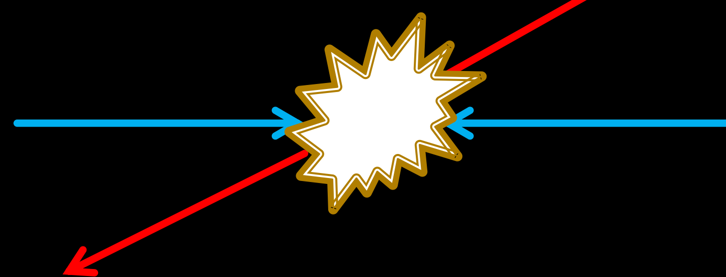
The XXIII International Baldin Seminar
on High Energy Physics Problems
"Relativistic Nuclear Physics and Quantum Chromodynamics"
19-24 September 2016 Dubna

Sizes and Distances in High-Energy Physics



Vladimir A. Petrov
Institute for High-Energy Physics
"NRC Kurchatov Institute"
Protvino, Russia

Space-time characteristics in use

- “Charge radii” of hadrons → “hadron sizes”
- Spatio-temporal interaction ranges in high-energy collisions
- Space-time sizes of the particle “emission source”

"Charge Radii" and Hadron Sizes

$$d\sigma \sim F^2(\boldsymbol{q})$$

$$F(\boldsymbol{q}) = \sum e_k \int dx\ e^{i\boldsymbol{q}\boldsymbol{x}} \rho_k(\boldsymbol{x})$$

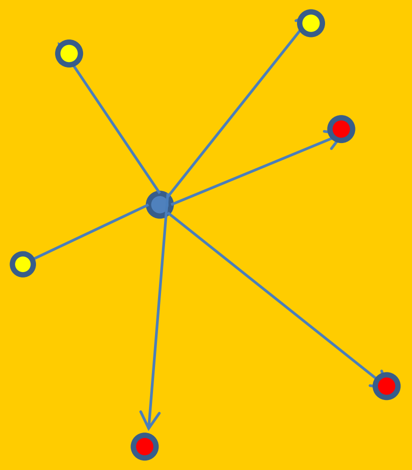
$$\rho_k(\boldsymbol{x}) = N_k \int d\boldsymbol{x}_2 \ldots d\boldsymbol{x}_{N_k} \left|\psi(\boldsymbol{x}, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_{N_k})\right|^2$$

$\sum_k e_k N_k$ = net system's charge = $F(0)$

"Charge Radius" vs the True One

$$F(\boldsymbol{q}) = F(0) - \frac{\boldsymbol{q}^2}{6}\langle \boldsymbol{r}^2\rangle_{charge} + \cdots$$

$$\langle \boldsymbol{r}^2\rangle_{charge} = -\frac{\partial F(\boldsymbol{q})}{\partial \boldsymbol{q}^2}\Big|_{\boldsymbol{q}^2=0}$$



$$\langle \boldsymbol{r}^2\rangle_{charge} = \sum_{k=1}^{\nu} e_k N_k \langle \boldsymbol{x}^2\rangle_k \ngeq 0$$

$$\langle \boldsymbol{r}^2\rangle_{true} = \frac{1}{\nu}\sum_{k=1}^{\nu} \langle \boldsymbol{x}^2\rangle_k \geq 0$$

$$r_e^2(proton)(ep\ \text{CODATA}) = 0.7700 \pm 0.0089 fm^2 = (\mathbf{0.8775} \pm \mathbf{0.0051}\ \boldsymbol{fm})^2$$

$$\langle \boldsymbol{r}^2\rangle_{true}(\boldsymbol{proton}) = 0.6539 \pm 0.0092 fm^2 = (0.8086 \pm 0.0070 fm)^2$$

Interaction Region

Transverse size of the IR

$$\langle b^2 \rangle \equiv 2B(s) = 2\,\partial[\ln(d\sigma/dt)]/\partial t|_{t=0}$$

$$\langle b^2 \rangle^{1/2}(LHC) \approx 1.3\ Fm$$

U-70 (IHEP (1971)) 0.01 TeV:

$$\langle b^2 \rangle^{1/2} \cong 0.88 \pm 0.02\ (fm)$$

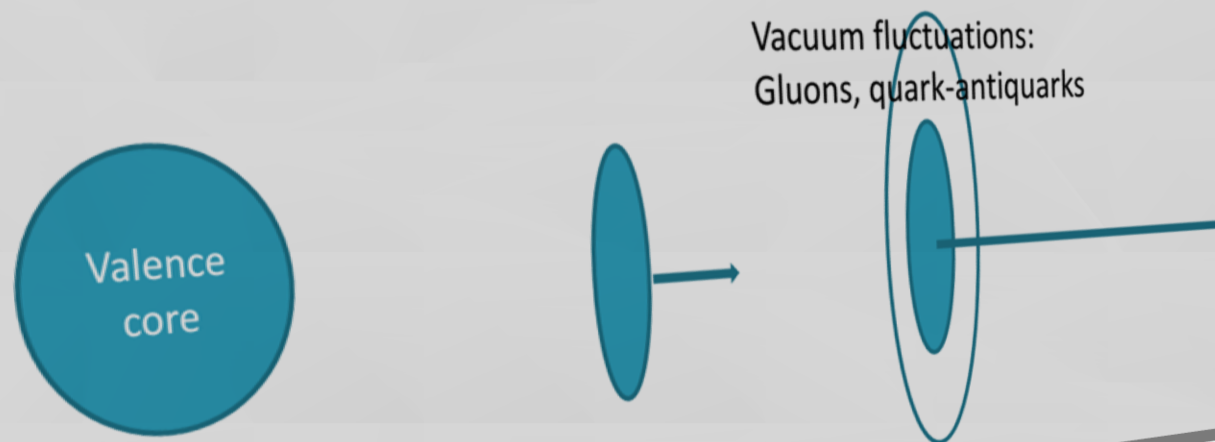
Longitudinal size of the IR

$$\langle L^* \rangle \equiv 4k^* \langle \partial\Phi(s,t)/\partial t \rangle$$

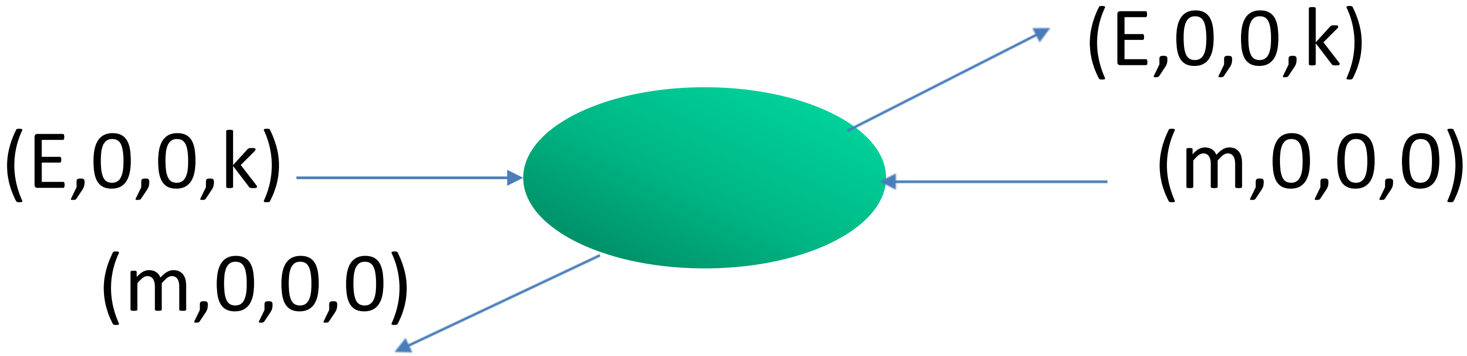
$$T(s,t) = e^{i\Phi(s,t)}|T(s,t)|$$

$$R_L \sim \Delta x_L \geq \frac{\sqrt{s}}{\sqrt{< t^2 > - < t >^2}}$$

- **Moving nucleon's spatial structure**



Interaction Time



$$T(E) = i \int d^4x \, e^{iEt-ikz} \, \theta(t) \langle p | [J(0), J^{+}(x)] | p \rangle$$

$$x = (t, \boldsymbol{x}_{\perp}, z)$$

$$\langle \boldsymbol{e^{iEt-ikz}} \rangle \neq \boldsymbol{0}, \quad \boldsymbol{0 < t < \frac{E}{m^2}} \quad |\boldsymbol{z}| < \frac{\boldsymbol{E}}{\boldsymbol{m^2}} \qquad |\vec{x}_{\perp}| < \frac{1}{m}$$

Correlations at Large Times and Distances

Regge regime:

$$T(s,t)|_{s\to\infty; t\ fixed} \sim s^{\alpha(t)}$$

Configuration space:

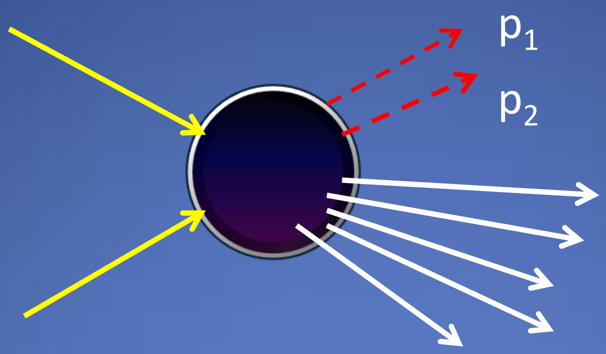
$$\langle p|[J(0),J^{+}(x)]_R|p\rangle|_{px\to\infty; x^2 fixed} \approx g(x^2)(px)^{\alpha(0)}$$

$$px = mt$$

Causality:

$$t > |\vec{x}|$$

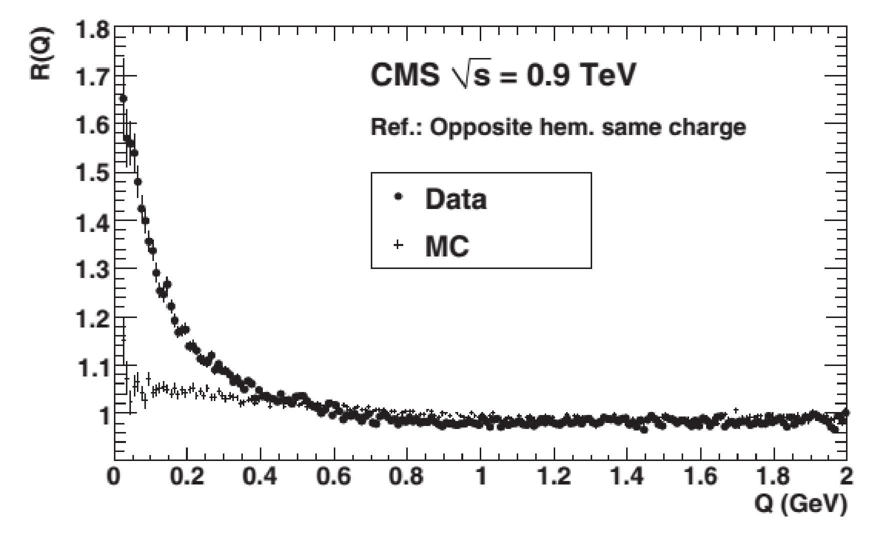
Bose-Einstein Correlations in Proton-Proton Collisions



$$R(Q) = (dN/dQ)/(dN_{\text{ref}}/dQ)$$

$$Q = \sqrt{-(p_1 - p_2)^2} = \sqrt{M^2 - 4m_\pi^2}$$

$$R(Q) = C[1 + \lambda\Omega(Qr)](1 + \delta Q)$$

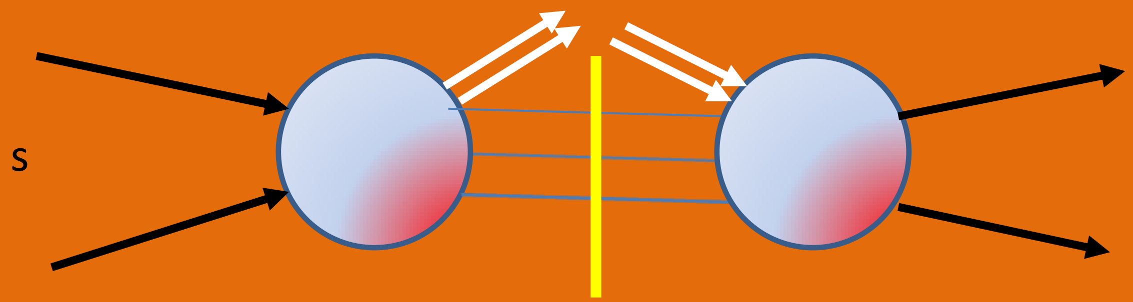


$$\Omega(Qr) = e^{-Qr}$$

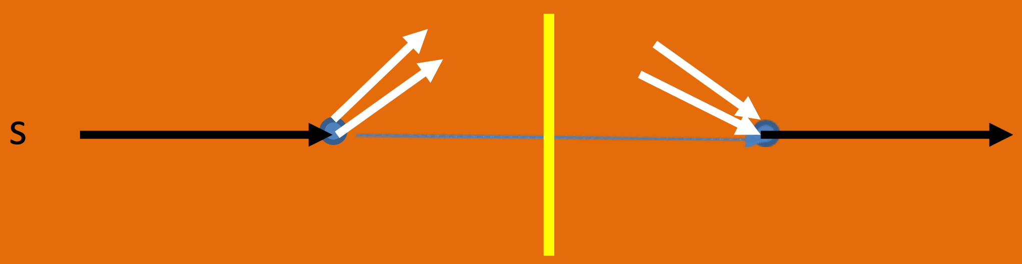
r (fm)= $2.76 \pm 0.25 \pm 0.44$

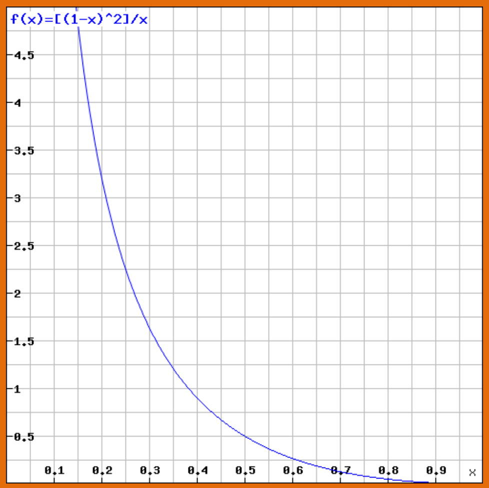
$$r_{pp}^{total}|_{LHC} \approx 1.5\ fm$$

A Toy Model



$$Q = \sqrt{-(p_1 - p_2)^2}$$





$$\frac{dN}{dQ} \sim \left(1 - \frac{Q^2}{s}\right)_+$$

$$\frac{dN_{ref}}{dQ} \sim \frac{Q^2/s}{(1 - Q^2/s)}$$

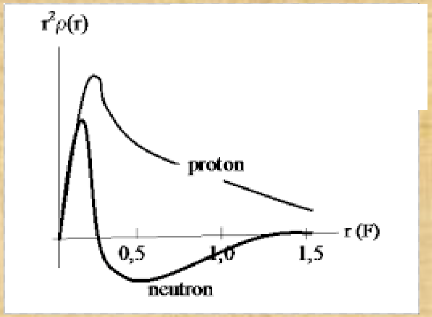
$$R(Q) \sim \frac{(1 - Q^2/s)^2}{Q^2/s}$$

$$x = Q^2/s$$

$$\langle q+p;\pi|J_\mu(0)|p;\pi\rangle = (2p_\mu + q_\mu)F(t),\, t = q^2$$

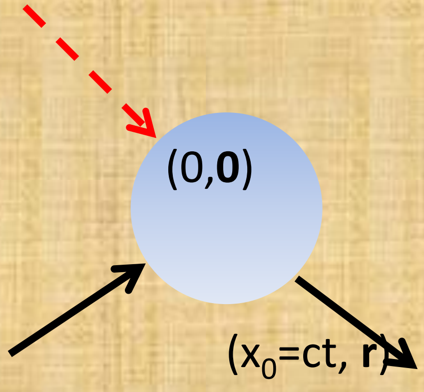
$\langle r^2\rangle_{charge} = 6\,F'(0)$

$$\langle r^2\rangle_{charge} = \int d\boldsymbol{r}\boldsymbol{r}^2\,\rho(\boldsymbol{r})$$



$$F_\pi(t) = \\ = \frac{2m_\pi}{4m_\pi^2 - t}\int d^4x e^{i(q+p)x}\left\langle\Omega\left|\frac{\delta J_0(0)}{\delta\phi_\pi^+(x)}\right|\vec{0};\pi\right\rangle$$

$$\frac{\delta J_0(0,\boldsymbol{0})}{\delta\phi_\pi^+(x_0,\boldsymbol{r})} = 0 \quad at\ x_0 < |\boldsymbol{r}|$$



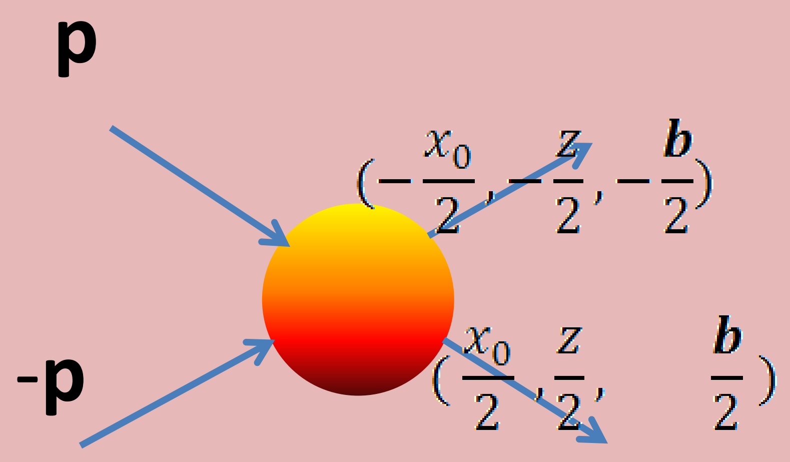
$$\rho(\boldsymbol{r}) = \\ = \frac{1}{2m_\pi}\int dx_0 e^{im_\pi x_0}\left\langle\Omega\left|\frac{\delta J_0(0,\boldsymbol{0})}{\delta\phi_\pi^+(x_0,\boldsymbol{r})}\right|\vec{0};\pi\right\rangle$$

NR limit: $$\lim_{\substack{x_0=ct\\ c\to\infty}}\left[c\cdot\left\langle\Omega\left|\frac{\delta J_0(0,\boldsymbol{0})}{\delta\phi_\pi^+(x_0,\boldsymbol{r})}\right|\vec{0};\pi\right\rangle\right] = f(\boldsymbol{r})\,\delta(t)$$

“Interaction Radius”

$$\langle b^2 \rangle = \int d^2b \, \boldsymbol{b}^2 \rho(\boldsymbol{b}; s)$$

$$\rho(\boldsymbol{b}; s) = 4Re\left[\frac{1}{T(s,0)}\int dz dx_0 \cos(|\boldsymbol{p}|z)\left\langle \Omega \left| \frac{\delta J(-\frac{x_0}{2}, -\frac{z}{2}, -\frac{\boldsymbol{b}}{2})}{\delta\phi(\frac{x_0}{2}, \frac{z}{2}, \frac{\boldsymbol{b}}{2})} \right| \boldsymbol{p}, -\boldsymbol{p} \right\rangle\right]$$



Pion "Emission Source"

$$R(Q) = (dN/dQ)/(dN_{\text{ref}}/dQ)$$

$$R(Q) = C[1 + \lambda\Omega(Qr)](1 + \delta Q)$$

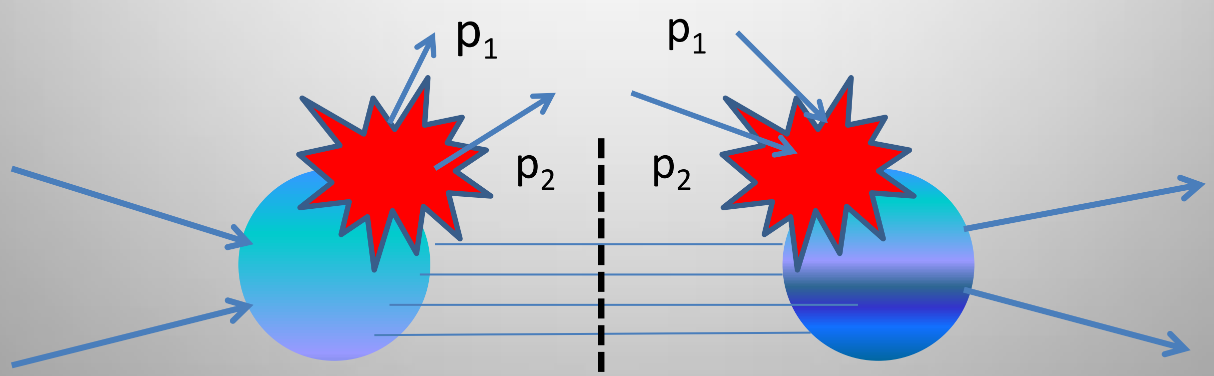
$$\Omega(Qr) = e^{-Qr}$$

$$Q = \sqrt{-(p_1 - p_2)^2}$$

$$P = (p_1+p_2)/2$$

$$r = |\xi - \eta|$$

$$R(Q;s) \sim \left[\int dX d\xi d\eta\, e^{iPX} e^{i(p_1-p_2)(\xi-\eta)} \left\langle \text{pp, in} \left| \frac{\delta J(X-\frac{\xi}{2})}{\delta\phi(X+\frac{\xi}{2})} \frac{\delta J(-\frac{\eta}{2})}{\delta\phi(\frac{\eta}{2})} \right| \text{pp, in} \right\rangle\right]$$



Inconclusive Conclusions

- Spatio-temporal correlations do not die-off at large distances and times
- Spatial scales in use in HEP are of a derivative character and both definition- and model-dependent
- Spatial scales aren't instantaneous snapshots
- Theoretically sound elaboration of these notions is badly needed