The Asymmetry of the $\langle A^2 \rangle$ Condensate and Propagators in the $SU(2)$ Gluodynamics at $T > T_c$

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- Motivation
  - Why $\langle A^2 \rangle$ is of interest?
  - Do we need propagators at $T \neq 0$?
  - Does semi-QGP makes sense?
- Definitions and details of simulation
- Gribov-copy and finite-volume effects
- Temperature dependence of the asymmetry
- Propagators and transition from electric to magnetic dominance
- Conclusions

Dimension 2 operators in gauge theories.

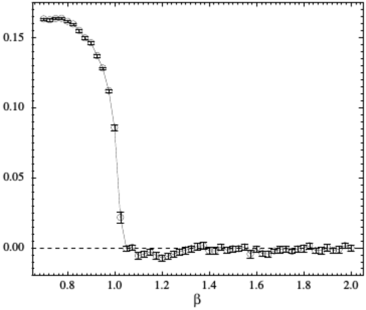
We begin with the Semenov-tyan-Shanskii–Franke functional:

$$\frac{1}{V}\int_V A^a_\mu A^a_\mu dx = min \qquad \Longrightarrow \qquad \partial_\mu A^a_\mu = 0$$

The operator $A^a_\mu A^a_\mu$, where $\partial_\mu A^a_\mu = 0$, is

- local in the Landau gauge;
- Lorentz and BRST invariant;
- needed in the Operator Product Expansion in PT
- $\langle A^2 \rangle$ is gauge-invariant [A.A.Slavnov, 2004]

Interest in $A_\mu^a A_\mu^a$ aroused in 2001

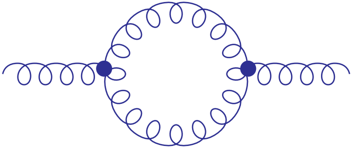


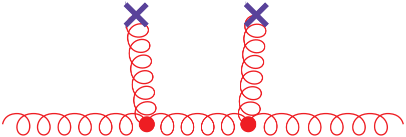
F.V.Gubarev, L.Stodolsky, V.I.Zakharov, Phys.Rev.Lett.(2001)

Rapid change of

$$\langle A^2 \rangle_{noncompact} - \langle A^2 \rangle_{compact}$$

is correlated with the confinement-deconfinement transition in the compact $U(1)$ theory.





$$D^{ab}_{\mu\nu}(p) \simeq \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \left(D_{pert}(p^2) + \frac{3g^2 \langle A^2 \rangle}{4(N_c^2 - 1)p^2} + \dots \right)$$

$$\alpha_s^{MOM}(q^2) \simeq \left(\alpha_s^{MOM}(q^2) \right)_{pert} \left[1 + \frac{9g^2 \langle A^2 \rangle}{4(N_c^2 - 1)q^2} + \dots \right]$$

$\langle A^2 \rangle$ was computed numerically
from fits to lattice data for the gluon and ghost propagators.

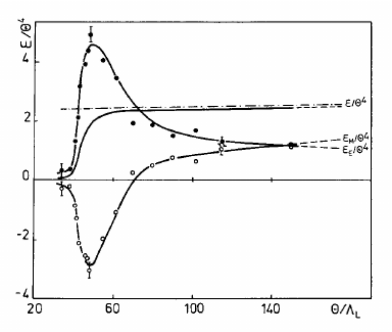
For example, in the Taylor renormalization scheme
(defined by zero incoming ghost momentum) at $\mu = 10$ GeV,
the following values for the $N_f = 2 + 1 + 1$ QCD were found
[Blossier, Boucaud et al., 2013]:

$$\langle A^2 \rangle = 2.8(8) \text{ GeV}^2 \qquad (\text{OPE up to } \frac{1}{p^4})$$

$$\langle A^2 \rangle = 3.8(6) \text{ GeV}^2 \qquad (\text{OPE up to } \frac{1}{p^6})$$

in order to obtain the QCD coupling constant
$\alpha_{\overline{MS}}(M_Z) = 0.1198(4)(8)(6)$!

Postconfinement domain



[Mitrjushkin, Zadorozhny, Zinoviev 1988]

- Polaykov loop behavior
- Failure of PT and old effective theories to evaluate pressure at $T_c < T < 2 \div 3 T_c$.
- Monopole density (condensate-liquid-solid)

Partition function

$$Z = Tr\ \exp\left(-\frac{\hat{H}}{T}\right)\ , \qquad\qquad F = -T\ln Z$$

and density matrix

$$\varrho(x'', x') = \frac{1}{Z}\langle x''|\exp\left(-\frac{\hat{H}}{T}\right)|x'\rangle$$

can be expressed in terms of path integral:

$$Z = \int_{x(0)=x(1/T)} Dx(t)\ \exp(-S_E[x])$$

$$\varrho(x'', x') = \frac{1}{Z}\int_{x(0)=x', x(1/T)=x''} Dx(t)\ \exp(-S_E[x])$$

One can consider temperature-dependent fields

$$\hat{x}(\tau) = \exp(\tau H)\hat{x}(0)\exp(-\tau H)$$

and their Green's functions

$$D(\tau', \tau'') = \langle\langle T\hat{x}(\tau'')\hat{x}(\tau')\rangle\rangle \qquad (0 < \tau', \tau'' < 1/T)\ ,$$

where

$$\langle\langle \hat{A} \rangle\rangle = \int dx' dx'' \varrho(x'', x') A(x', x'')\ .$$

Thus

$$D(\tau', \tau'') = \int_{x(0)=x(1/T)} Dx\ x(\tau')x(\tau'') \exp(-S_E[x])$$

Why should one deal with such a monster?

Interest in such monsters is provided by the

Linear response theory:

We consider a Heisenberg operator $Y_{tot}(t)$ when the Hamiltonian involves a small time-dependent term caused by an external force,

$$H_{tot} = H + h(t), \qquad \dot{Y} = i\,[H_{tot}, Y_{tot}] \ .$$

If

$$Y_{tot} = Y + \hat{y}, \qquad \dot{Y} = i\,[H, Y] \ , \qquad \hat{y}(t_0) = 0$$

and

$$H|n\rangle = E_n|n\rangle$$

then it is straightforward matter to derive

$$\langle n|\hat{y}(t)|n\rangle = i\int_{t_0}^{t} dt' \ \langle n|[h(t'), Y(t)]|n\rangle + \underline{O}(h^2)$$

An example: $H = \dfrac{\hat{p}^2}{2m} + V(\hat{x}), \qquad h(t) = j(t)\hat{x}, \qquad Y = \hat{x}$

Assuming thermodynamical equilibrium, $t_0 \to -\infty$ and we arrive at

$$\langle\langle \delta x(t) \rangle\rangle = \int_{-\infty}^{\infty} dt' \; D_R(t-t') j(t') \; + \; \underline{O}(h^2)$$

where

$$G_R(t-t') \; = \; i \sum_n e^{-E_n/T} \, \theta(t-t') \, \langle n | [\hat{x}(t'), \hat{x}(t)] | n \rangle \; .$$

The retarded Green's function is related to the temperature Green's function as follows:

$$\tilde{G}_R \left(-i \, \frac{2\pi n}{T} \; - \; i\epsilon \right) = - \, \tilde{D}_n \; ,$$

where

$$\tilde{D}_n = \int_0^{1/T} d\tau \exp\left(- \, \frac{2i\pi\tau n}{T} \right) \langle\langle \hat{x}(\tau) \hat{x}(0) \rangle\rangle$$

Propagators in the gauge theory at $T \neq 0$

We compute the “gluon” propagator in the $SU(2)$ theory at the temperature $T = \frac{1}{\beta}$:

$$D^{bc}_{\mu\nu}(\tau,\vec{x}) = \frac{1}{\mathcal{Z}} \int_{A_\mu(0,\vec{x}) = A_\mu(\beta,\vec{x})} DA^a_\mu(x) A^b_\nu(\tau,\vec{x})\, A^c_\mu(0,0) e^{-S_E[A]} |\det\ M_{FP}(A)| \tag{1}$$

$$S_E[A] = \int_0^\beta d\tau \int_V d^3\vec{x}\ \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2\alpha}(\partial_\mu A^a_\mu)^2\right) \tag{2}$$

$$D^{bc}_{\mu\nu}(\tau,\vec{x}) = \delta^{bc} D_{\mu\nu}(\tau,\vec{x})$$

$$(D(p))^{-1}{}_{\mu\nu} = (D^0(p))^{-1}{}_{\mu\nu}(p) + \Pi_{\mu\nu}(p)$$

$$p_\mu \, \Pi_{\mu\nu}(p) = 0 \tag{3}$$

$$p_\mu p_\nu D_{\mu\nu}(p) = \alpha$$

$$P^T_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \frac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P^L_{\mu\nu} = \frac{1}{p^2} \begin{pmatrix} |\vec{p}|^2 & p_4 \vec{p} \\ p_4 \vec{p} & \frac{p_4^2}{|\vec{p}|^2} p_i p_j \end{pmatrix}$$

$$P^{L,T}_{\mu\nu} P^{L,T}_{\nu\lambda} = P^{L,T}_{\mu\lambda}, \qquad P^L_{\mu\nu} P^T_{\nu\lambda} = 0$$

$$p_\mu P^L_{\mu\nu} = p_\mu P^T_{\mu\nu} = 0, \qquad P^L_{\mu\mu} = 1, \qquad P^T_{\mu\mu} = 2$$

$$D_{\mu\nu}(p) = D_L(p) P^L_{\mu\nu} + D_T(p) P^T_{\mu\nu} + \alpha \frac{p_\mu p_\nu}{p^4}$$

$$D_L(p) = \frac{1}{p^2 + F(p)}, \qquad D_T(p) = \frac{1}{p^2 + G(p)}$$

$$\Pi_{\mu\nu} = F(p) P^L_{\mu\nu} + G(p) P^T_{\mu\nu}$$

The quantities under study:

$$D_{ii}(|\vec{p}|^2) = 2 D_T(0, \vec{p}), \qquad D_{44}(|\vec{p}|^2) = D_L(0, \vec{p}),$$

We consider $\alpha = 0$ (the Landau gauge)

Screening mass in QED

We consider two charges in QED plasma,

$$\vec{E}_1^{cl} = -i \frac{\vec{p}}{|\vec{p}|^2} Q_1 e^{-i\vec{p}\vec{x}_1} \qquad \vec{E}_2^{cl} = -i \frac{\vec{p}}{|\vec{p}|^2} Q_2 e^{-i\vec{p}\vec{x}_2}$$

Each of them can be considered as a small perturbation in the linear response theory:

$$h = \int d\vec{x}\ \vec{E}^{cl}(\vec{x}) \vec{E}(\vec{x}) \ ,$$

the resulting field has the form

$$E_i^{tot}(\vec{p}) = E_i^{cl} + \langle\langle \delta E_i \rangle\rangle = \frac{p_i p_j E_j^{cl}(\vec{p})}{|\vec{p}|^2 + F(0, \vec{p})} \ .$$

$$
\begin{aligned}
V &\simeq \frac{1}{2}\int d\vec{x}\ \left(\langle\!\langle \vec{E}_1^{tot}\rangle\!\rangle \vec{E}_2^{cl} + \langle\!\langle \vec{E}_2^{tot}\rangle\!\rangle \vec{E}_1^{cl}\right) \\
&= Q_1 Q_2 \int \frac{d\vec{k}}{(2\pi)^3}\ \frac{e^{i\vec{k}(\vec{x}_1-\vec{x}_2)}}{|\vec{k}|^2 + F(0,\vec{k})} \\
&\simeq \frac{Q_1 Q_2}{4\pi}\ \frac{e^{-\,m_e|\vec{x}_1-\vec{x}_2|}}{|\vec{x}_1 - \vec{x}_2|}
\end{aligned}
$$

$$
m_e = \frac{eT}{\sqrt{3}}
$$

Screening in $SU(N_c)$ theories to one loop

Feynman gauge:

$$\begin{aligned} F(0,\vec{p}\to 0) &= \frac{1}{3}g^2T^2N_c - \frac{1}{4}g^2TN_c|\vec{p}| \\ G(0,\vec{p}\to 0) &= -\frac{3}{16}g^2TN_c|\vec{p}| \end{aligned}$$

Temporal axial gauge ($A_0 = 0$ with the PV pole prescription):

$$\begin{aligned} F(0,\vec{p}\to 0) &= \frac{1}{3}g^2T^2N_c - \frac{1}{4}g^2TN_c|\vec{p}| - \frac{11g^2}{48\pi^2}N_c|\vec{p}|^2\ln\left(\frac{|\vec{p}|^2}{T^2}\right) \\ G(0,\vec{p}\to 0) &= -\frac{5}{16}g^2TN_c|\vec{p}| \end{aligned}$$

Lattice settings

$$S = \frac{4}{g^2} \sum_{P=x,\mu,\nu} \left(1 - \frac{1}{2} \mathrm{Tr}\; U_P \right)$$

where

$$U_P = U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}$$

$$U_{x,\mu} \;=\; u_0 \;+\; i \sum_{a=1}^{3} u_a \sigma_a, \qquad (4)$$

$$A^a_\mu = -\,\frac{2Z\; u^a_\mu}{ga}, \qquad (5)$$

$$\Lambda : \quad U_{x,\mu} \to \Lambda^{\dagger}_x U_{x,\mu} \Lambda_{x+\hat{\mu}},$$

We fix the absolute Landau gauge by finding the global maximum of the functional

$$\mathcal{F}[\,\mathcal{U}\,] = \frac{1}{2} \sum_{x,\mu} \mathrm{Tr}\, U_{x,\mu}, \qquad (6)$$

Stationarity condition:

$$\partial_\nu A^a_\nu = 0.$$

We use the simulated annealing algorithm with subsequent overrelaxation

At nonzero temperatures there are two condensates,

$$\langle A_E^2 \rangle = g^2 \langle A_4^a(x) A_4^a(x) \rangle, \qquad \langle A_M^2 \rangle = g^2 \langle \sum_{i=1}^{3} A_i^a(x) A_i^a(x) \rangle.$$

The A^2 asymmetry is defined by the formula

$$\langle \Delta_{A^2} \rangle \equiv \langle A_E^2 \rangle - \frac{1}{3} \langle A_M^2 \rangle \qquad \mathcal{A} = \frac{\langle \Delta_{A^2} \rangle}{T^2}.$$

The asymmetry in terms of the propagators:

$$\mathcal{A} = \frac{4N_t}{\beta a^2 N_s^3} \left[3(D_L(0) - D_T(0)) + \sum_{p \neq 0} \left(\frac{3|\vec{p}|^2 - p_4^2}{p^2} D_L(p) - 2 D_T(p) \right) \right]$$

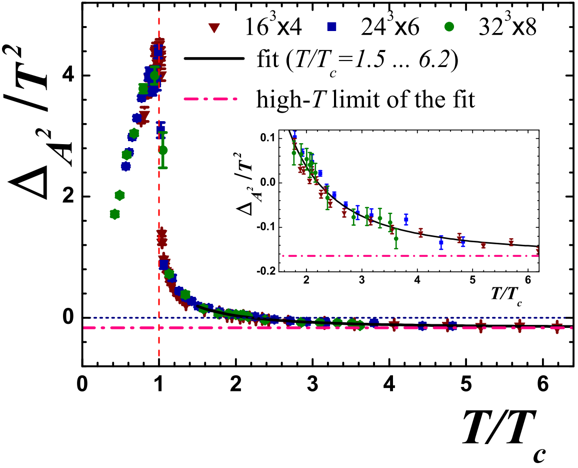
We consider propagators only for *soft modes* $p_4 = 0$, where

$$P^T_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \dfrac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P^L_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$D_L(0)$ — chromoelectric forces
$D_T(0)$ — chromomagnetic forces
One-loop estimate at high temperatures [Vercauteren *et al.*, 2010]

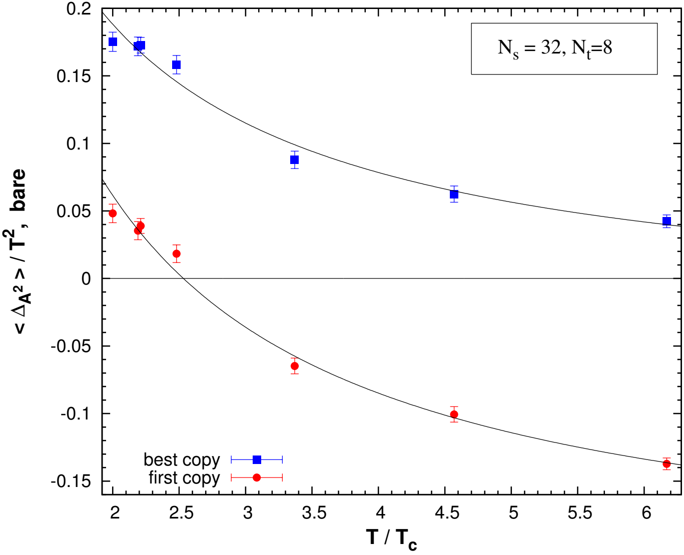
$$\langle \Delta_{A^2} \rangle \simeq c\, g^2 T^2 \left(1 - \frac{g}{3\pi}\sqrt{\frac{2}{3}} \right) \tag{7}$$

- Perturbation theory (2010): c>0
- Lattice simulations (2008): c<0



M.N. Chernodub and E.-M. Ilgenfritz, Phys.Rev.D (2008)

main result



Lattice size decreases from 1.3 fm to 0.4 fm

our result

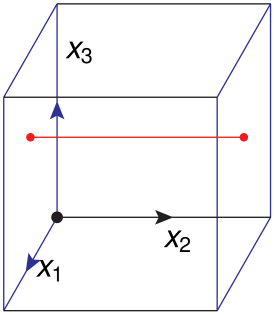
$$A_\mu \to A_\mu^\Lambda = (\Lambda Z)^\dagger A_\mu (\Lambda Z) + \frac{i}{g} (\Lambda Z)^\dagger \partial_\mu (\Lambda Z).$$

$$A_\mu \to A_\mu^\Lambda = \Lambda^\dagger A_\mu \Lambda + \frac{i}{g} \Lambda^\dagger \partial_\mu \Lambda.$$

For $SU(3)$, as an example:

$$Z \in \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} e^{\frac{2i\pi}{3}} & 0 & 0 \\ 0 & e^{\frac{2i\pi}{3}} & 0 \\ 0 & 0 & e^{\frac{2i\pi}{3}} \end{pmatrix}, \begin{pmatrix} e^{\frac{4i\pi}{3}} & 0 & 0 \\ 0 & e^{\frac{4i\pi}{3}} & 0 \\ 0 & 0 & e^{\frac{4i\pi}{3}} \end{pmatrix} \right\}$$

Gauge transformation is the same on both sides!



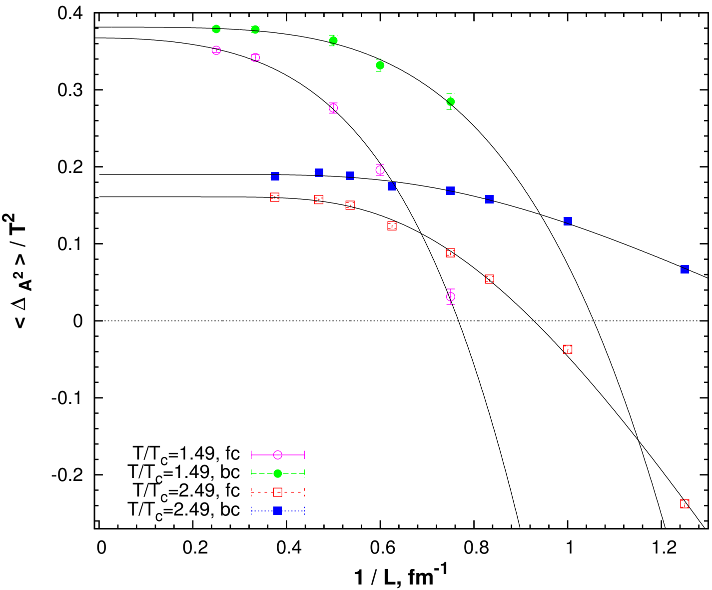
We extend the gauge group by nonperiodic gauge transformations:

$$\Lambda(x_1, b, x_3) = Z\Lambda(x_1, 0, x_3) \text{ etc.}$$

$$P\exp\left(ig\int_0^b A_2(x_1, z, x_3)dz\right) =$$

$$= L(x_1, x_3) \longrightarrow L(x_1, x_3)Z$$

Thus the Hilbert space is broken into 8 superselection sectors



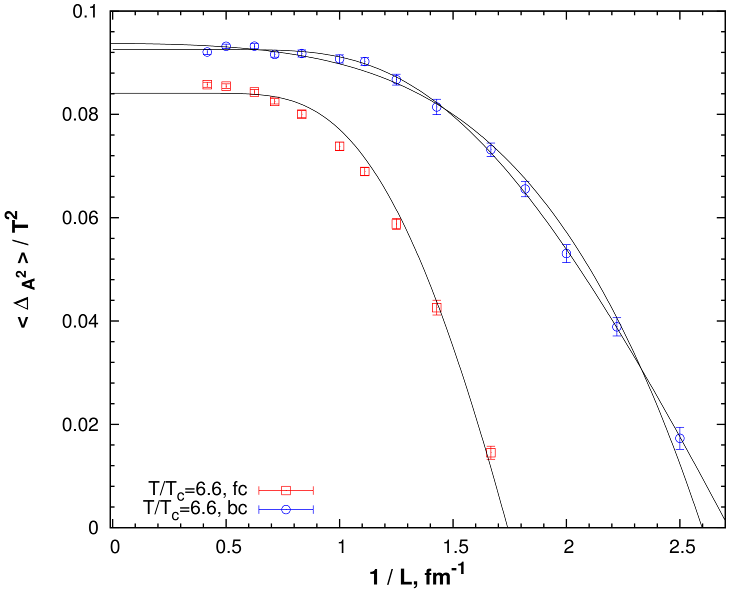
Finite-volume effects

$$\overline{\mathcal{A}}(L) = \overline{\mathcal{A}}_{\infty}^{pol} - \frac{c_2}{L^2} - \frac{c_4}{L^4} \,,$$

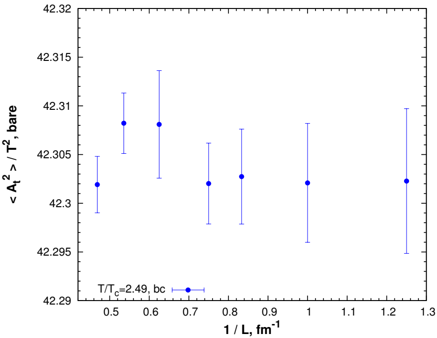
$$\overline{\mathcal{A}}(L) \simeq \overline{\mathcal{A}}_{\infty}^{exp} - c \exp\left(- \; L/L_0 \right)$$

$\frac{T}{T_c}$	Gauge fixing algorithm	$\overline{\mathcal{A}}_{\infty}^{exp}$	c	L_0 (fm)	$\frac{\chi^2}{N_{dof}}$
1.49	*bc*	0.380(2)	1.7(1.0)	0.41(5)	0.34
1.49	*fc*	0.352(1)	4.7(1.0)	0.47(8)	0.06
2.49	*bc*	0.190(2)	1.7(5)	0.31(3)	1.71
2.49	*fc*	0.161(2)	5.6(5)	0.31(1)	2.60
6.60	*bc*	0.09254(21)	1.06(11)	0.151(5)	0.89
6.60	*bc*	0.0937(6)	$N_t = 4$	pol. fit	2.93

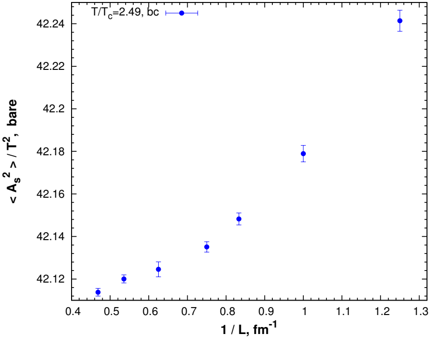
Table : Fit parameters.



0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2
2.5
< Δ A² > / T²
1 / L, fm⁻¹
T/T_c=6.6, fc
T/T_c=6.6, bc

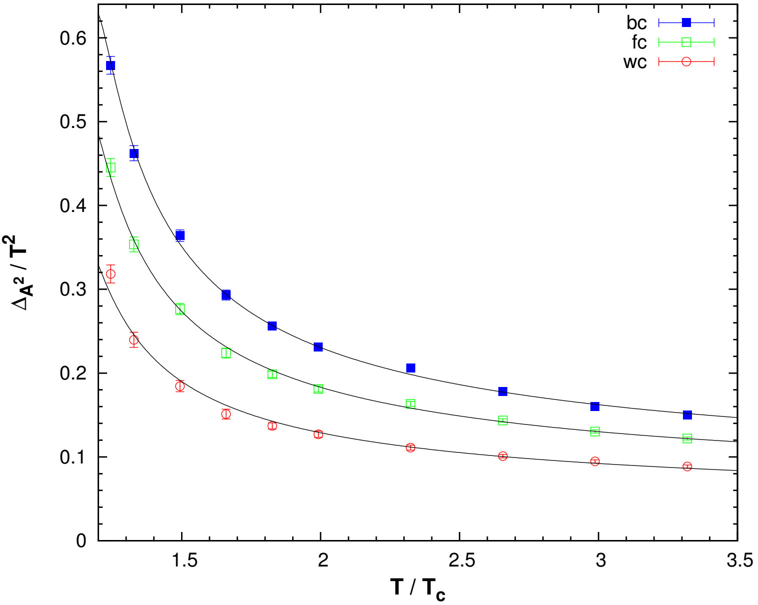


“Chromoelectric” condensate



“Chromomagnetic” condensate

$$\langle \Delta_{A^2} \rangle \equiv \langle A_t^2 \rangle - \langle A_s^2 \rangle \ .$$


bc
fc
wc
Δ_A^2 / T^2
0.6
0.5
0.4
0.3
0.2
0.1
0
1.5
2
2.5
3
3.5
T / T_c

Fitting high-temperature behavior

$$\bar{\mathcal{A}} \simeq b_0 + b_2 \left(\frac{T_c}{T}\right)^2$$

Gauge fixing algorithm	b_0	b_2	$\frac{\chi^2}{N_{dof}}$
bc	0.1036(27)	0.517(16)	1.40
fc	0.0893(22)	0.372(13)	0.92
wc	0.0682(5)	0.231(3)	0.05

Table : $1.65 < T/T_c < 3.32$, fixed lattice size $L = 2$fm.

$b_0 > 0$ in all cases in agreement with perturbation theory

$$\bar{\mathcal{A}} \simeq \frac{zg^2(T)}{4}\left(1 - \frac{g(T)}{3\pi}\sqrt{\frac{2}{3}}\right) ,$$

where the running coupling is taken in the two-loop approximation,

$$\frac{1}{g^2(T)} = \frac{1}{4\pi^2}\ \left(\frac{11}{6}\ln\left(\frac{T^2}{\Lambda^2}\right) + \frac{17}{11}\ln\ln\left(\frac{T^2}{\Lambda^2}\right)\right) ,$$

z and Λ are the fit parameters, $1.24 < \dfrac{T}{T_c} < 3.32$.

$$z = 0.1284(14), \quad \Lambda/T_c = 0.845(7), \quad \frac{\chi^2}{N_{dof}} = 1.50$$

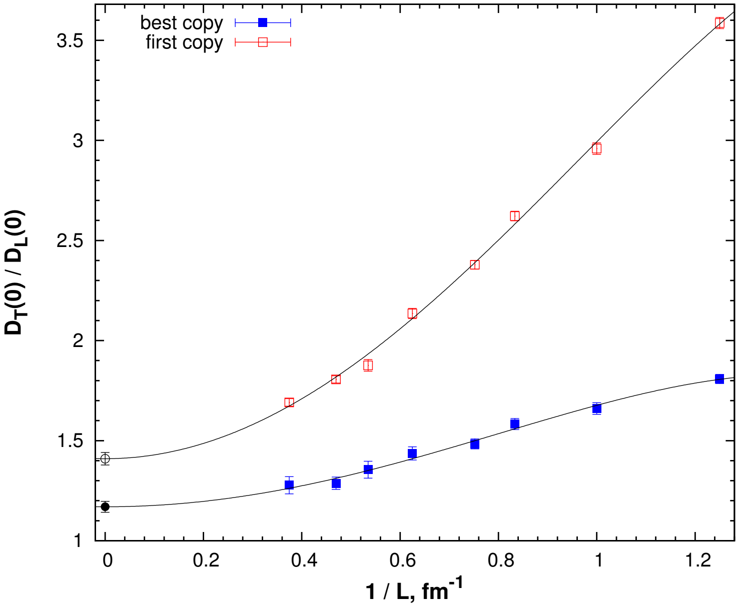
Another definition of screening masses [Heller, Karsch, Rank 97]:

$$\begin{aligned}\tilde{D}_L(p_\perp = 0, x_3) &\sim \exp(-m_e|x_3|),\\ \tilde{D}_T(p_\perp = 0, x_3) &\sim \exp(-m_m|x_3|),\ |x_3| \to \infty\end{aligned}$$

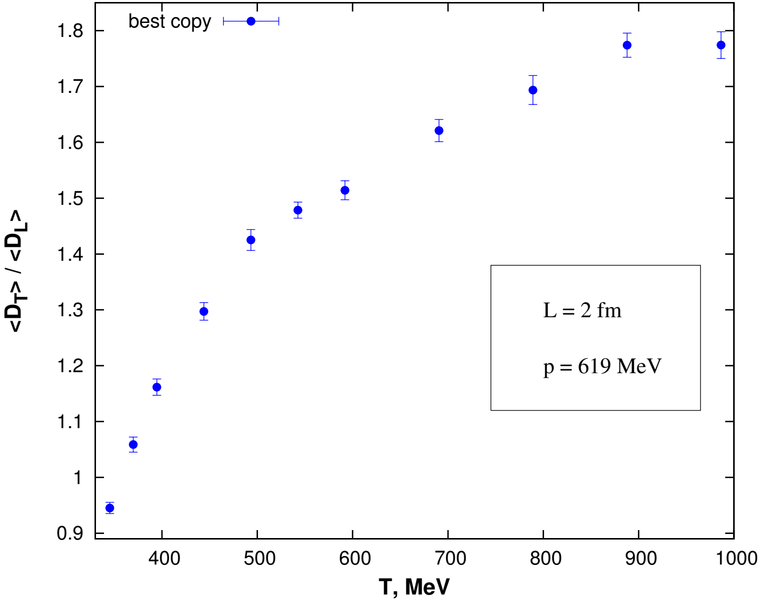
Approximations $m_e = \sqrt{\frac{2}{3}} g(T)T + ...$ and $m_m \sim g^2(T)T$ suggest the fit function ($T > 2T_c$)

$$\frac{m_e^2(T)}{m_m^2(T)} = \frac{C}{g^2(T)} = 1 \quad \text{at} \quad \frac{T}{T_c} = 0.9(1)$$

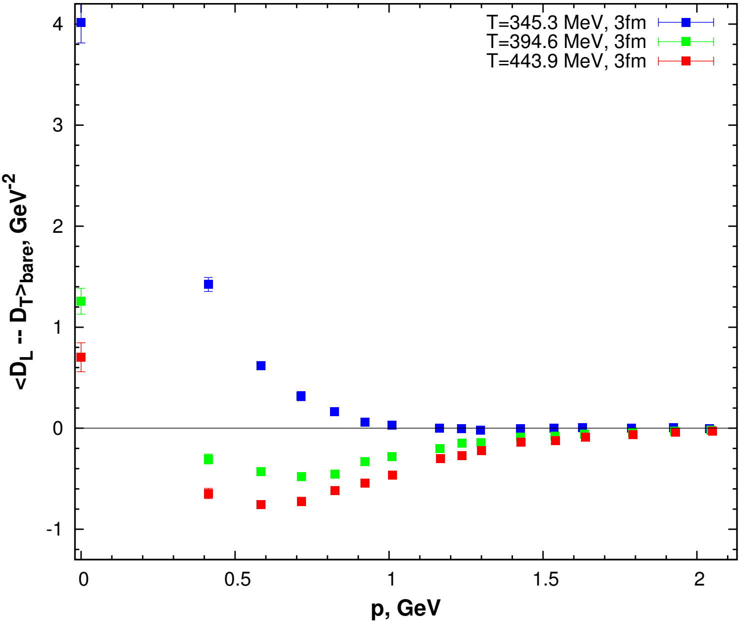
we consider the ratio $r(T) = \frac{D_T(0)}{D_L(0)}$ instead of $\frac{m_e^2}{m_m^2}$



best copy
first copy
$D_T(0) / D_L(0)$
$1 / L, fm^{-1}$
1
1.5
2
2.5
3
3.5
0
0.2
0.4
0.6
0.8
1
1.2



Ratio of the “magnetic” to the “electric” propagator at $p = p_{min}$



T=345.3 MeV, 3fm
T=394.6 MeV, 3fm
T=443.9 MeV, 3fm
$\langle D_L - D_T \rangle_{bare}$, GeV^{-2}
p, GeV

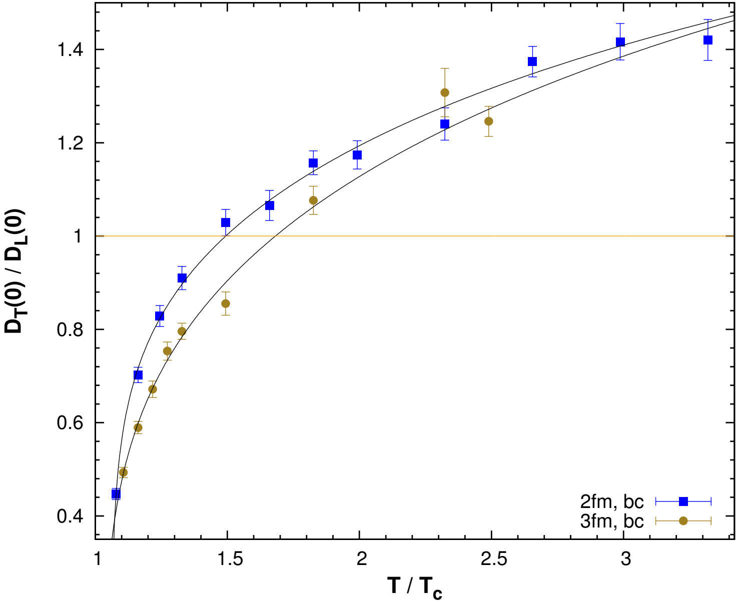
$$r(T) \simeq r_0 + \frac{r_1}{g^2(T)}$$

where

$$\frac{1}{g^2(T)} = \frac{1}{4\pi^2} \left(\frac{11}{6} \ln\left(\frac{T^2}{\Lambda^2}\right) + \frac{17}{11} \ln\ln\left(\frac{T^2}{\Lambda^2}\right)\right) ,$$

Lattice size	r_0	r_1	Λ/T_c	T_p/T_c	$\frac{\chi^2}{N_{dof}}$
2 fm	0.94(1)	3.78(12)	1.060(3)	1.494(30)	0.64
3 fm	0.79(3)	4.59(37)	1.02(2)	1.68(12)	1.42

Table : Fit parameters for the best-copy values of $r(T)$.



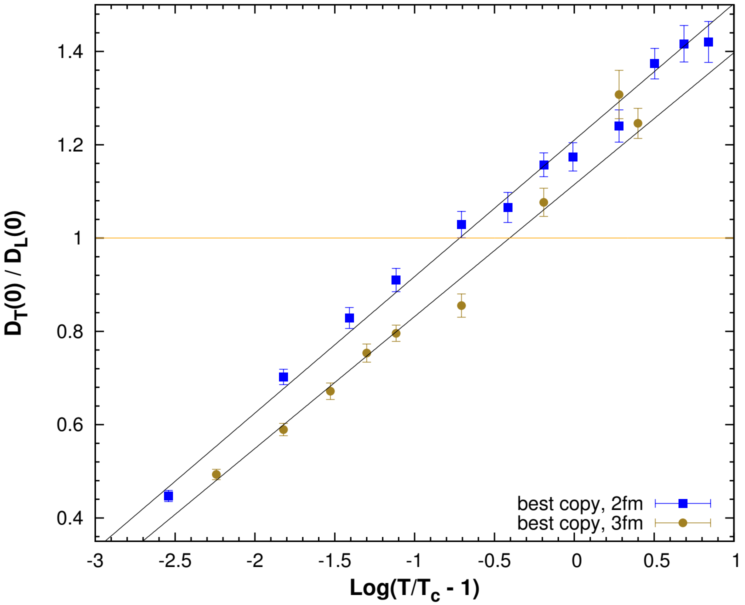
$D_T(0) / D_L(0)$
1.4
1.2
1
0.8
0.6
0.4
1
1.5
2
2.5
3
T / T_c
2fm, bc
3fm, bc

$$r(T) \simeq R_0 + R_1 \ln\left(\frac{T}{T_c} - 1\right) ,$$

$$r(T) \simeq R_1 \ln\left(\frac{T - T_c}{Q}\right) .$$

Lattice size	R_0	R_1	T_p/T_c	$\frac{\chi^2}{N_{dof}}$
2 fm	1.21(1)	0.293(6)	1.488(13)	1.35
3 fm	1.115(15)	0.283(9)	1.667(27)	1.92

Table : Fit parameters for the best-copy values of $r(T)$.



$D_T(0) / D_L(0)$
1.4
1.2
1
0.8
0.6
0.4
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
$\mathrm{Log}(T/T_c - 1)$
best copy, 2fm
best copy, 3fm

Conclusions

- The flip-sector effect is substantial at $L \simeq 2$ fm and crucial at $L < 1$ fm. In the latter case, it dramatically changes the behavior of the asymmetry.
- Finite-volume effects for $\overline{\mathcal{A}}$ and r are significant at lattice sizes < 2 fm .
- The data can be fitted to the function motivated by perturbation theory down to temperatures as low as $1.25 T_c$
- Contrary to the conclusions by Chernodub and Ilgenritz (2008), $\overline{\mathcal{A}} > 0$ at all temperatures under consideration
- Boundary of the postconfinement domain T_p is indicated by the condition $D_T(0)/D_L(0) = 1$ rather than by criteria based on $\overline{\mathcal{A}}$. At $L = 3 fm \quad T_p = 1.68(12) T_c$.