Исследование решеточной КХД при ненулевом барионном химическом потенциале

В.Г. Борняков

21 марта 2017
Семинар ОТФ

Outline

- QCD in lattice regularization

- Some results at T>0

- Problem at $\mu > 0$ and methods to solve it

- Results at $\mu > 0$

Литература

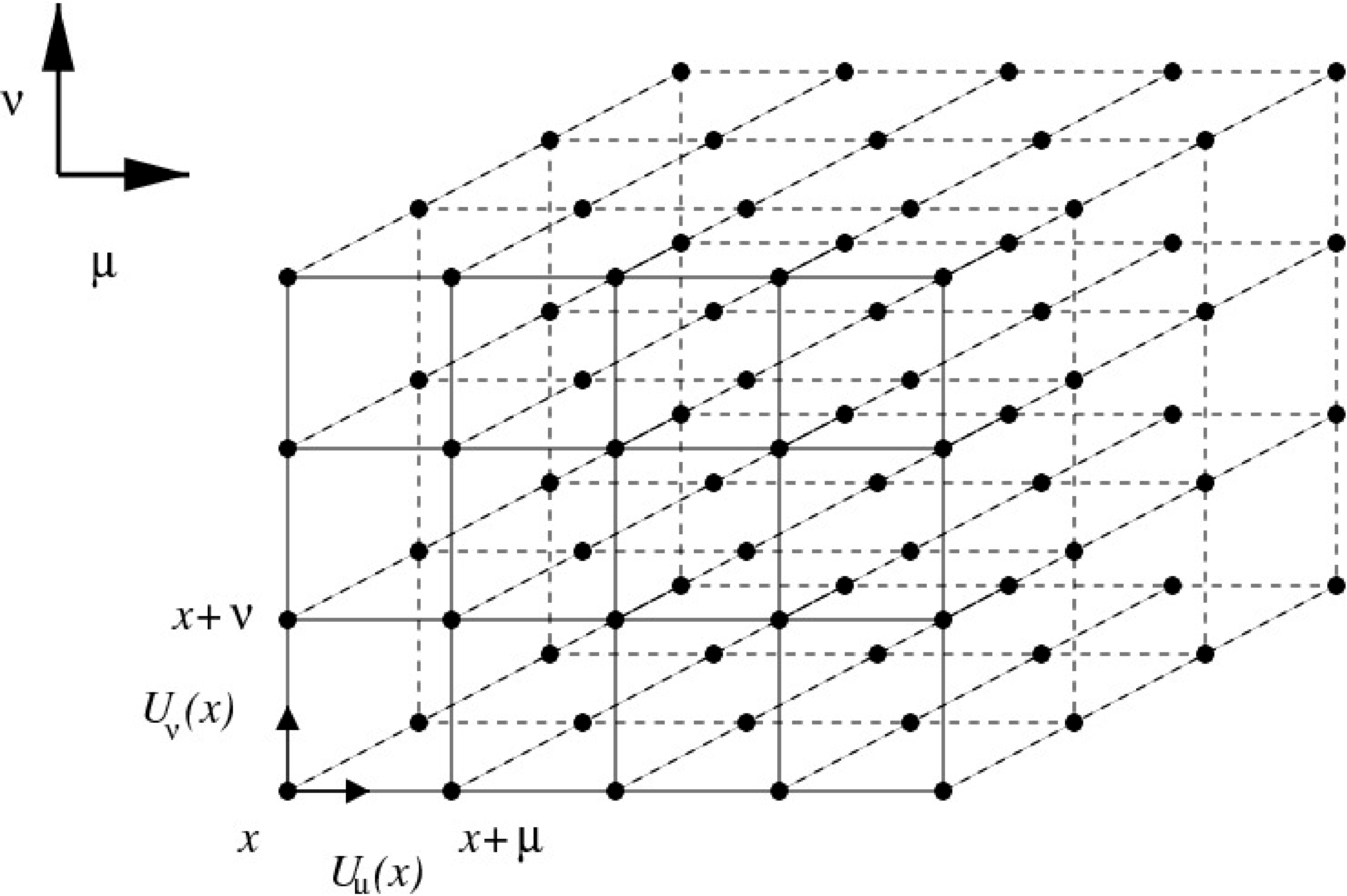
- M. Creutz, Quarks, gluons and lattices, Cambrigde Univ. Press (1983), translated into Russian

- H. Rothe, Lattice gauge theories - An introduction, World Scientific (4th ed. 2012)

- I. Montvay, G. Munster, Quantum fields on a lattice, Cambrigde Univ. Press (1996)

- T. DeGrand, C. E. DeTar, Lattice methods for quantum chromodynamics, World Scientific (2006)

- C. Gattringer, C.B. Lang, Quantum Chromodynamics on the Lattice - An introductory presentation, Springer (2010)



ν
μ
x+ν
$U_\nu(x)$
x
x+μ
$U_\mu(x)$

Lattice action

$$S_W^G = \beta \sum_P \left(1 - \frac{1}{3} \mathrm{Re}\ \mathrm{Tr} U_P \right)$$

$$U_{\mu\nu}(s) = U_\mu(s) U_\nu(s + \hat{e}_\mu) U_\mu^\dagger(s + \hat{e}_\nu) U_\nu^\dagger(s)\,.$$

$$S_W^F = \bar{\psi}\, M(U)\, \psi$$

$$S_W^G \xrightarrow{a\to 0} \frac{1}{2\,g^2}\int \mathrm{Tr}\,F_{\mu\nu}^2(x)\,d^4x + O(a^2)\,,$$

$$S_W^F \xrightarrow{a\to 0} \int \bar{\psi}_f(x)(\gamma_\mu D_\mu + m_f)\psi_f(x)\,d^4x + O(a)$$

Интегрирование по фермионным полям

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{-\bar{\psi} M(U) \psi} = \det M(U)$$

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi}\;\; \bar{\psi}^a(s')\psi^b(s)\; e^{-\bar{\psi} M(U) \psi}$$

$$= \left(M^{-1}_{s,s'}(U)\right)^{ab} \det M(U)$$

$$< \mathcal{O} >= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \, \mathcal{O}(U) \, e^{-S_{\text{eff}}(U)}$$

$$\mathcal{Z} = \int \mathcal{D}U \, e^{-S_{\text{eff}}(U)}$$

$$S_{\text{eff}}(U) = S_W^G(U) - \sum_f \ln\det M_f(U)$$

$$\mathcal{D}U = \prod_{s,\mu} dU_\mu(s)$$

Вычисления в решеточной КХД состоят из нескольких этапов.

Первый этап заключается в генерации конфигураций калибровочного поля. Эти конфигурации генерируются последовательно и образуют марковскую цепь с распределением вероятности, пропорциональным

$$e^{-S_{eff}(U)}$$

$$< \mathcal{O} > \approx \frac{1}{N_{\mathrm{conf}}} \sum_{i=1}^{N_{\mathrm{conf}}} \mathcal{O}_i(U)$$

QCD in lattice regularization (aka Lattice QCD) has statistical and number of systematic uncertainties

These are controlled uncertainties

They can be estimated and decreased

QCD at T>0

$$S_G[A] = \int_0^{1/T} dx_4 \int d^3x \, \frac{1}{2} \mathrm{Tr}\, F_{\mu\nu}(x) F_{\mu\nu}(x)$$

$$S_F[\bar{\psi}, \psi, A] = \int_0^{1/T} dx_4 \int d^3x \sum_f \bar{\psi}_f(x) \left(\gamma_\mu D_\mu + m_f - \mu_f \gamma_0\right) \psi_f(x)$$

$T = 1/L_4$, L_4 - length in 4th direction

$L_4 = a N_4$, a – lattice spacing

$$T = \frac{1}{aN_4}$$

Simple estimation of $1/a$:

If $T \sim T_c$ (take T_c about 180 MeV)

$$1/a \sim N_4\, T_c \sim \begin{array}{l} 720\ MeV,\ N_4 = 4 \\ 1.44\ GeV,\ N_4 = 8 \\ 2.16\ GeV,\ N_4 = 12 \end{array}$$

Sign problem

$\det(\not{D} + m + \mu\gamma_0)$ -- In the integral

We use $\gamma_5 \not{D} \gamma_5 = \not{D}^\dagger$

$$\gamma_5(\not{D} + m + \mu\gamma_0)\gamma_5 = \not{D}^\dagger + m - \mu\gamma_0$$
$$= (\not{D} + m - \mu^*\gamma_0)^\dagger$$

$$\det(\not{D} + m + \mu\gamma_0) = \det{}^*(\not{D} + m - \mu^*\gamma_0)$$

$$\det(\not{D} + m + \mu\gamma_0) = \det{}^*(\not{D} + m - \mu^*\gamma_0)$$

Determinant is real only for μ =0 and $\mu = i\mu_I$

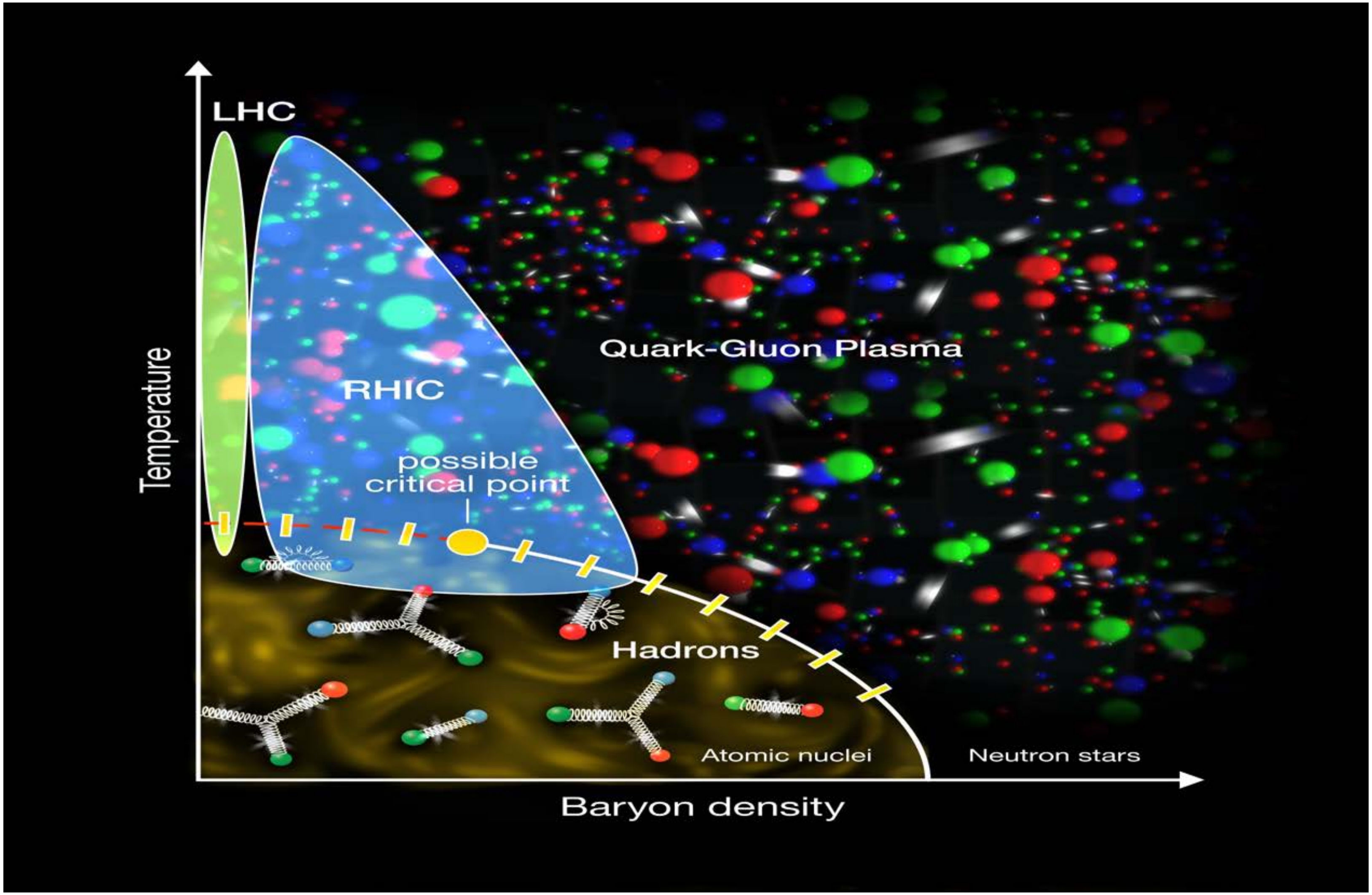
This makes impossible to apply usual MCMC algorithm In case of real μ

Note, that for imaginary μ this problem is absent

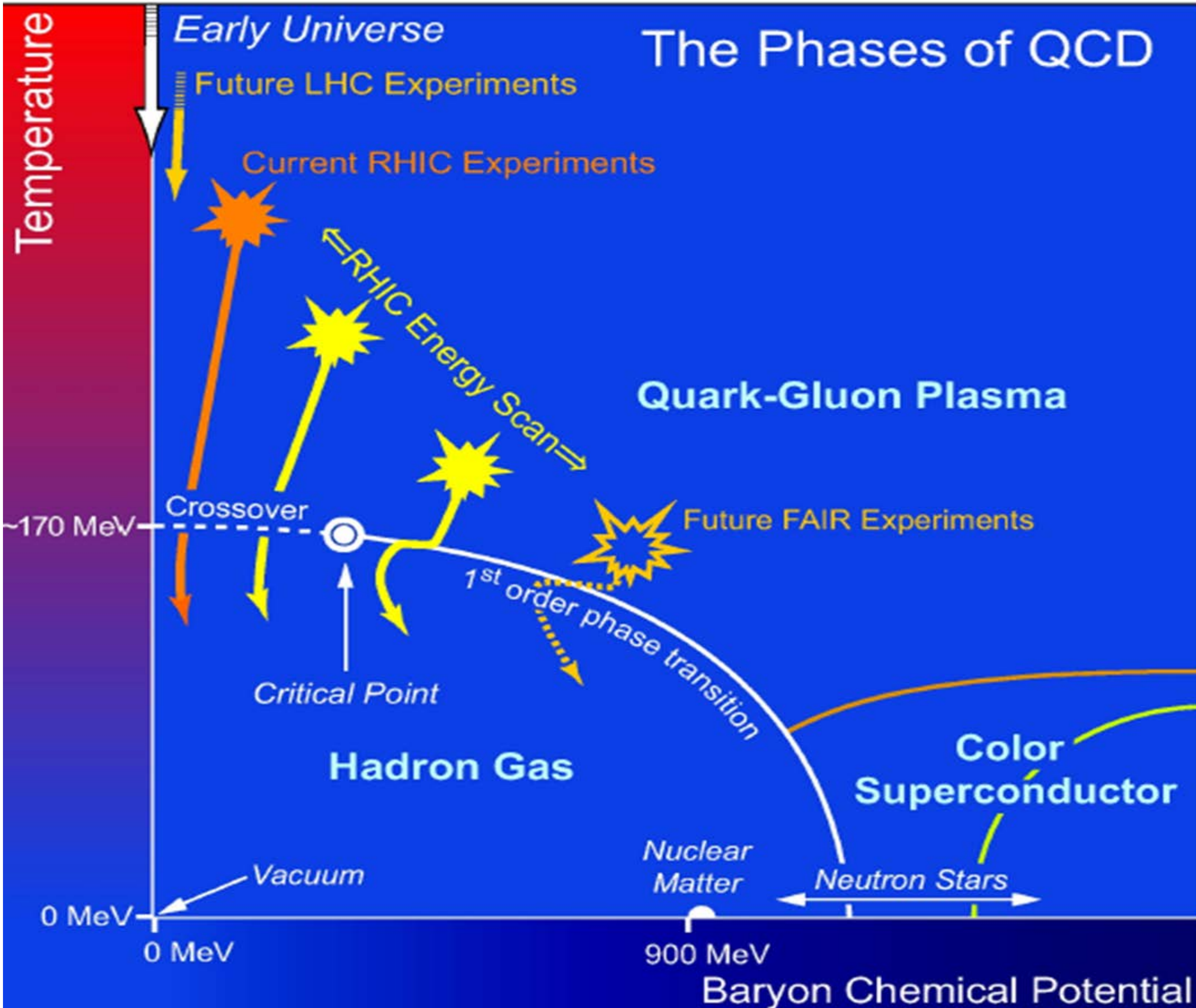
LQCD action with μ:

$$S_F(\psi, \bar{\psi}, U) = \bar{\psi}\, M(U)\psi$$

where $U_4 \to U_4 e^{a\mu}, \quad U_4^+ \to U_4^+ e^{-a\mu}$



There is a curve in the plane of temperature T versus baryon chemical potential μ representing a line of first-order phase transitions. This curve terminates in a second-order phase transition at some (T_c, μ_c)



The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
Crossover
~170 MeV
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Nuclear Matter
Vacuum
Neutron Stars
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

The expectation is that Tc is less than 160 MeV and μc is greater than a few hundred MeV.

Results of LQCD for $\mu = 0$ (T_c, EoS) are used to fix parameters in various phenomenological models.
But results at $\mu > 0$ are desirable

Methods to solve sign problem

- Multi-Parameter Reweighting

Fodor, Katz, 2002

- Taylor expansion

Gottlieb et al. Phys.Rev.Lett. 59, 2247 (1987) (up to μ^2)
Allton et al., Phys.Rev. D71, 054508 (2005) (up to μ^6)

- Imaginary Chemical Potential

D'Elia, Lombardo, 2002

- Canonical ensemble approach

de Forcrand, Philipsen, 2002

For free quark-gluon gas (Stefan-Boltzmann limit):

$$\frac{p_{SB}}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,..} \left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4 \right]$$

This is valid for very high T

For low T – Hadron resonance gas (HRG) model

$$\frac{p}{T^4} = G(T) + F(T)\cosh\left(\frac{3\mu_q}{T}\right)$$

Notations

pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \log Z(V, T, \mu)$$

Quark number density

$$n_f/T^3 = \frac{\partial p/T^4}{\partial \mu_f/T}$$

Susceptibility

$$\chi_{ff}/T^2 = \frac{\partial n_f/T^3}{\partial \mu_f/T}$$

Taylor expansion for pressure:

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T}\right)^n$$

$$\mu_u = \mu_d = \mu_q$$

arXiv:1701.04325v2 [hep-lat] 17 Jan 2017

The QCD Equation of State to $\mathcal{O}(\mu_B^6)$ from Lattice QCD

A. Bazavov,[1] H.-T. Ding,[2] P. Hegde,[3] O. Kaczmarek,[2,4] F. Karsch,[4,5] E. Laermann,[4] Y. Maezawa,[6] Swagato Mukherjee,[5] H. Ohno,[5,7] P. Petreczky,[5] H. Sandmeyer,[4] P. Steinbrecher,[4,5] C. Schmidt,[4] S. Sharma,[5] W. Soeldner,[8] and M. Wagner[9]

[1] *Department of Computational Mathematics, Science and Engineering and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA*
[2] *Key Laboratory of Quark & Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*
[3] *Center for High Energy Physics, Indian Institute of Science, Bangalore 560012, India*
[4] *Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*
[5] *Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*
[6] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8317, Japan*
[7] *Center for Computational Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan*
[8] *Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[9] *NVIDIA GmbH, D-52146 Würselen, Germany*

We calculated the QCD equation of state using Taylor expansions that include contributions from up to sixth order in the baryon, strangeness and electric charge chemical potentials. Calculations have been performed with the Highly Improved Staggered Quark action in the temperature range $T \in [135\ \text{MeV}, 330\ \text{MeV}]$ using up to four different sets of lattice cut-offs corresponding to lattices of size $N_\sigma^3 \times N_\tau$ with aspect ratio $N_\sigma/N_\tau = 4$ and $N_\tau = 6-16$. The strange quark mass is tuned to its physical value and we use two strange to light quark mass ratios $m_s/m_l = 20$ and 27, which in the continuum limit correspond to a pion mass of about 160 MeV and 140 MeV respectively. Sixth-order results for Taylor expansion coefficients are used to estimate truncation errors of the fourth-order expansion. We show that truncation errors are small for baryon chemical potentials less then twice the temperature ($\mu_B \le 2T$). The fourth-order equation of state thus is suitable for the modeling of dense matter created in heavy ion collisions with center-of-mass energies down to $\sqrt{s_{NN}} \sim 12$ GeV. We provide a parametrization of basic thermodynamic quantities that can be readily used in hydrodynamic simulation codes. The results on up to sixth order expansion coefficients of bulk thermodynamics are used for the calculation of lines of constant pressure, energy and entropy densities in the T-μ_B plane and are compared with the crossover line for the QCD chiral transition as well as with experimental results on freeze-out parameters in heavy ion collisions. These coefficients also provide estimates for the location of a possible critical point. We argue that results on sixth order expansion coefficients disfavor the existence of a critical point in the QCD phase diagram for $\mu_B/T \le 2$ and $T/T_c(\mu_B = 0) > 0.9$.

Data for basic thermodynamic observables calculated for baryon chemical potentials $\mu_B/T \le 2.2$ in the temperature range $135\ \text{MeV} \le T \le 280\ \text{MeV}$ for the cases (I) of vanishing strangeness and electric charge chemical potentials and (II) vanishing strangeness density, $n_S = 0$ and fixed electric charge to net baryon-number ratio, $n_Q/n_B = 0.4$ are provided as two ancillary files.

January 18, 2017

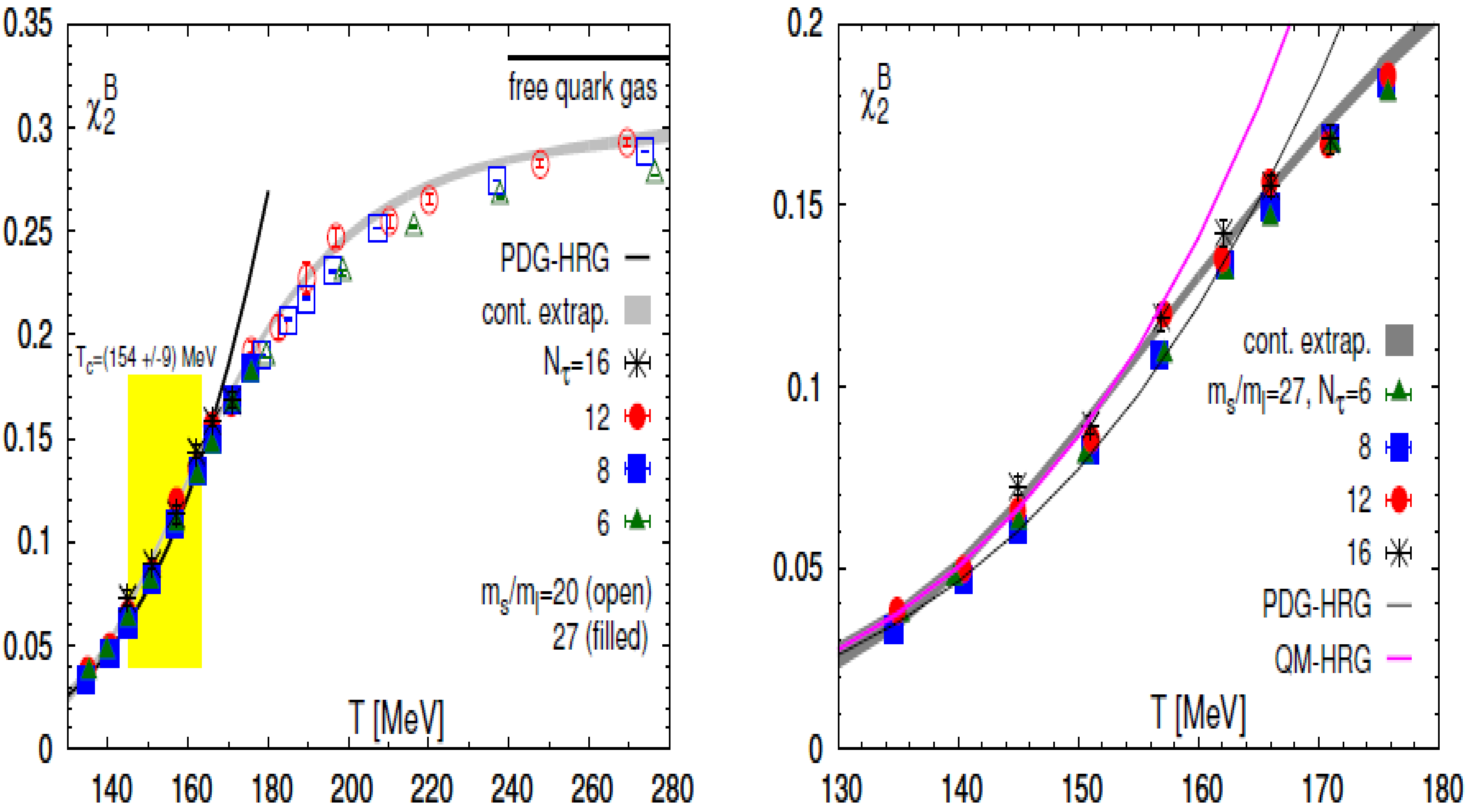
PACS numbers: 11.10.Wx, 12.38.Gc, 12.38Mh

$$N_f = 2 + 1$$

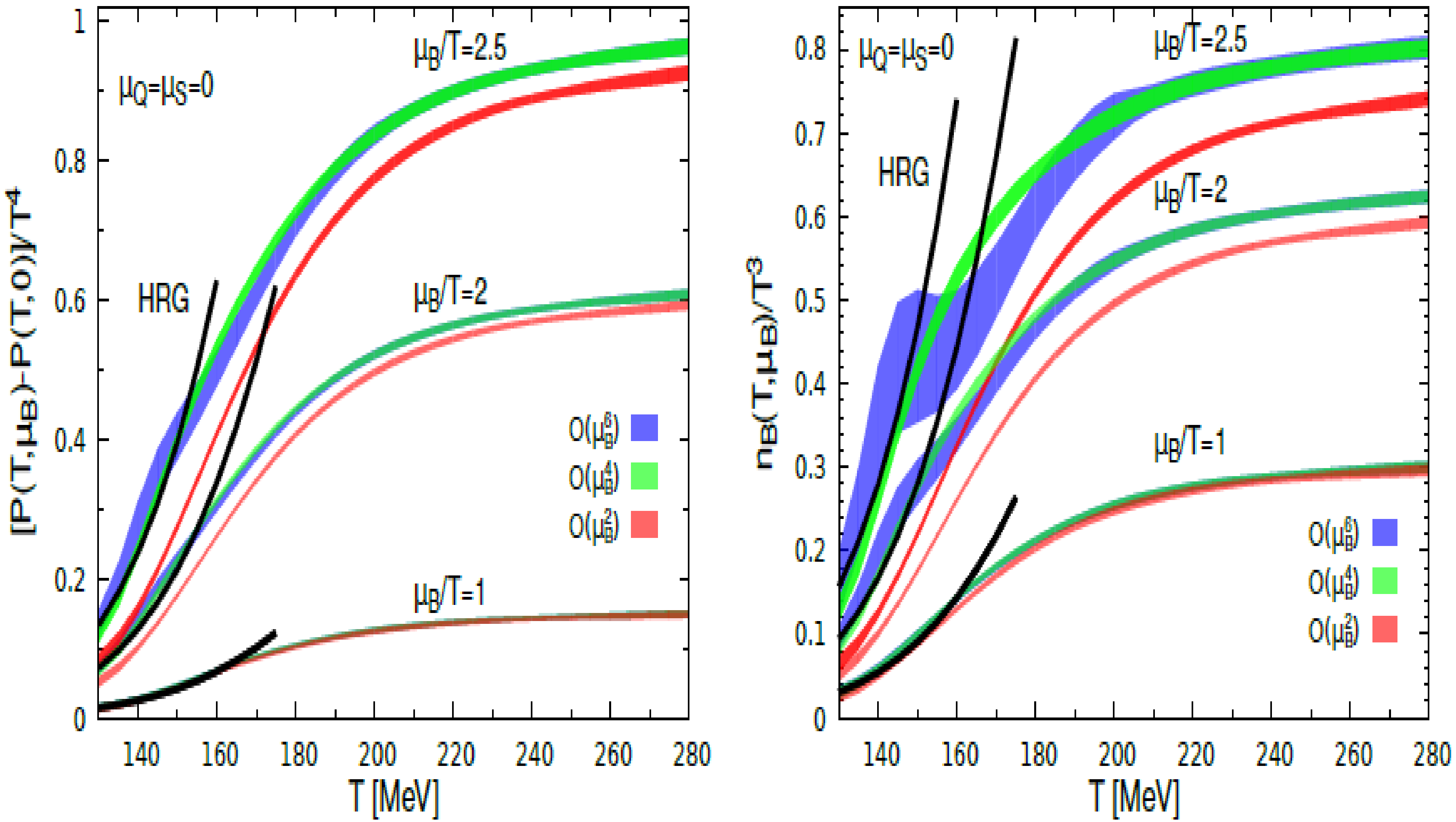
$$\frac{m_s}{m_l} = 27 \quad (m_\pi = 140\text{ MeV})$$

$$N_t = 8,10,12,16$$

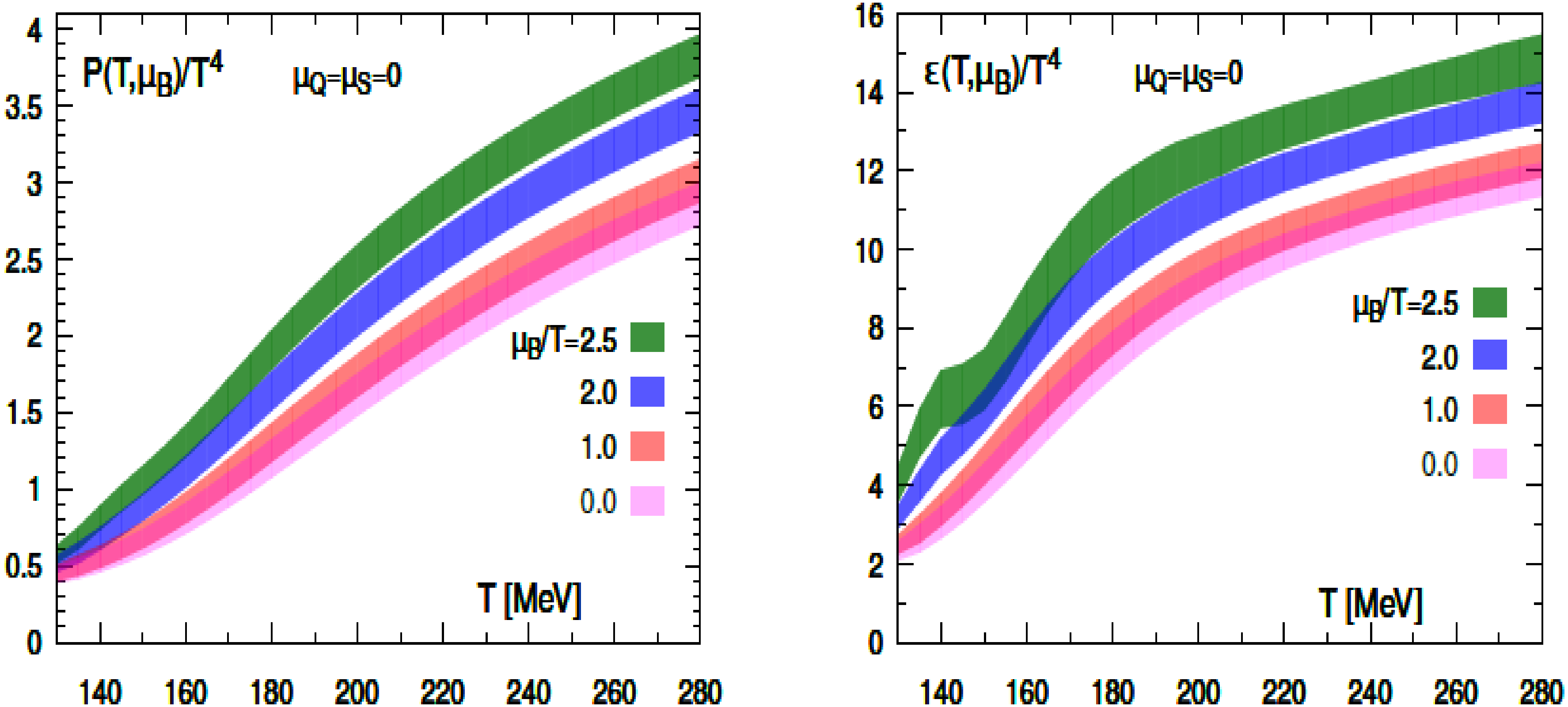
$$\frac{N_s}{N_t} = 4$$



χ_2^B
free quark gas
PDG-HRG
cont. extrap.
N_τ=16
12
8
6
m_s/m_l=20 (open)
27 (filled)
T_c=(154 +/-9) MeV
T [MeV]
χ_2^B
cont. extrap.
m_s/m_l=27, N_τ=6
8
12
16
PDG-HRG
QM-HRG
T [MeV]



[P(T,μB)-P(T,0)]/T4
μQ=μS=0
μB/T=2.5
μB/T=2
μB/T=1
HRG
O(μB6)
O(μB4)
O(μB2)
T [MeV]
nB(T,μB)/T3
μQ=μS=0
μB/T=2.5
μB/T=2
μB/T=1
HRG
O(μB6)
O(μB4)
O(μB2)
T [MeV]



P(T,μB)/T4
μQ=μS=0
μB/T=2.5
2.0
1.0
0.0
T [MeV]
ε(T,μB)/T4
μQ=μS=0
μB/T=2.5
2.0
1.0
0.0
T [MeV]

Assuming that the current results obtained with expansion coefficients up to 6th order are indicative for the behavior of higher order expansion coefficients and taking into account the current errors on 6th order expansion coefficients we concluded that at temperatures T > 135 MeV the presence of a critical point in the QCD phase diagram for $\mu_B < 2T$ is unlikely.

Imaginary μ_q

At imaginary chemical potential $\mu_q = i\mu_{qI}$ the sign problem is absent and standard Monte Carlo algorithms can be applied to simulate Lattice QCD.   Can we use this?

Study of QCD at nonzero μ_{qI} can provide us with information about physical range of μ_q

- extrapolation to $\mu_q = 0$ or analytical continuation to nonzero real μ_q

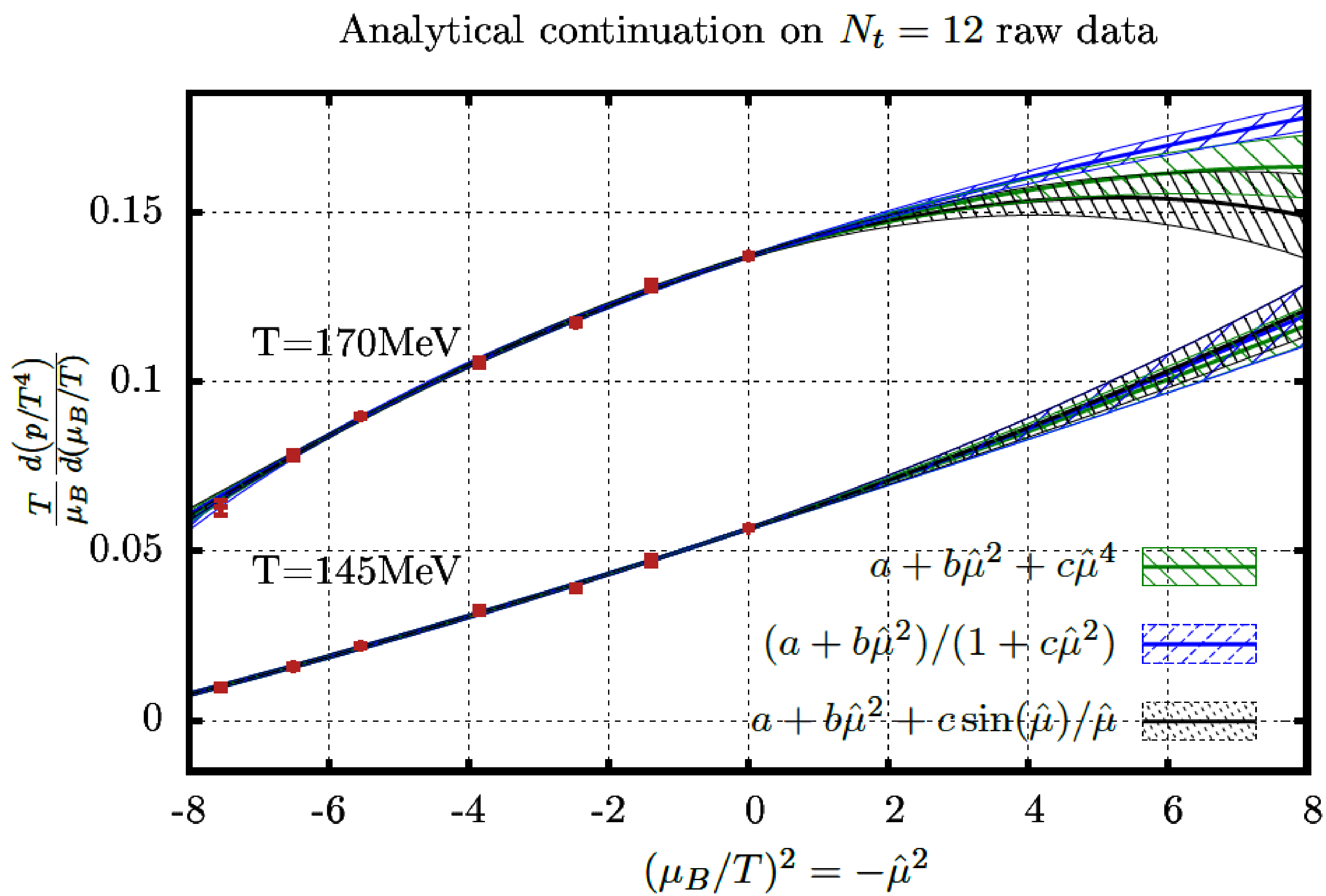
The QCD partition function Z is a periodic function of $\theta = \mu_{qI}/T$:

$$Z(\theta) = Z(\theta + 2\pi k/3)$$

There are 1st order phase transitions at $\theta = (2k+1)\frac{\pi}{3}$

This symmetry is called Roberge-Weiss symmetry

N_f=2+1, N_t=10,12,16, N_s=40,48,64
Fodor et al., 2016



QCD at nonzero baryon density

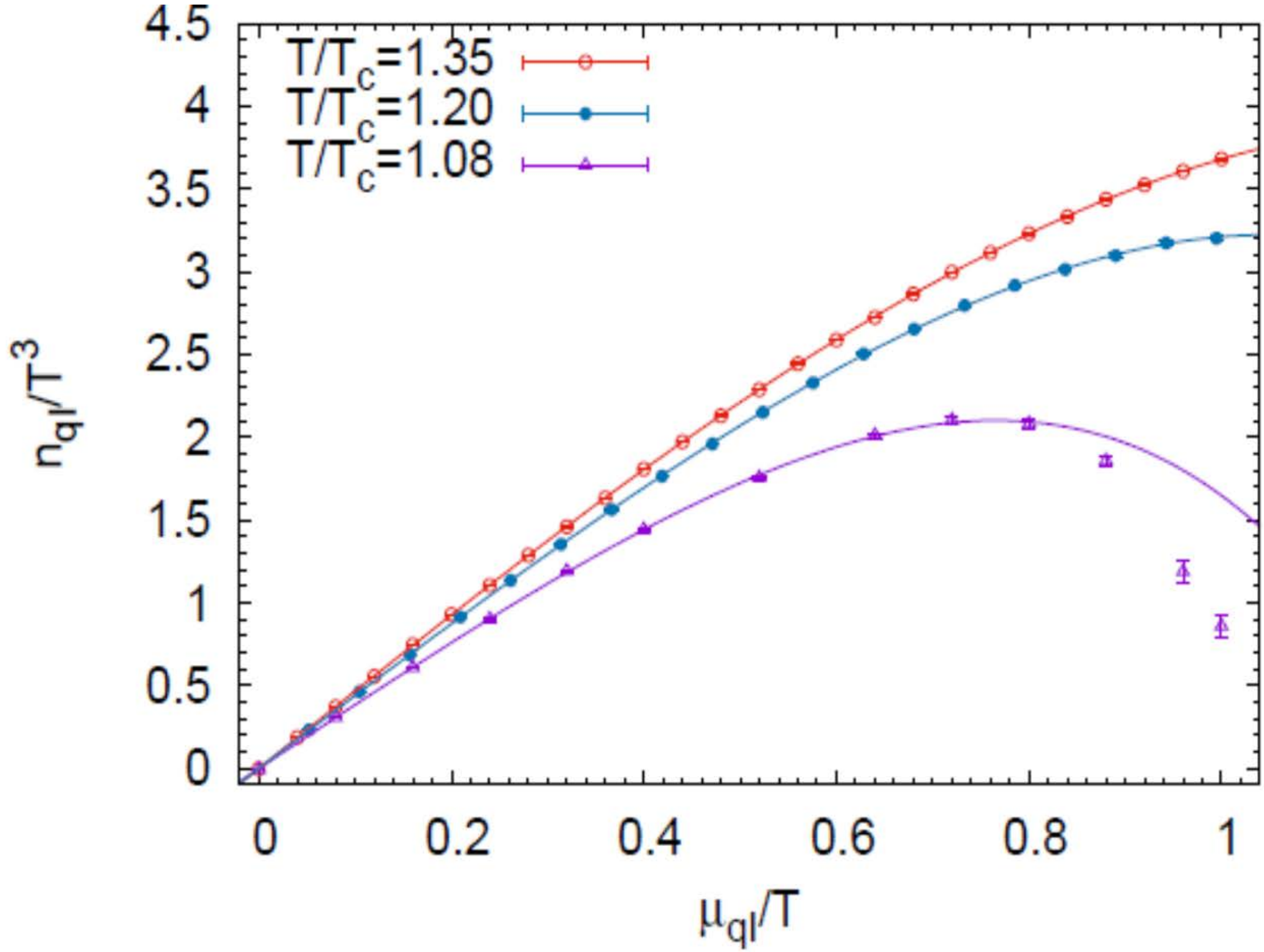
A. Nakamura, V.B., A. Molochkov, D. Boyda, V. Goy, A. Nikolaev, V. Zakharov

Simulation settings

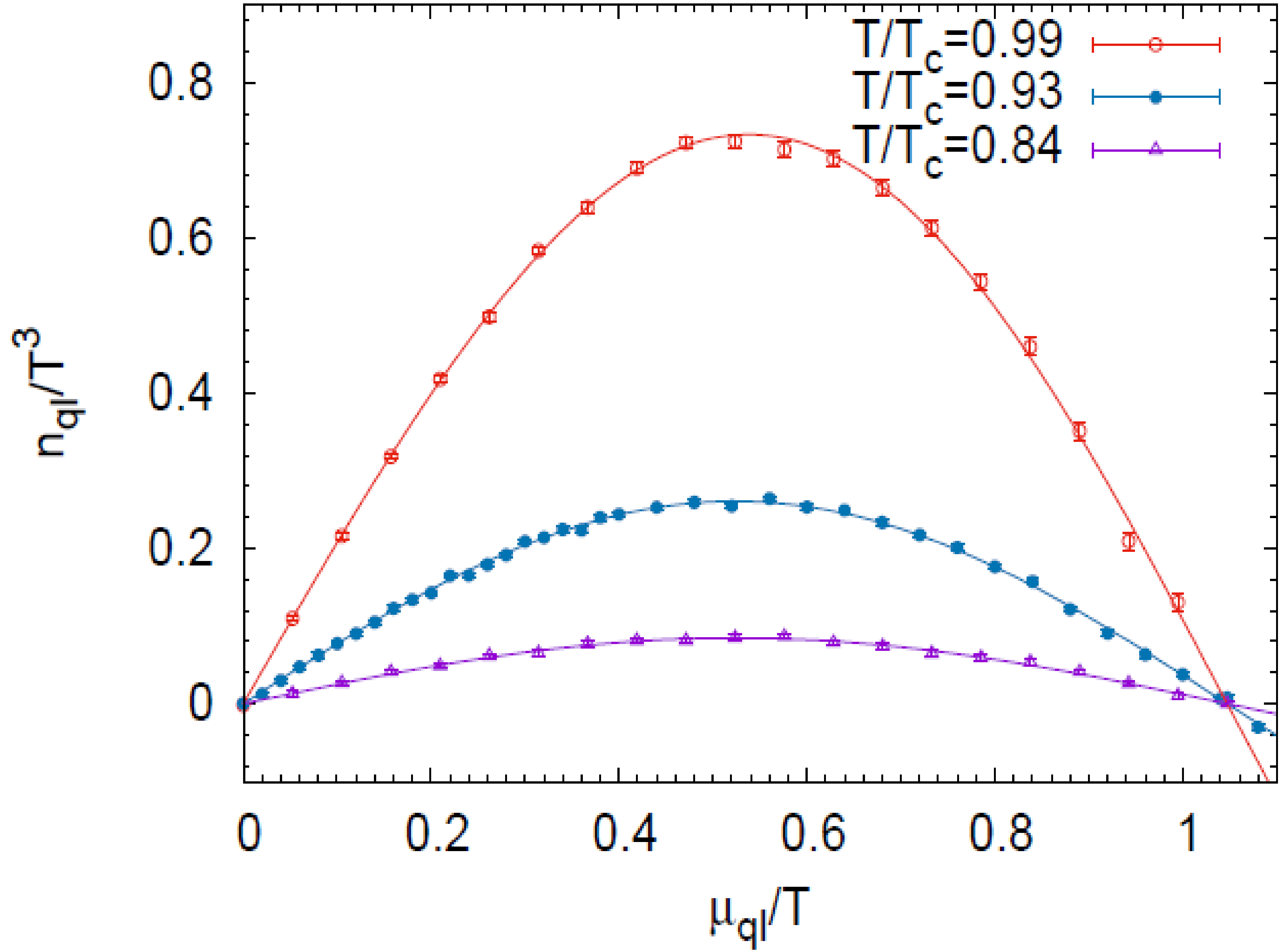
We simulate $N_f = 2$ lattice QCD with clover improved Wilson fermions and Iwasaki improved gauge field action

To fix parameters (lattice spacing a, temperature T, quark mass m_q) we use $T = 0$ results of WHOT QCD collaboration

Currently quark mass is defined by ratio $m_\pi / m_\rho = 0.8$ ($m_\pi \approx 0.7$ GeV
Lattice size: $16^3 \times 4$
large lattice spacing: $a \approx 0.2$ fm
large volume: $L \approx 3.2$ fm
We simulate at imaginary chemical potential $\mu_q = i\mu_q^I$
At $T > T_c$ ($T/T_c = 1.08$; 1.35, 1.20)
at $T < T_c$ ($T/T_c = 0.84$; 0.93; 0.99)



Imaginary number density at $T > T_{RW}$ and $T_c < T < T_{RW}$



T/T_c=0.99
T/T_c=0.93
T/T_c=0.84
0.8
0.6
0.4
0.2
0
n_{ql}/T^3
0
0.2
0.4
0.6
0.8
1
μ_{ql}/T

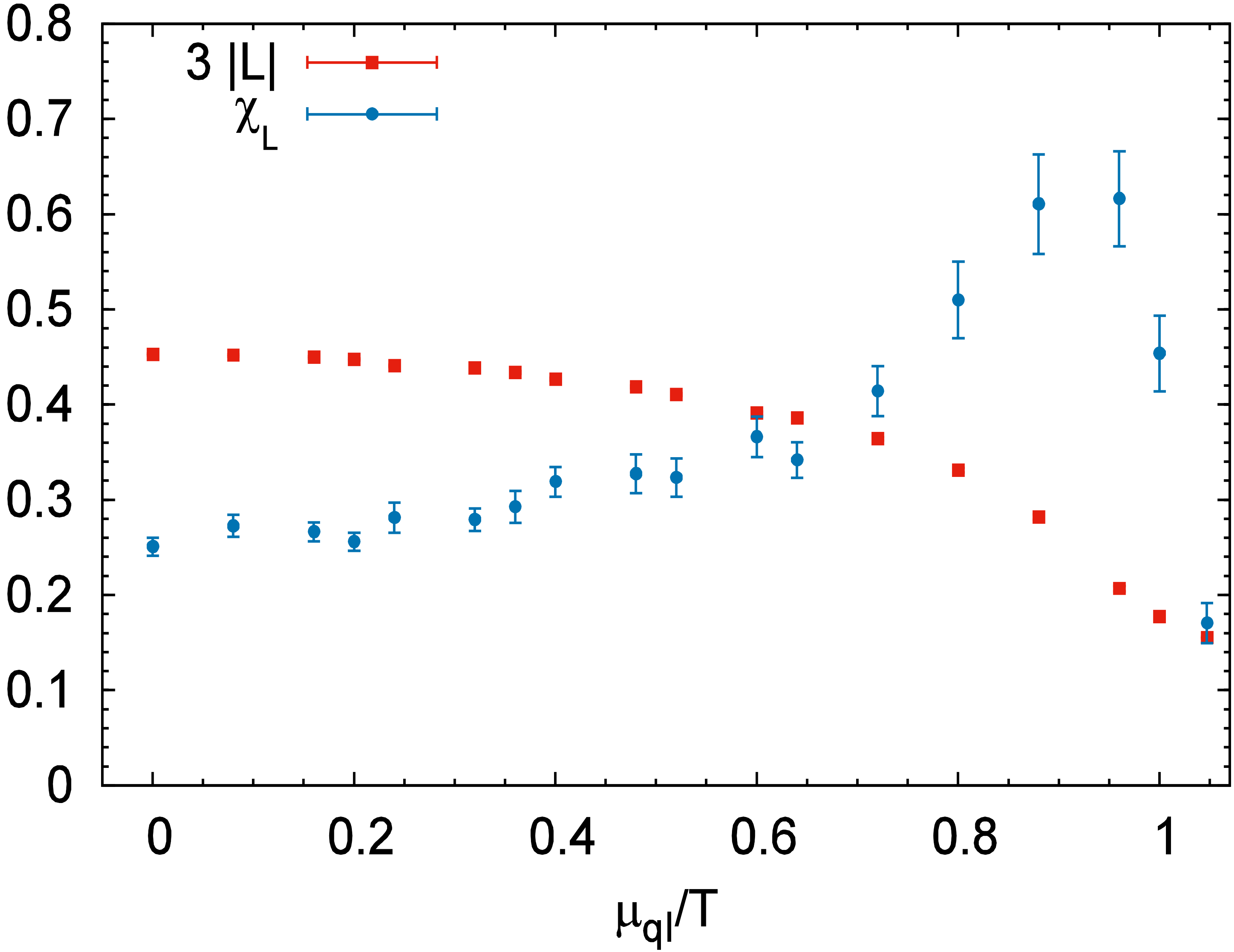
In the deconfining phase we fit the data for n_{ql} to a polynomial of θ

$$n_{ql}(\theta) = \sum_{n=1}^{n_{max}} a_n \theta^{2n-1} \tag{10}$$

while in the confining phase (below T_c) we fit it to a Fourier expansion

$$n_{ql}(\theta) = \sum_{n=1}^{n_{max}} f_{3n} sin(3n\theta) \tag{11}$$

Similar fits were used in Takahashi et al, 2014
Gunther et al., 2016 arXiv:1607.02493



3 |L|
χL
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
μql/T

T/T_c	a_1	a_3	a_5	χ^2/N_{dof}	$2c_2$	$4c_4$
1.35	4.671(2)	-0.991(4)	-	0.67	4.68(1)	0.97(8)
1.20	4.409(6)	-1.03(3)	-0.17(3)	0.70	4.40(1)	1.3(1)
1.08	3.86(2)	-1.46(16)	-0.75(25)	0.91	3.88(2)	1.3(2)

Results of fitting of n_{ql}/T^3 in the deconfinement phase.

Comparison with Taylor expansion — Ejiri et al., 2010

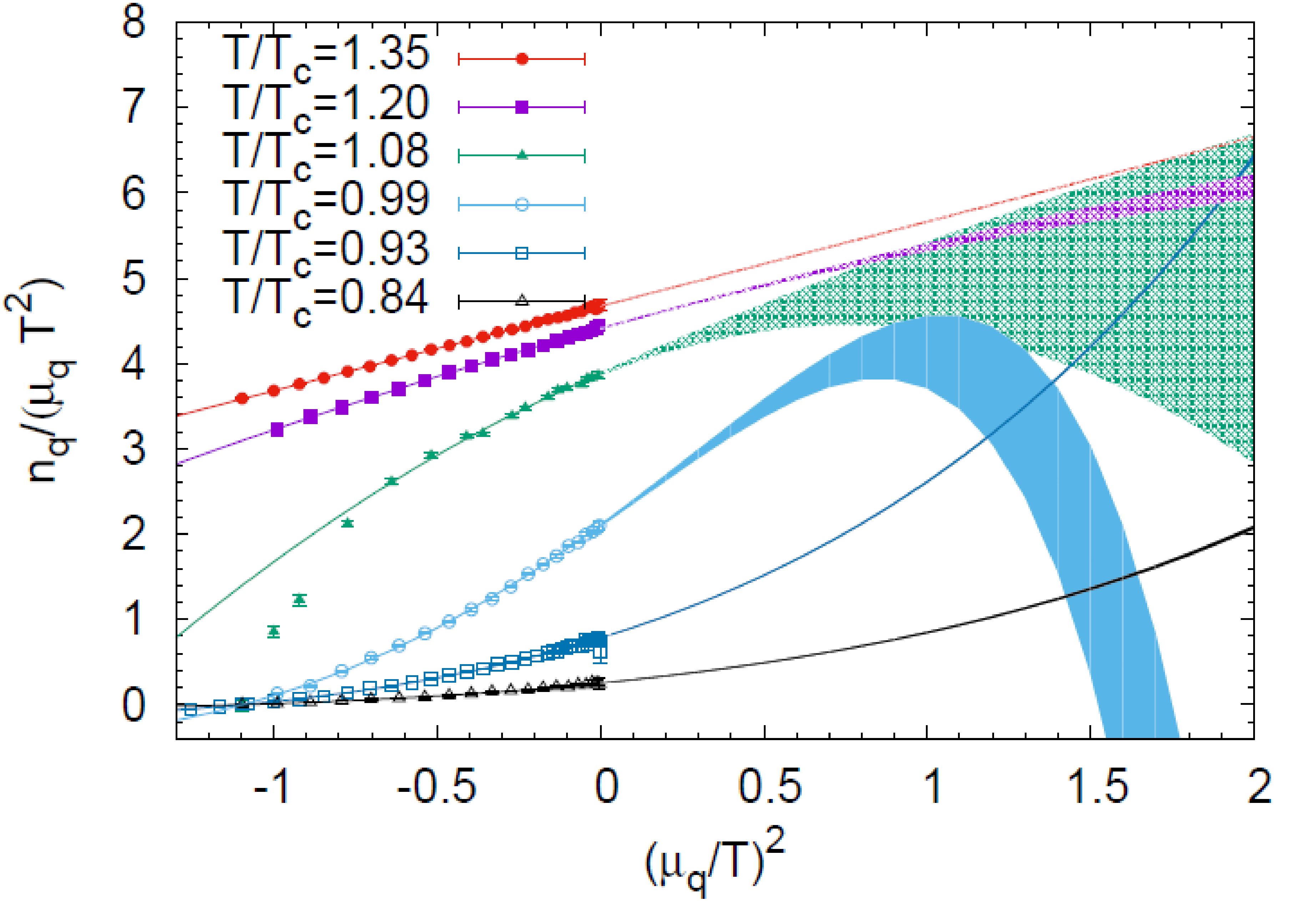
note, $a_1 = 2c_2; a_3 = -4c_4$

T/T_c	f_3	f_6	$\chi^2/N_{dof}, N_{dof}$
0.99	0.7326(25)	-0.0159(21)	0.83, 18
0.93	0.2608(8)	-	0.93, 37
0.84	0.0844(7)	-	0.41, 18

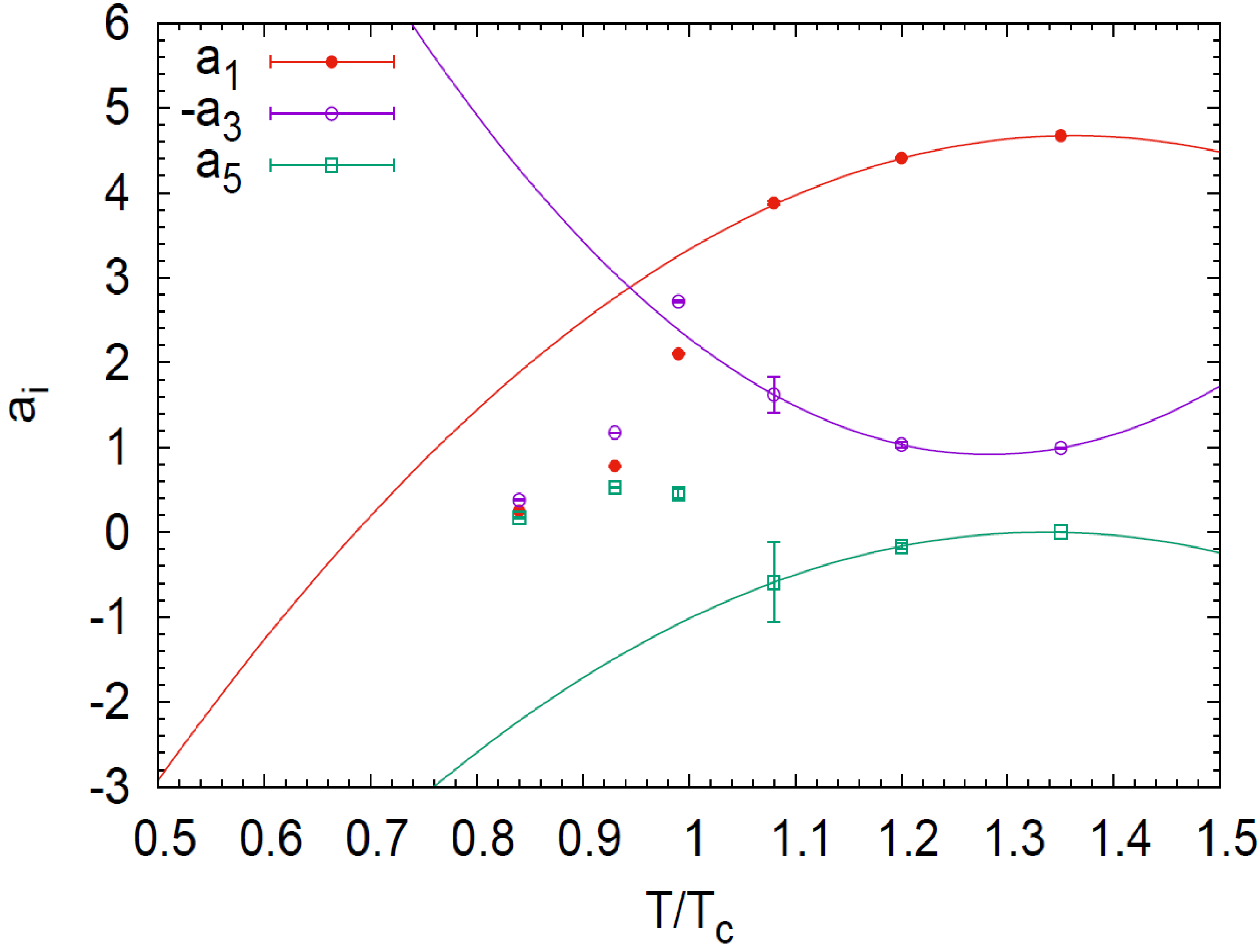
Results of fitting data for n_{ql}/T^3 in the confinement phase

T/T_c	a_1	a_3	a_5	χ^2/N_{dof}	$2c_2$	$4c_4$
0.99	2.10(1)	-2.72(2)	0.45(6)	0.83	2.07(3)	2.90(8)
0.93	0.782(3)	-1.174(4)	0.528(2)	0.93	0.71(4)	0.33(5)
0.84	0.253(2)	-0.380(3)	0.171(2)	0.41	0.25(4)	0.0(37)

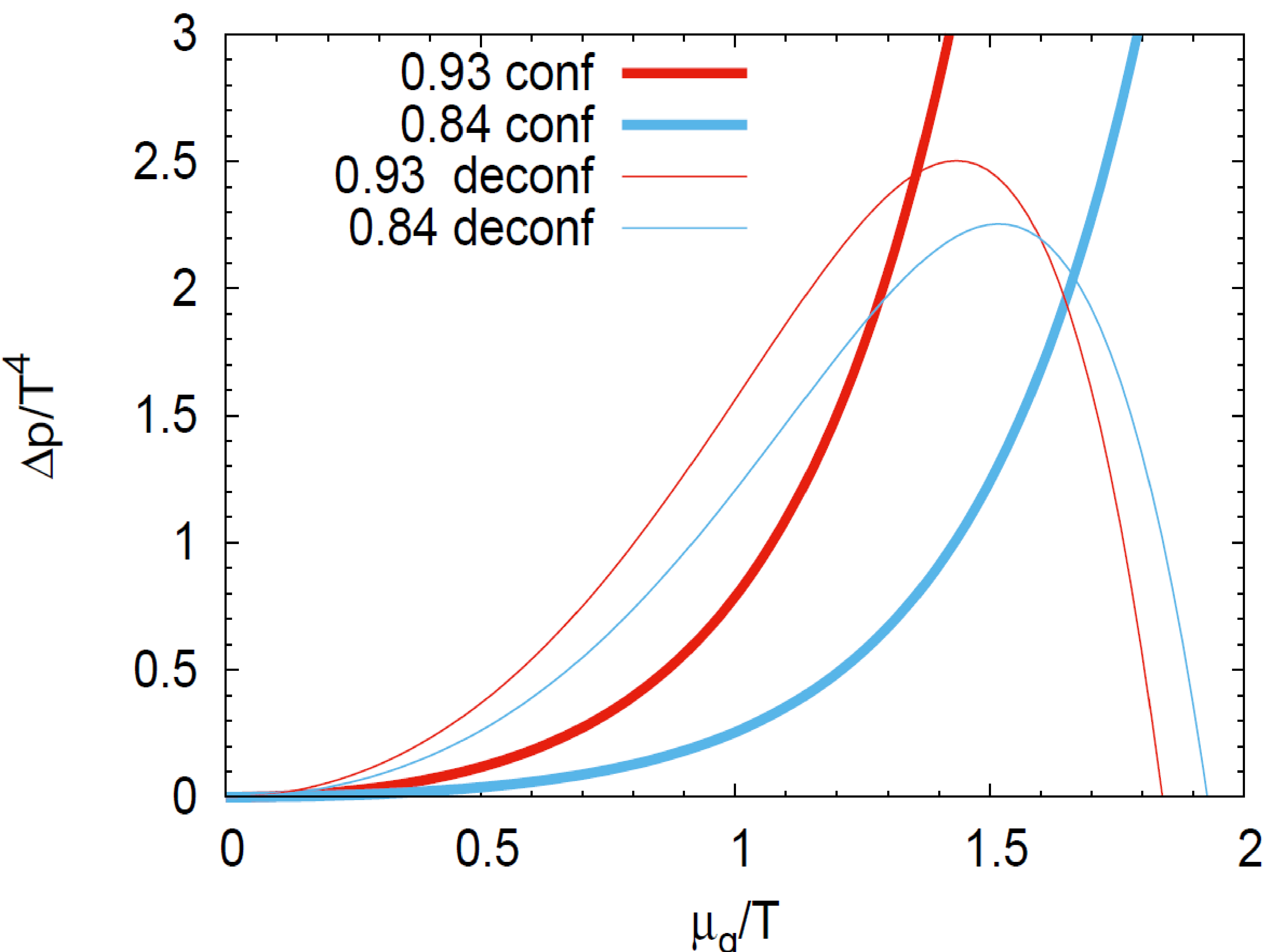
Taylor coefficients in the confinement phase.



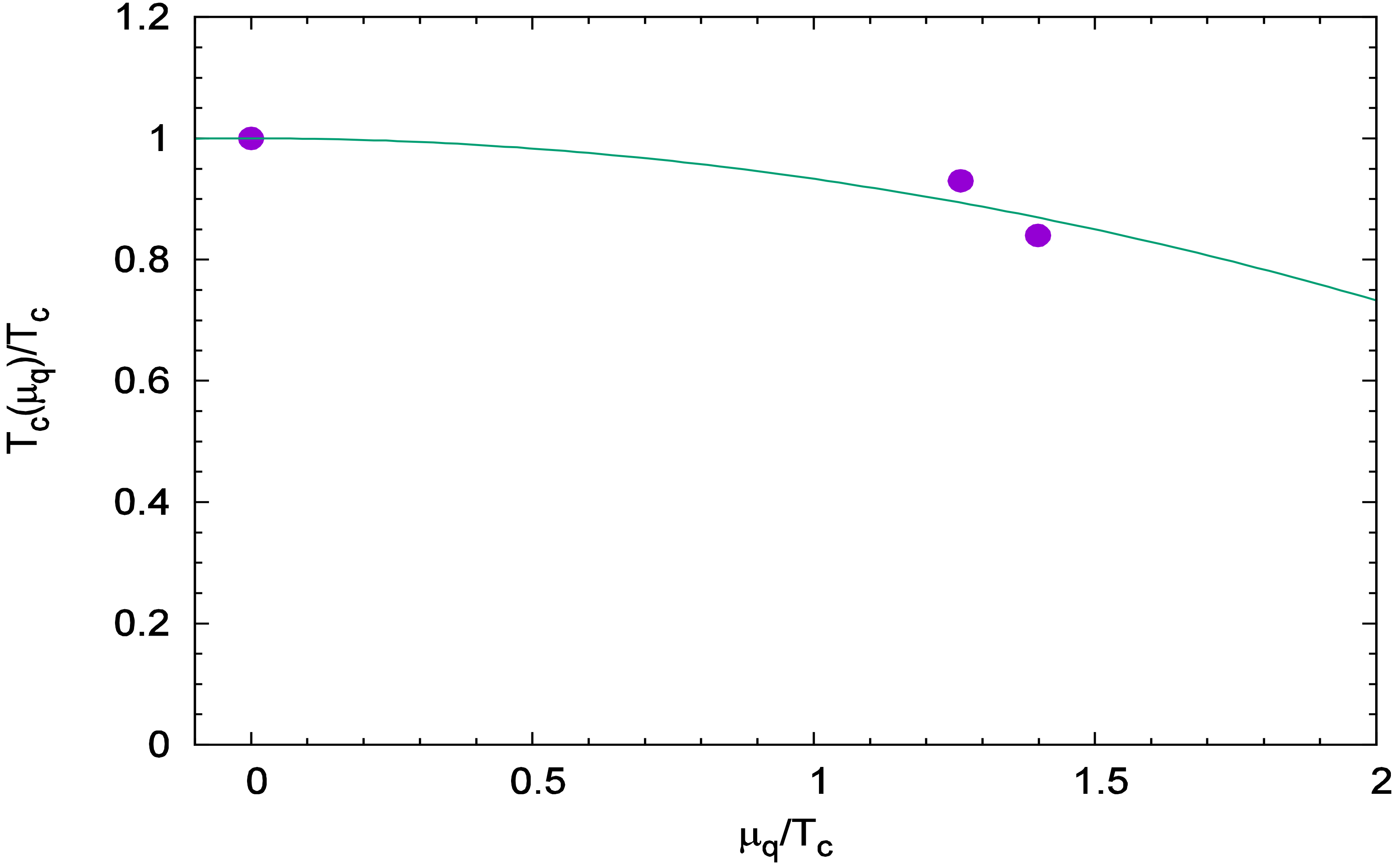
8
7
6
5
4
3
2
1
0
-1
-0.5
0
0.5
1
1.5
2
T/Tc=1.35
T/Tc=1.20
T/Tc=1.08
T/Tc=0.99
T/Tc=0.93
T/Tc=0.84
nq/(μq T2)
(μq/T)2



a_1
$-a_3$
a_5
a_i
T/T_c
6
5
4
3
2
1
0
-1
-2
-3
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5



0.93 conf
0.84 conf
0.93 deconf
0.84 deconf
Δp/T^4
μ_q/T
0
0.5
1
1.5
2
2.5
3



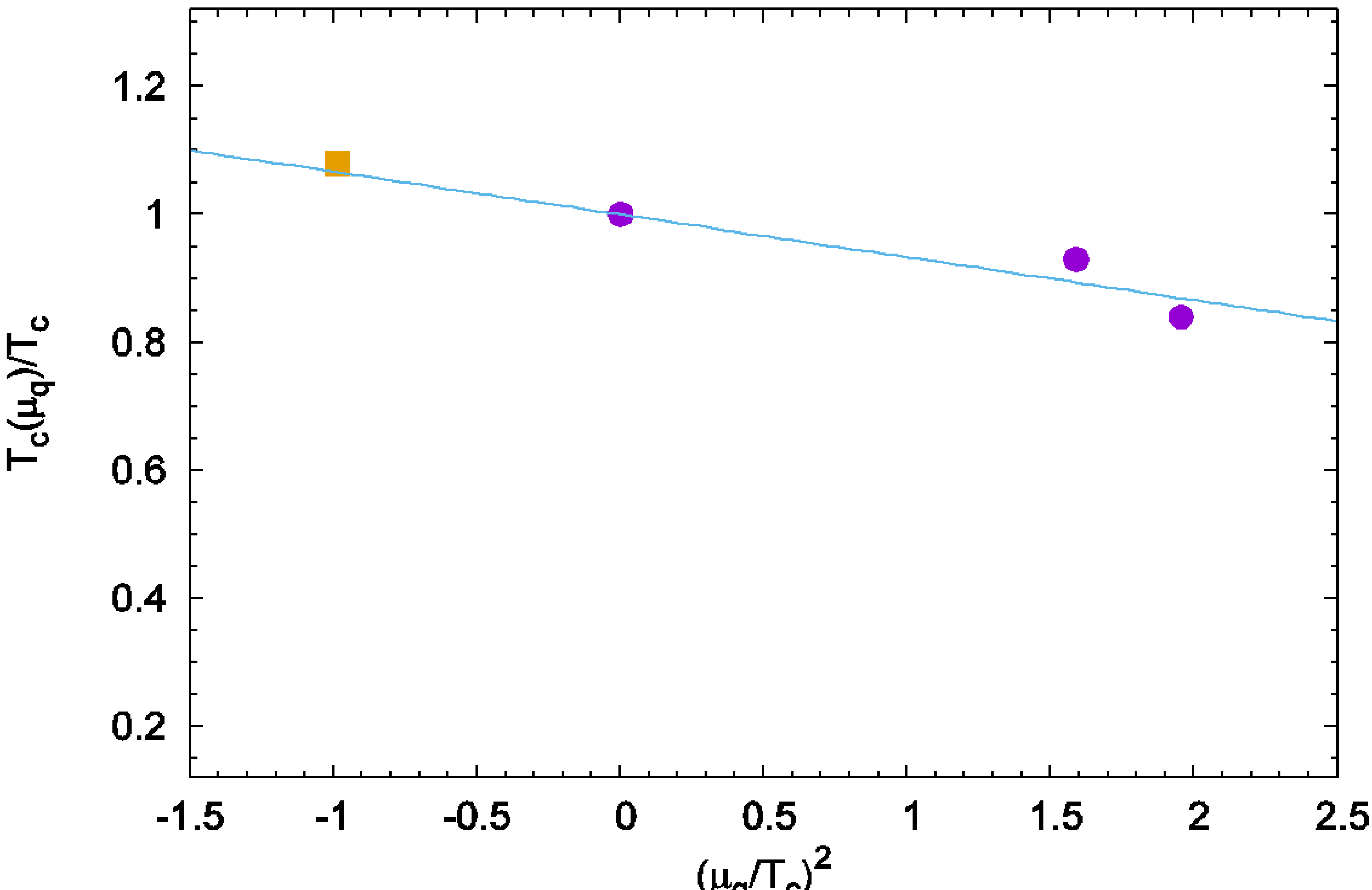
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
$T_c(\mu_q)/T_c$
μ_q/T_c

$$T_c(\mu_q)/T_c = 1 - C\left(\mu_q/T_c\right)^2$$

C=0.07 in nice agreement with other results for N_f=2 lattice QCD:

C=0.051(3) De Forcrand, Philipsen, 2002

C=0.065(7) Wu, Luo, Chen, 2007



1.2
1
0.8
0.6
0.4
0.2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
$T_c(\mu_q)/T_c$
$(\mu_q/T_c)^2$

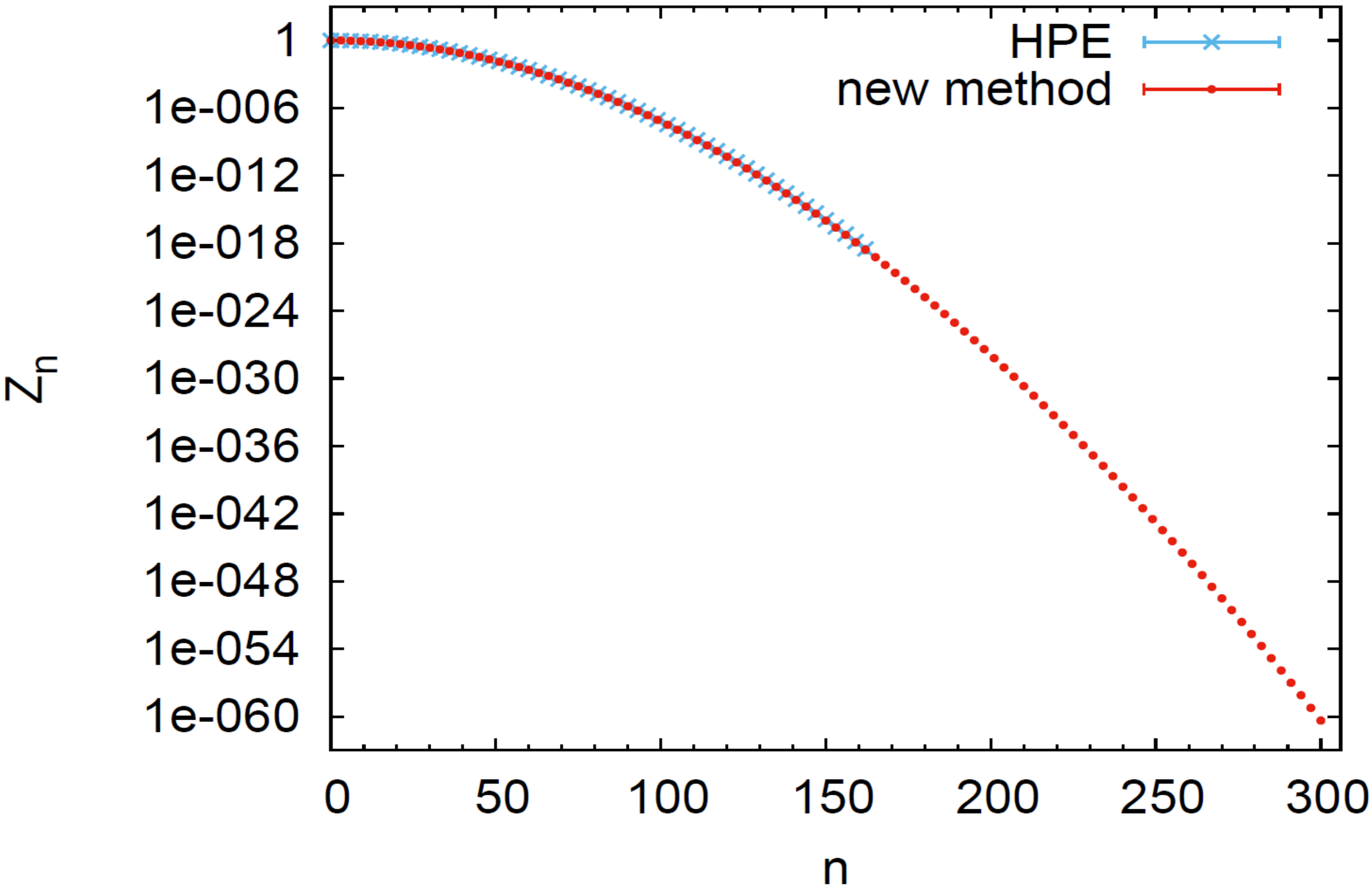
Canonical ensemble

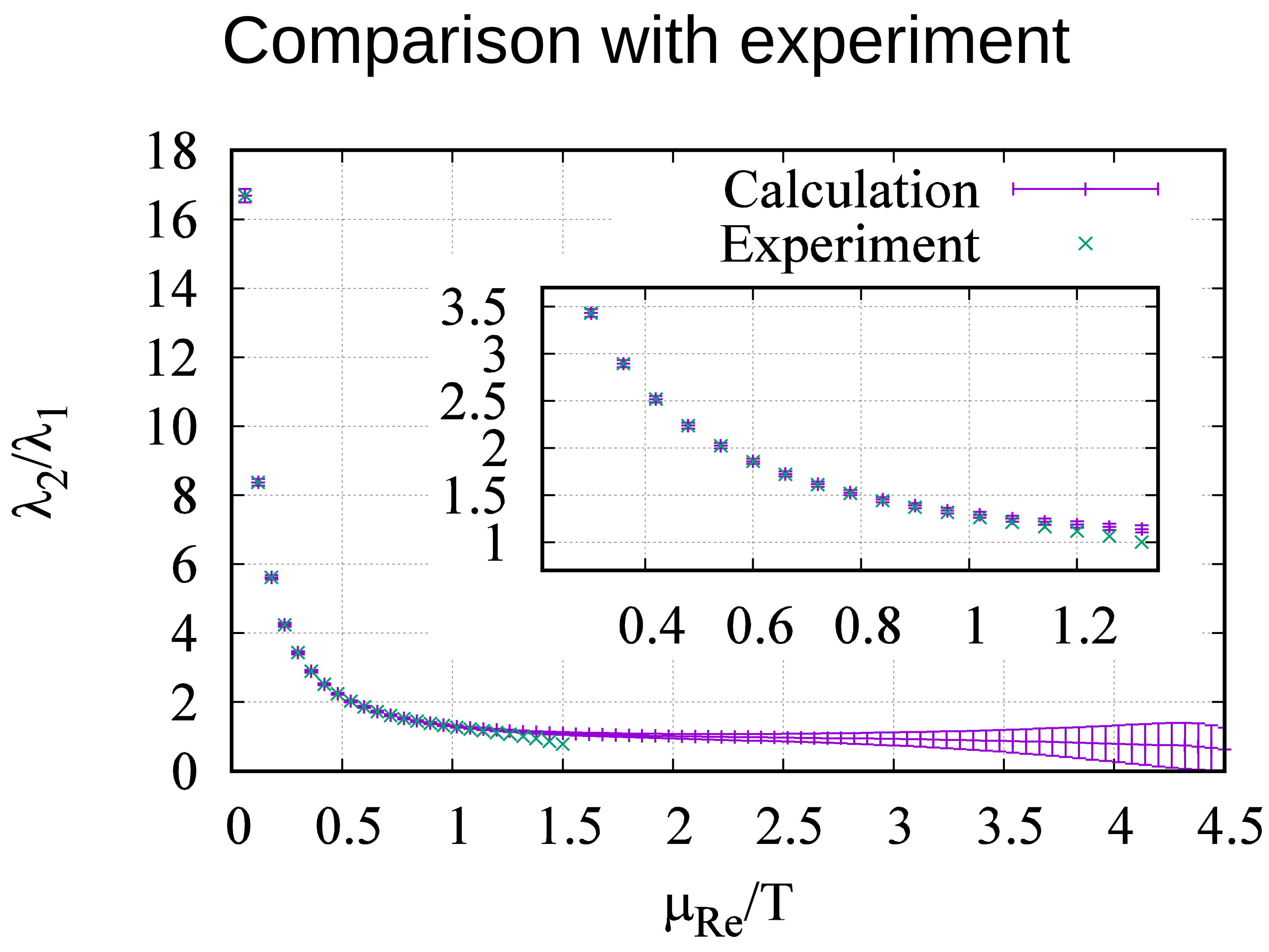
$$Z_{GC}(\mu, T, V) = \mathrm{Tr}\left(e^{-\frac{\hat{H}-\mu\hat{N}}{T}}\right) = \sum_{n=-\infty}^{\infty} \langle n| e^{-\frac{\hat{H}}{T}} |n\rangle e^{\frac{\mu n}{T}}$$

$$= \sum_{n=-\infty}^{\infty} Z_C(n, T, V) e^{\frac{\mu n}{T}} = \sum_{n=-\infty}^{\infty} Z_n e^{\theta n} = \sum_{n=-\infty}^{\infty} Z_n \xi^n.$$

$$Z_n = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-in\theta} Z_{GC}(\mu = i\mu_{Im}, T, V).$$

Comparison with another method

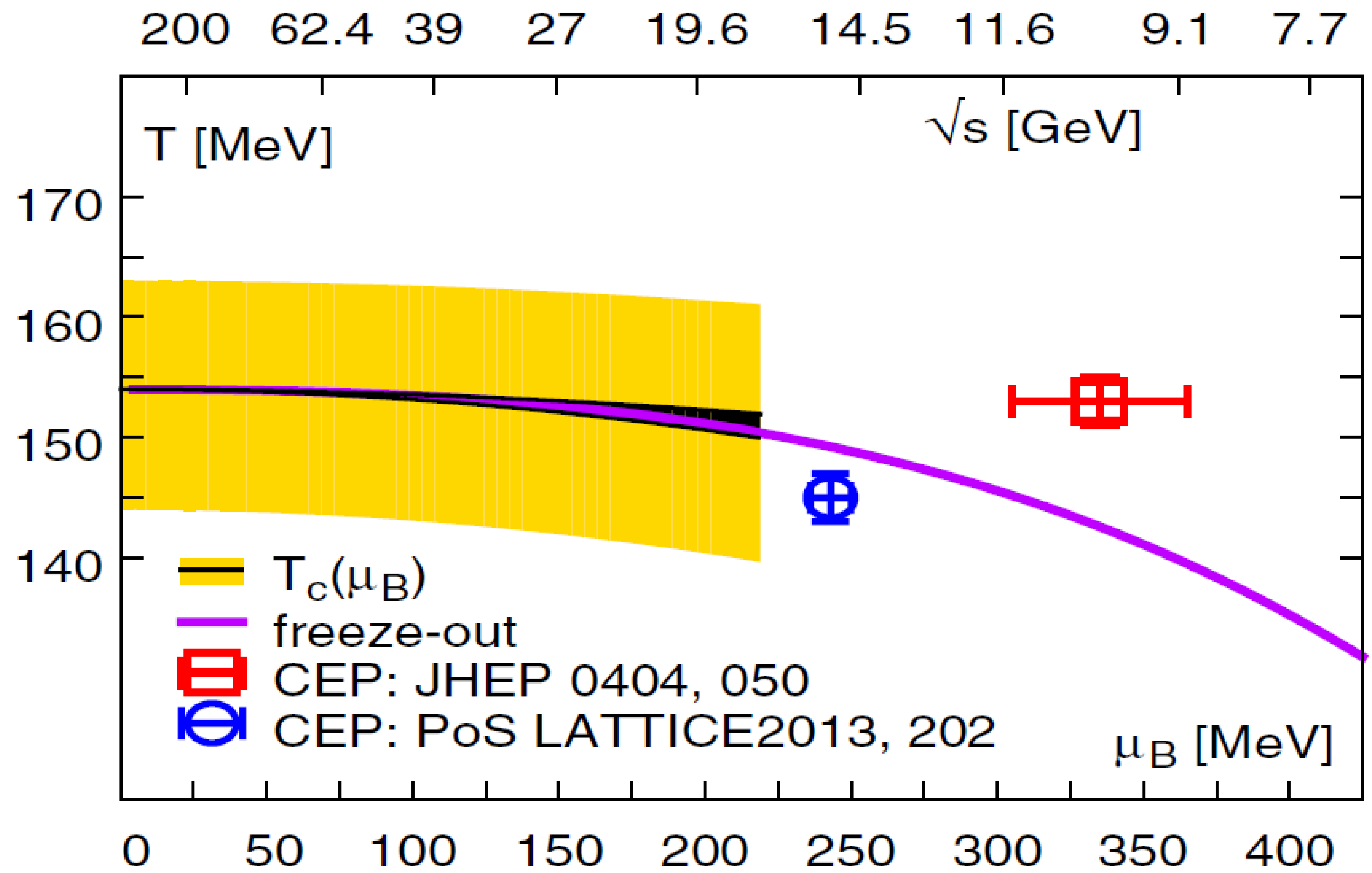




Comparison with experiment
Calculation
Experiment
18
16
14
12
10
8
6
4
2
0
3.5
3
2.5
2
1.5
1
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
λ_2/λ_1
μ_{Re}/T

- coefficients of Taylor expansion agree with direct computation, smaller errors
- in the deconfining phase our results are in nice agreement with hopping parameter expansion
- in the confining phase agreement is not so good but there is hope for improvement
- contrary to hopping parameter expansion - no limitations on quark mass
- observed agreement with the hopping parameter expansion means method works beyond Taylor expansion validity range.
- Pressure, number density and higher cumulants can be computed beyond Taylor expansion

$T_c(\mu)$



200
62.4
39
27
19.6
14.5
11.6
9.1
7.7
T [MeV]
√s [GeV]
170
160
150
140
Tc(μB)
freeze-out
CEP: JHEP 0404, 050
CEP: PoS LATTICE2013, 202
μB [MeV]
0
50
100
150
200
250
300
350
400

We use hopping parameter expansion (HPE) to evaluate the determinant ($\xi = e^{\mu_B a N_t} = e^{\mu_B/T}$):

$$Tr\left[ln\,\Delta\right] = Tr\left[ln\left(I - \kappa Q\right)\right] = -\sum_{n=1}^{\infty} \frac{\kappa^n}{n} Tr\left[Q^n\right] = \sum_{n=-\infty}^{\infty} W_n \left(e^{\mu_B a N_\tau}\right)^n$$

$$\det \Delta(U) = e^{Tr[ln\Delta]} = exp\left[\sum_{n=-N_{cut}}^{N_{cut}} W_n[U]\xi^n\right]$$

$W_n[U]$ may be calculated using stochastic estimators for $Tr\left[Q^n\right]$.

$$\det \Delta(U) = \sum_{n=-2N_x N_y N_z N_c}^{2N_x N_y N_z N_c} z_n[U]\xi^n$$