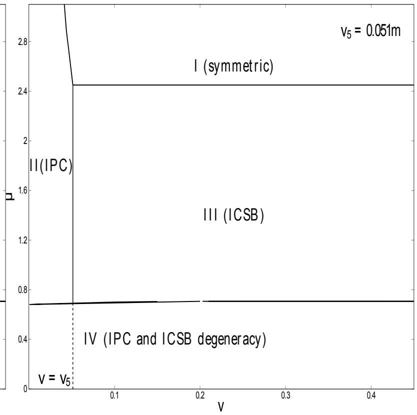

Рис.: (μ, ν) phase diagram at $\nu_5 = 0.051m$

Phase portrait (μ, ν)

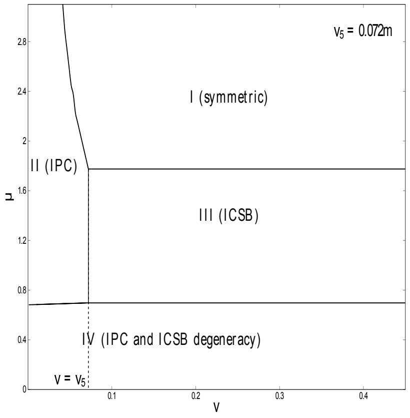

Рис.: (μ, ν) phase diagram at $\nu_5 = 0.072m$

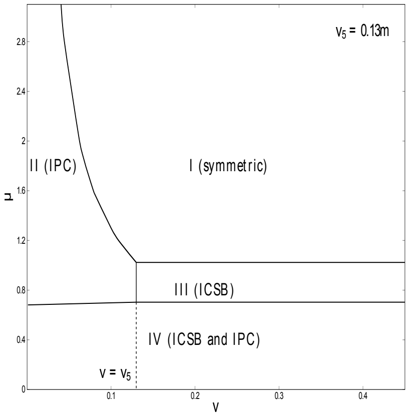

Рис.: (μ, ν) phase diagram at $\nu_5 = 0.13m$

Phase portrait

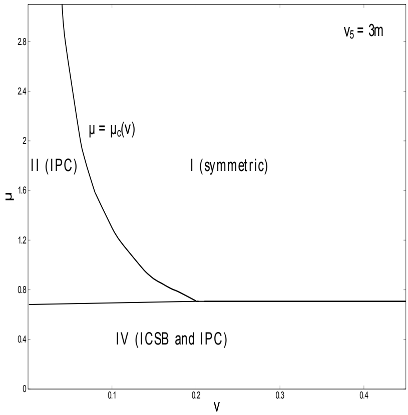

Рис.: (μ, ν) phase diagram at $\nu_5 = 3m$

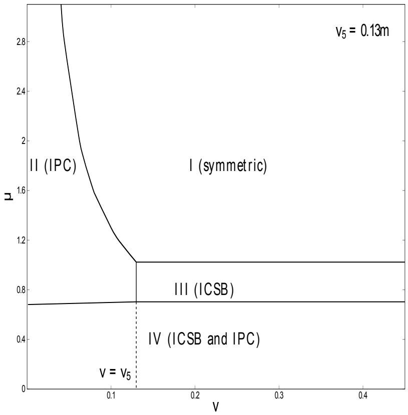

Рис.: (μ, ν_5) phase diagram at $\nu = 0.13m$

Phase portrait (ν, ν_5)

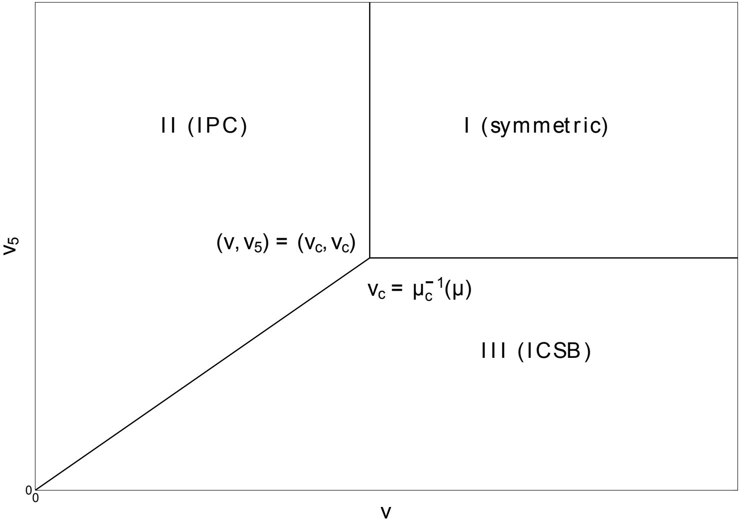

Рис.: (ν, ν_5) phase diagram at $\mu > 0.707$

Phase portrait in homogeneous case

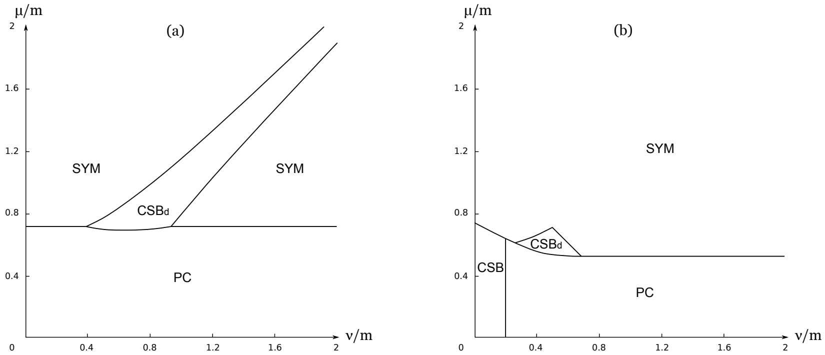

Рис.: The (ν, μ)-phase portrait of the model for different values of the chiral chemical potential ν_5: (a) The case $\nu_5 = 0$. (b) The case $\nu_5 = 0.2m$.

Phase portrait in homogeneous case

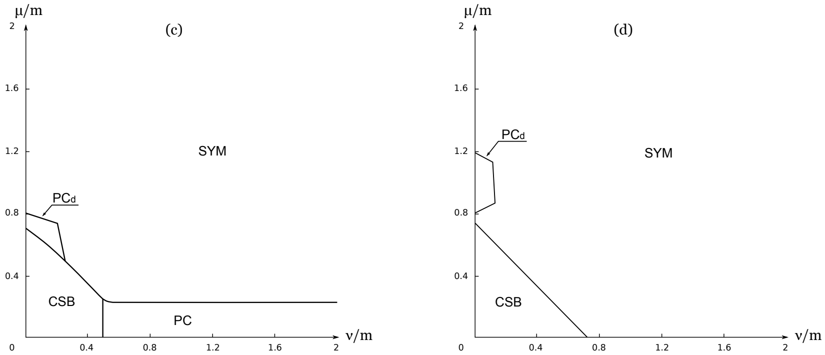

Рис.: The (ν, μ)-phase portrait of the model for different values of the chiral chemical potential ν_5:(a) The case $\nu_5 = 0.5m$. (b) The case $\nu_5 = m$.

Conclusions

Our consideretion aims at study of the properties of chirally ($\mu_{I5} \neq 0$) and isotopically ($\mu_I \neq 0$) asymmetric dense ($\mu_B \neq 0$) quark matter with inhomogeneous condensates.

- At $\mu_{I5} \neq 0$ even in homogeneous case there is charged PC phase with nonzero baryon density.
- Charged PC phase realises at any nonzero $\mu_{I5} \neq 0$ in contrst to homogeneous case where it realises only for rather large values of μ_{I5}(large than some value). It means that cahrged pion condensation happens at even small chiral asymmetry.
- In the leading order of the large-N_c approximation in the framework of the NJL_2 model (1) there is a duality correspondence between CSB and charged PC phenomena.