Energy dependence of the slope parameter

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Overview

Introduction

- Unitarization procedure is an economy and convenient way to construct a true scattering amplitude obeying unitarity.
- Regge model with linear trajectory $\sim (s/s_0)^{\alpha(t)}$ includes diffraction cone shrinkage ab initio, $B(s)$ logarithmically increases with the energy , $B(s) \sim \alpha'(0) \ln(s/s_0)$, $\alpha'(0) \neq 0$. Unitarity requires its double logarithmic asymptotic growth, $B(s) \sim \ln^2(s/s_0)$ if the total cross-section saturates the Froissart-Martin bound.
- Interpretation of the origin of $B(s)$ growth when it is a unitarity effect alone. Model-dependent result and it correlates with a form for the input amplitude used under the unitarization. The set of models is wide and includes all the models assuming a factorized s- and t-dependence of the input amplitude: geometrical models operating with the amplitudes in the impact parameter representation.

Unitarization of the factorized input

- We discuss the slope of the diffraction cone

$$B(s) = \frac{d}{dt} \ln \frac{d\sigma}{dt}|_{t=0}. \quad (1)$$

  It is determined by the mean value of the impact parameter squared b^2,

$$\langle b^2 \rangle = \frac{\int_0^\infty b^3 dbf(s,b)}{\int_0^\infty bdbf(s,b)}. \quad (2)$$

- In our qualitative consideration we suppose that the real part of the elastic scattering amplitude is vanishingly small and can be neglected

Reminder of unitarity

S-matrix elastic scattering element

Elastic scattering S-matrix element is related to the amplitude $f(s,b)$ by the relation $S(s,b) = 1 - 2f(s,b)$ $f(s,b)$ – in the region $0 \leq f \leq 1$

- The unitarization schemes can provide an output amplitude f limited by the unitarity itself (U–matrix) or by the black disc limiting value of $1/2$ (eikonal, method of continued unitarity) .
- Mechanisms generating the diffraction cone slope increase with the energy are similar for the different schemes.
- Due to the above similarity, we consider a particular scheme, namely, U–matrix . In the U–matrix approach (in the pure imaginary case) the relation between the scattering amplitude and the input quantity u is simple:

$$f(s,b) = u(s,b)/[1 + u(s,b)], \tag{3}$$

where u is non-negative.

Factorization of the input amplitude

- The geometrical models asssume that $u(s, b)$ has a factorized form

$$u(s, b) = g(s)\omega(b), \tag{4}$$

$g(s) \sim s^\lambda$ at the large values of s, and the power dependence guarantees asymptotic growth of the total cross–section $\sigma_{tot} \sim \ln^2 s$. Then

$$B(s) \sim \ln^2 s. \tag{5}$$

- A convolution of the two matter distributions in transverse plane as it was proposed by Chou and Yang:

$$\omega(b) \sim D_1 \otimes D_2 \equiv \int d\mathbf{b}_1 D_1(\mathbf{b}_1) D_2(\mathbf{b} - \mathbf{b}_1). \tag{6}$$

- The form of $\omega(b)$ consistent with the analyticity is a linear exponent at large values of b

$$\omega(b) \sim \exp(-\mu b). \tag{7}$$

The parameter μ is related to a physics model, it can be assumed that $\mu = 2m_\pi$.

Generation of the energy dependence

Slope parameter of an input is energy-independent

Diffraction cone slope B^0 corresponding to the factorized input amplitudes does not depend on the collision energy. It is determined by the geometrical radii of colliding particles. The geometrical radius of particle is determined by the minimal mass of the exchanged quanta responsible for the scattering.

- The energy dependence of final slope $B(s)$ is generated by the unitarization itself. Physical interpretation based on the analogy with bremsstrahlung- L.D. Soloviev and A.V. Schelkachev.
- Unitarization leads to an energy dependence of $B(s)$ at any value of the collision energy, it is $\sim \ln^2 s$ at the asymptotics. At small energies, where $g(s)$ is small (and linearly increases with $\sqrt{s}$), the energy dependence of $B(s)$ becomes:

$$B(s) \simeq \frac{6}{\mu^2}(1+\frac{3}{16}g(s)). \tag{8}$$

Conclusion

- The factorization of the input amplitude for the fast particles — manifestation of the independence of transverse and longitudinal dynamics in the first approximation. Interrelation is a consequence of the unitarization. The generation of the $B(s)$ energy growth can be treated due to unitarity alone, unitarization transforms energy independent slope into the one increasing like $\ln^2 s$ at $s \to \infty$.
- The most recent experimental data of the TOTEM and ATLAS-ALFA are in favor of the double logarithmic increase of the parameter $B(s)$.
- The unitarization procedure leads also to slowing down the asymptotic increase of the total cross–section.
- If unitarity generates the energy dependence of the diffraction cone slope parameter, one could expect energy independence of $B(s)$ in the case of the off–shell particle scattering. Contrary, the dominance of the Regge mechanism and Pomeron contribution with $\alpha'(0) \neq 0$ assumes ab initio similarity for the on–shell and off-shell scattering processes.