Dense baryon matter with isospin and chiral imbalance in the framework of NJL_4 model at large N_c: duality between chiral symmetry breaking and charged pion condensation

R.N. Zhokhov in collaboration with T.G. Khunjua K.G. Klimenko

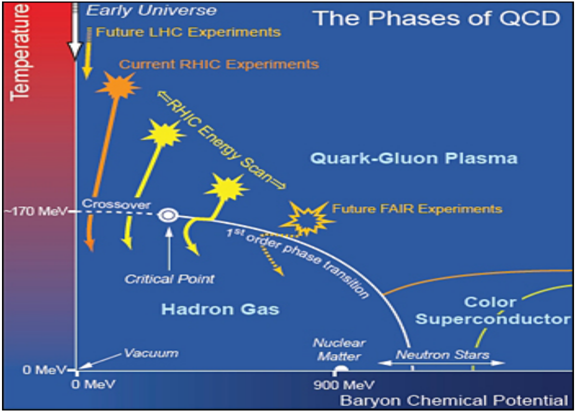
30 октября 2017 г.

QCD at finite temperature and nonzero chemical potential

QCD at nonzero temperature and baryon chemical potential plays a fundamental role in many different physical systems.
QCD at extreme conditions

- neutron stars
- heavy ion collision experiments
- Early Universe

QCD Phase Diagram



Two main phase transition

- confinement-deconfinement
- chiral symmetry breaking phase—chriral symmetric phase

Methods of dealing with QCD

Methods of dealing with QCD

- perturbative QCD, pQCD, high energy
- First principle calcaltion – lattice Monte Carlo simulations, LQCD
- Effective models

  Chiral pertubation theory χPT
  Nambu–Jona-Lasinio model NJL
  Polyakov-loop extended Nambu–Jona-Lasinio model PNJL
  Quark meson model
- $1/N$ expansion (large number of colors) G.t'Hooft.
  the predictions of $\frac{1}{N_c}$ expansions for QCD are mostly of a qualitative nature
- Holographic methods, Gauge/gravity or gauge/string duality AdS/CFT conjecture

Nambu–Jona-Lasinio model

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^\nu \mathrm{i}\partial_\nu q + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\right]$$

$$q \to e^{i\gamma_5\alpha} q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\left[\gamma^\rho \mathrm{i}\partial_\rho - \sigma - \mathrm{i}\gamma^5\pi\right]q - \frac{N_c}{4G}\left[\sigma^2 + \pi^2\right].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

$$\langle \bar{q}q \rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\left[\gamma^\rho \mathrm{i}\partial_\rho - \langle\sigma\rangle\right]q$$

Isotopic and axial isotopic chemical potentials

Isotopic chemical potential

Dense matter with isotopic imbalance in neutron stars, heavy ion collision experiments

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

axial chemical potential

Systems with chiral imbalance have attracted some interest in recent years.
Chiral imbalance is a nonzero difference between densities of left- and right-handed fermions,

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

stems from highly nontrivial interplay of chiral symmetry of QCD, axial anomaly, and the topology of gluon configurations and leads to the chiral magnetic effect.

Axial isotopic chemical potential

Axial isotopic chemical potential

$$\text{Term in the Lagrangian} \quad - \quad \bar{q}\frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 q$$

$$\mu_{I5} = \mu_{uR} - \mu_{uL} + \mu_{dL} - \mu_{dR}$$

So the corresponding density is

$$n_{I5} = n_{uR} - n_{uL} + n_{dL} - n_{dR}$$

Axial isotopic chemical potential leads to the chiral imbalance.

The same conditions in (1+1)- dim Gross-Neveu model

(1+1)-dimensional Gross-Neveu (GN) model possess a lot of common features with QCD

- renormalizability
- asymptotic freedom
- sponteneous chiral symmetry breaking in vacuum
- dimensional transmutation
- have the similar $\mu_B - T$ phase diagrams

Relative simplicity, renormalizability $\rightarrow$ NJL_2 model can be used as a laboratory for the qualitative simulation of specific properties of QCD at arbitrary energies

We showed that chiral isospin chemical potential generates charged pion condensation in dense quark matter

Model and its Lagrangian

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5\Big]q + \qquad (1)$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets; τ_k $(k = 1, 2, 3)$ are Pauli matrices.

Symmetries of Lagrangian

Lagrangian is invariant with respect to the abelian $U_B(1)$, $U_{I_3}(1)$ and $U_{AI_3}(1)$ groups,

$$U_B(1): \;\; q \to \exp(\mathrm{i}\alpha/3)q; \tag{2}$$
$$U_{I_3}(1): \;\; q \to \exp(\mathrm{i}\alpha\tau_3/2)q; \tag{3}$$
$$U_{AI_3}(1): \;\; q \to \exp(\mathrm{i}\alpha\gamma^5\tau_3/2)q. \tag{4}$$

Lagrangian (1) is invariant with respect to the electromagnetic $U_Q(1)$ group,

$$U_Q(1): \;\; q \to \exp(\mathrm{i}Q\alpha)q, \tag{5}$$

where $Q = \mathrm{diag}(2/3, -1/3)$.

Equivalent Lagrangian

To find the thermodynamic potential of the system, we use a semi-bosonized version of the Lagrangian (1), which contains composite bosonic fields $\sigma(x)$ and $\pi_a(x)$ $(a = 1, 2, 3)$

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^1 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q$$

$$-\frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big]. \tag{6}$$

For bosonic fields one has

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q). \tag{7}$$

Condansates ansatzs: homogeneous and inhomogeneous Chiral density wave and pion wave

Chiral density wave and plane pion wave

$$\langle\sigma(x)\rangle = M\cos(2bx),\ \langle\pi_3(x)\rangle = M\sin(2bx),$$

$$\langle\pi_1(x)\rangle = \Delta\cos(2b'x),\ \langle\pi_2(x)\rangle = \Delta\sin(2b'x),$$
$$\langle\pi_+(x)\rangle = \Delta e^{2b'x},\ \langle\pi_-(x)\rangle = \Delta e^{-2b'x},$$

8 where M, b, b' and Δ are constant dynamical quantities.
We will use the following ansat: $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates x,

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0. \tag{8}$$

where M and Δ are already constant quantities.

Condansates ansatz: homogeneous case

We will use the following ansat: $\langle \sigma(x) \rangle$ and $\langle \pi_a(x) \rangle$ do not depend on spacetime coordinates x,

$$\langle \sigma(x) \rangle = M, \quad \langle \pi_1(x) \rangle = \Delta, \quad \langle \pi_2(x) \rangle = 0, \quad \langle \pi_3(x) \rangle = 0. \qquad (9)$$

where M and Δ are already constant quantities.

large N_c-expansion

In the leading order of the large N_c-expansion it is defined by the following expression:

$$\int d^4x\Omega(M,\Delta) = -\frac{1}{N_c}\mathcal{S}_{\rm eff}\{\sigma(x),\pi_a(x)\}\Big|_{\sigma(x)=\langle\sigma(x)\rangle,\pi_a(x)=\langle\pi_a(x)\rangle}, \tag{10}$$

thermodynamic potential

For the thermodynamic potential one can obtain

$$\Omega(M,\Delta) \;=\frac{M^2+\Delta^2}{4G}+\mathrm{i}\int\frac{d^4p}{(2\pi)^4}\ln\overline{D}(p), \qquad 07 \qquad (11)$$

where

$$\overline{D}(p)=\left(\eta^4-2a\eta^2-b\eta+c\right)\left(\eta^4-2a\eta^2+b\eta+c\right)\equiv P_-(p_0)P_+(p_0),$$

where $\eta=p_0+\mu$, $|\vec{p}|=\sqrt{p_1^2+p_2^2+p_3^2}$ and

$$\begin{aligned} a &= M^2+\Delta^2+|\vec{p}|^2+\nu^2+\nu_5^2;\quad b=8|\vec{p}|\nu\nu_5;\\ c &= a^2-4|\vec{p}|^2(\nu^2+\nu_5^2)-4M^2\nu^2-4\Delta^2\nu_5^2-4\nu^2\nu_5^2. \end{aligned} \quad (13)$$

$$\nu=\frac{\mu_I}{2} \qquad \nu_5=\frac{\mu_{I5}}{2}$$

Duality

The thermodynamic potential is invariant with respect to the so-called duality transformation

$$\mathcal{D}: \quad M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5, \tag{14}$$

If we change axes $\nu \longleftrightarrow \nu_5$ then we should exchange phases PC $\longleftrightarrow$ CSB. For projections of thermodynamic potential

$$F_1(M) \equiv \Omega(M, \Delta = 0)$$

$$F_2(\Delta) \equiv \Omega(M = 0, \Delta)$$

$$F_2(\Delta) = F_1(\Delta)\Big|_{\nu \longleftrightarrow \nu_5}.$$

Duality in other situations

- Dualities between chiral and superconducting condensates in (2+1) and (1+1) NJL like models.
- Orbifold equivalences connect gauge theories with different gauge groups and matter content in the large Nc limit. There has been found the dualities akin to ours in the framework of universality principle (large N_c orbifold equivalence) of phase diagrams in QCD and QCD-like theories in the limit of large N_c.

Condansates ansatz: inhomogeneous condensate chiral density wave and pion wave

Chiral density wave and plane pion wave

$$\langle\sigma(x)\rangle = M\cos(2bx),\ \langle\pi_3(x)\rangle = M\sin(2bx),$$

$$\langle\pi_1(x)\rangle = \Delta\cos(2b'x),\ \langle\pi_2(x)\rangle = \Delta\sin(2b'x),$$

$$\langle\pi_+(x)\rangle = \Delta e^{2b'x},\ \langle\pi_-(x)\rangle = \Delta e^{-2b'x},$$

where M, b, b' and Δ are constant dynamical quantities.

Duality in inhomogeneous case

The expression for thermodynamic potential is quite complicated, but one can prove that the thermodynamic potential is invariant with respect to the duality transformation

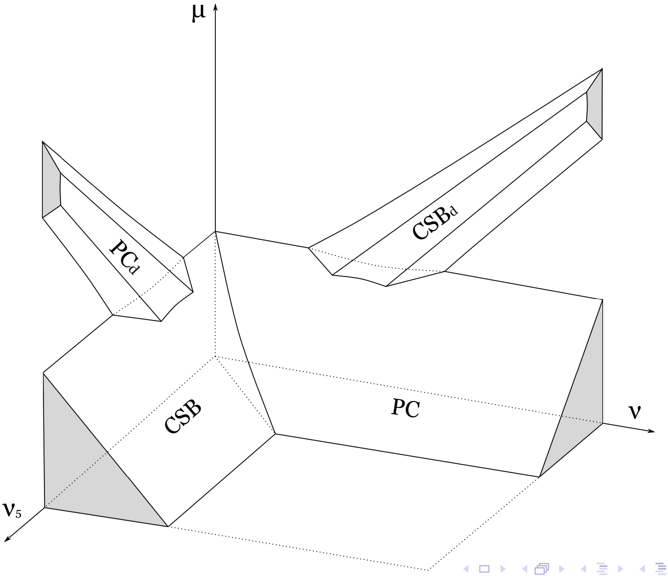
$$\mathcal{D}: \quad M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5, \quad b \longleftrightarrow b'. \tag{15}$$

$$\Omega(M, b, \Delta, b') \mid_{\mathcal{D}} = \Omega(M, b, \Delta, b') \tag{16}$$

If we change axes $\nu \longleftrightarrow \nu_5$ then we should exchange phases PC $\longleftrightarrow$ CSB.

Phase structure of the (1+1) dim NJL model

Phase portrait (μ, ν, ν_5) of NJL_2 in homogeneous case



Phase portrait of NJL_2 in homogeneous case

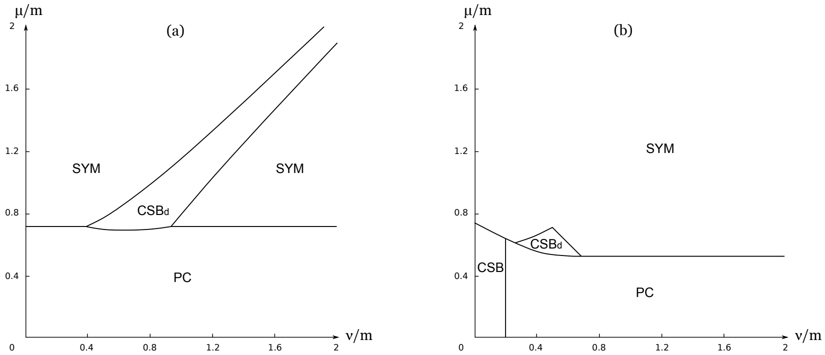


Рис.: The (ν, μ)-phase portrait of the model for different values of the chiral chemical potential ν_5: (a) The case $\nu_5 = 0$. (b) The case $\nu_5 = 0.2m$.

Phase portrait of NJL_2 in homogeneous case

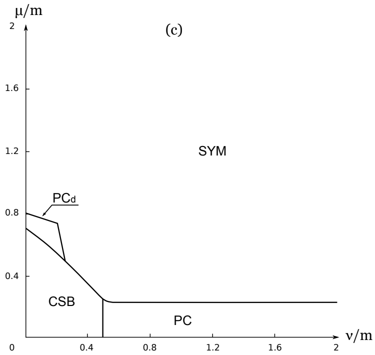
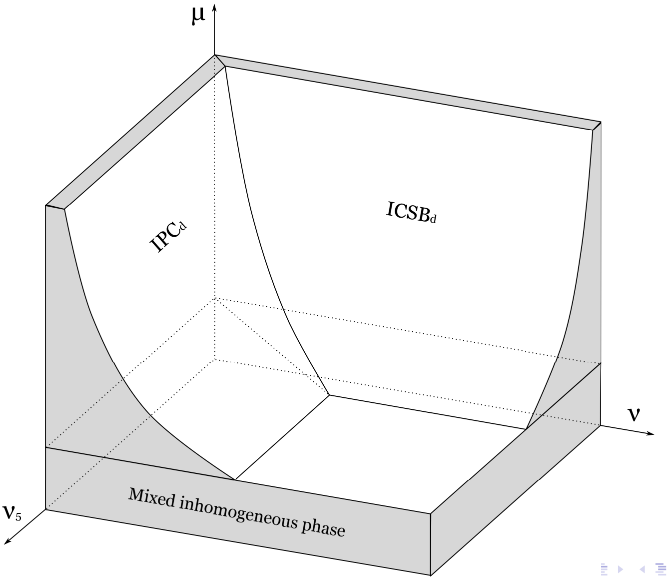


Рис.: The (ν, μ)-phase portrait of the model for different values of the chiral chemical potential ν_5:(a) The case $\nu_5 = 0.5m$. (b) The case $\nu_5 = m$.

Phase portrait (μ, ν, ν_5) in the framework of NJL_2 in inhomogeneous case



Phase structure of the (3+1)-dim NJL model

(ν, ν_5) phase portrait at different μ of NJL_4

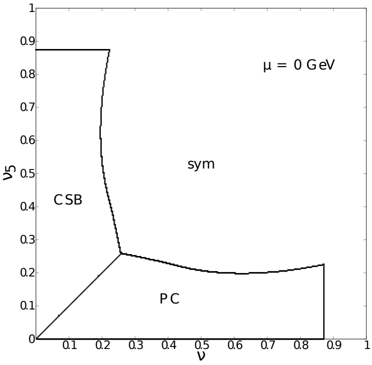


Рис.: (ν, ν_5) phase diagram at $\mu = 0$ GeV

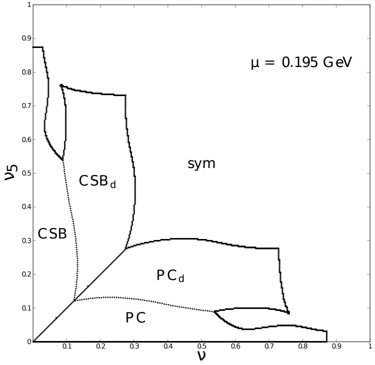


Рис.: (ν, ν_5) phase diagram at $\mu = 0.195$ GeV

comparison with lattice QCD

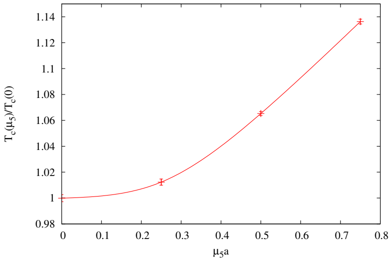


Рис.: critical temperature as a function of μ_5, SU(3) case arXiv:1512.05873 [hep-lat]

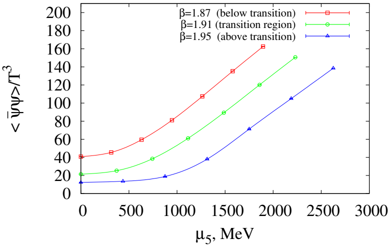


Рис.: chiral condensate as a function of μ_5, SU(2) case arXiv:1503.06670 [hep-lat]

comparison with lattice QCD

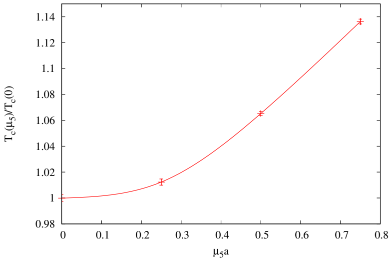


Рис.: critical temperature as a function of μ_5, SU(3) case arXiv:1512.05873 [hep-lat]

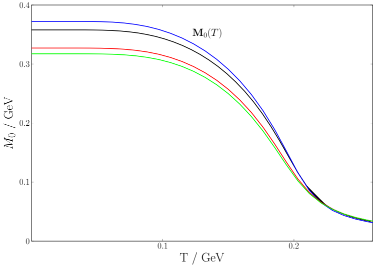


Рис.: chiral condensate as a function of μ_5, SU(2) case arXiv:1503.06670 [hep-lat]

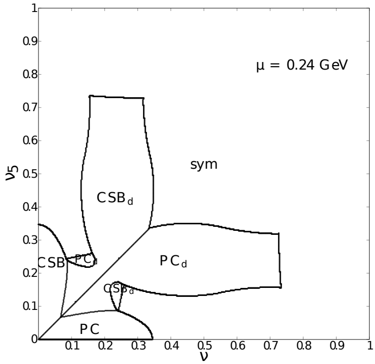


Рис.: (ν, ν_5) phase diagram at $\mu = 0.24$ GeV

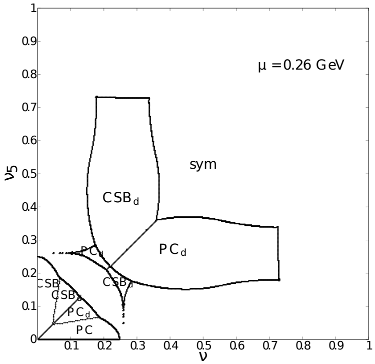


Рис.: (ν, ν_5) phase diagram at $\mu = 0.26$ GeV

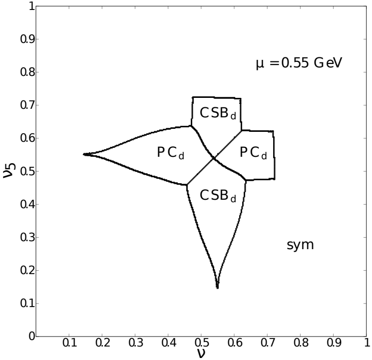


Рис.: (ν, ν_5) phase diagram at $\mu = 0.55$ GeV

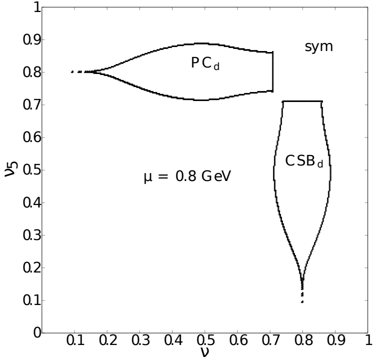


Рис.: (ν, ν_5) phase diagram at $\mu = 0.8$ GeV

(μ, ν) phase portrait at different ν_5 of NJL_4

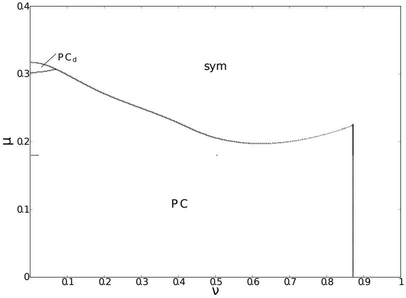


Рис.: (μ, ν) phase diagram at $\nu_5 = 0$ GeV

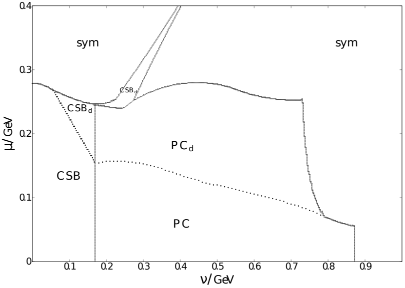
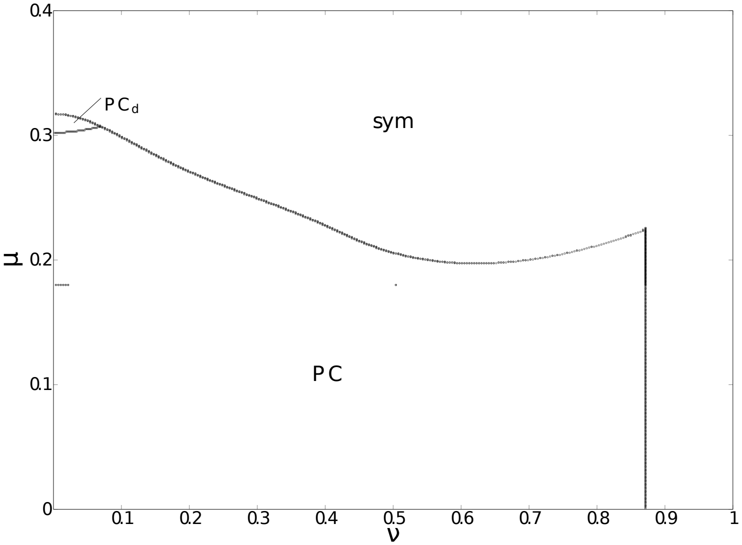
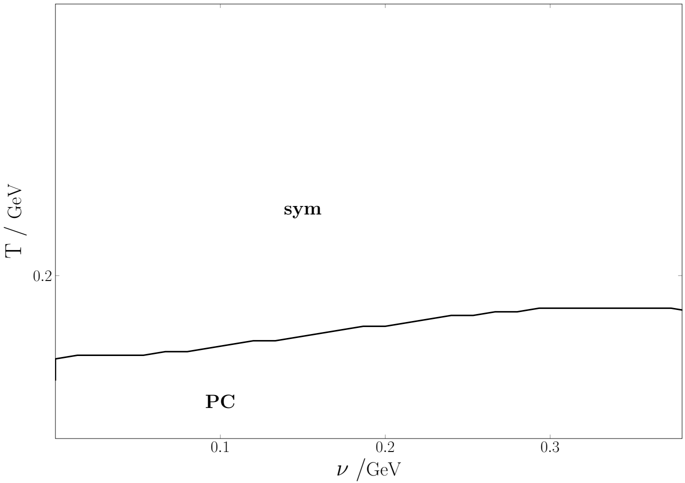


Рис.: (μ, ν) phase diagram at $\nu_5 = 0.195$ GeV

(μ, ν) phase portrait at $\nu_5 = 0$ GeV of NJL_4



(ν, T) phase portrait at $\nu_5 = 0$ GeV of NJL_4



(ν, T) phase portrait comparison with lattice QCD

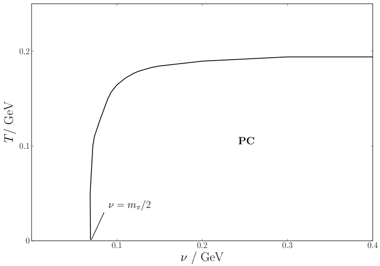


Рис.: (ν, T) phase diagram at $\mu = 0$ and $\nu_5 = 0$ GeV

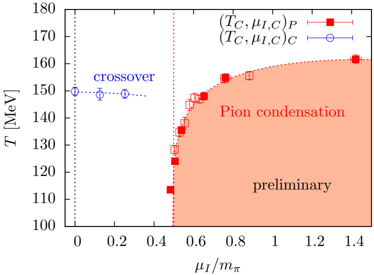


Рис.: (ν, T) phase diagram arXiv:1611.06758 [hep-lat]

(ν, T) phase portrait comparison with lattice QCD

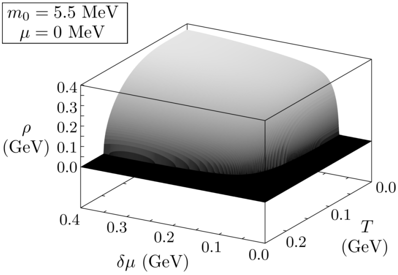


Рис.: (ν, T) phase diagram at $\nu_5 = 0$ GeV from J. Phys. G: Nucl. Part. Phys. 37 015003 (2010)

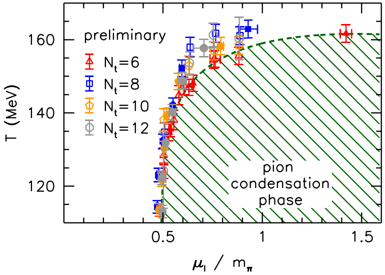
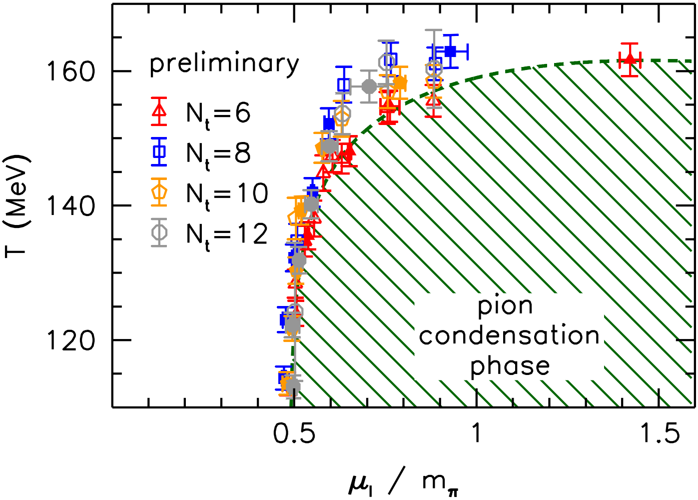


Рис.: (ν, T) phase diagram at $\nu_5 = 0$ GeV arXiv:1611.06758 [hep-lat]

(ν, T) phase portrait from arXiv:1611.06758 [hep-lat]



(μ, ν) and (μ, ν_5) phase portraits, use of duality

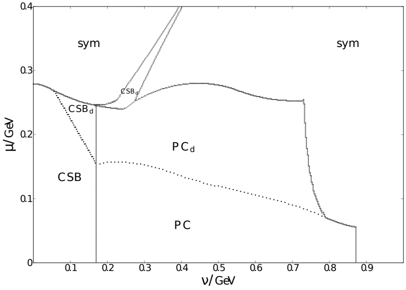


Рис.: (μ, ν) phase diagram at $\nu_5 = 0.17$ GeV

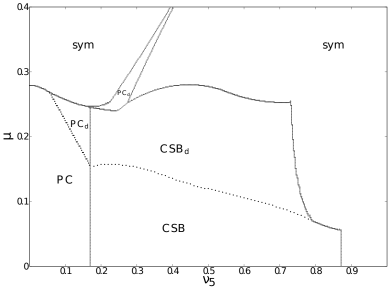
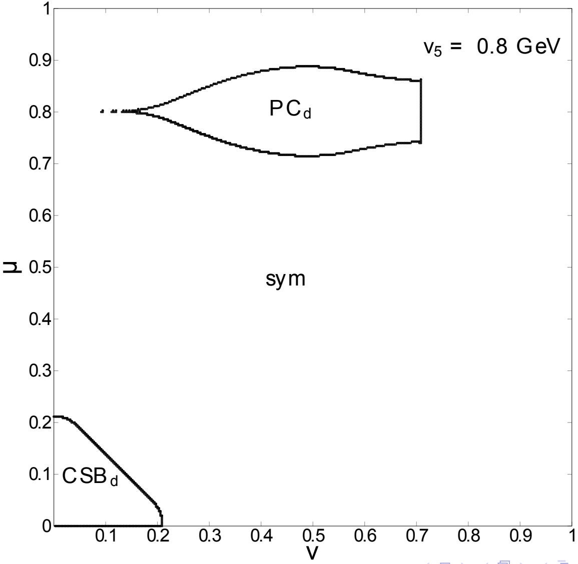


Рис.: (μ, ν_5) phase diagram at $\nu = 0.17$ GeV

(μ, ν) phase portrait at $\nu_5 = 0.8\ GeV$ of NJL_4



no chiral limit

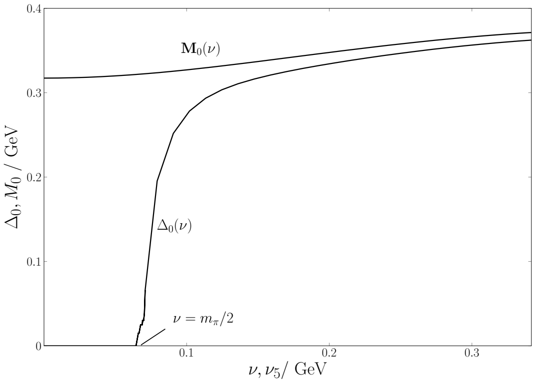


Рис.: M_0 and $\Delta_0$as a function of ν_5 and ν at $\nu = 0$ and $\nu_5 = 0$ respectively

Comparison of phase diagram of (3+1)-dim and (1+1)-dim NJL models

(μ,ν) phase portraits comparison, NJL_2 and NJL_4

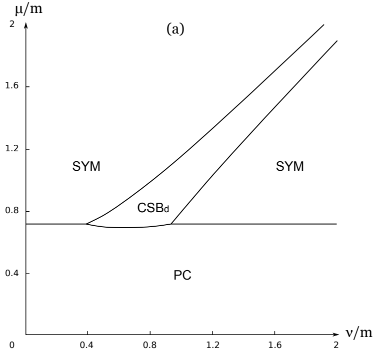


Рис.: (μ,ν) phase diagram in the framework of NJL_2 model at $\nu_5 = 0$ GeV

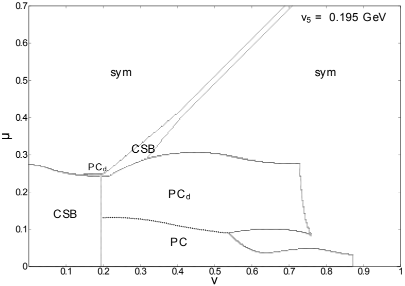


Рис.: (μ,ν) phase diagram in the framework of NJL_4 model at $\nu_5 = 0.195$ GeV

(μ, ν) phase portraits comparison, NJL_2 and NJL_4

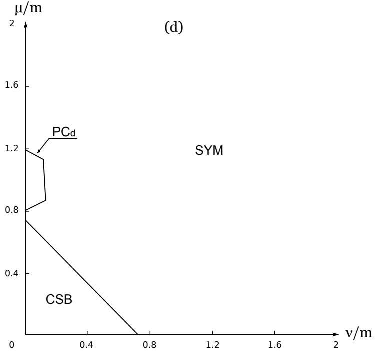


Рис.: (μ, ν) phase diagram in the framework of NJL_2 model at $\nu_5 = 0$ GeV

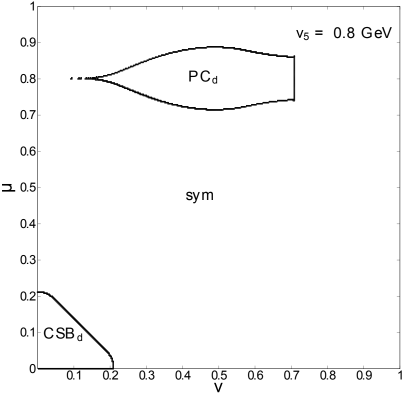


Рис.: (μ, ν) phase diagram in the framework of NJL_4 model at $\nu_5 = 0.15$ GeV

Conclusions

We studied chirally ($\mu_{I5} \neq 0$) and isotopically ($\mu_I \neq 0$) asymmetric dense ($\mu_B \neq 0$) quark matter in the framework of (3+1)-dim NJL model.

- Chiral isospin chemical potential generates charged pion condensation in (3+1)-dim NJL model. So generations of charged pion condensation due to chiral isospin chemical potential is predicted in two models (4D NJL and NJL_2) and might be the property of real QCD.
- It has been also demonstrated that in the framework of the (3+1)-dim NJL model duality correspondence between CSB and charged PC phenomena takes place in the leading order of the large-Nc approximation as in NJL_2 model.
- In contrast to NJL_2 model results in 4D NJL generation of PC_d requires not very large but nonzero isospin chemical potential. In order to generate PC_d phase one needs to have both nonzero isospin μ_I and chiral isospin μ_{I5} chemical potentials.