PRELIMINARY

Affine-Goldstone/quartet-metric gravity: emergent vs. existent

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Abstract

As a group-theoretic foundation of gravity, in an arbitrary world manifold there is consistently constructed a nonlinear model based on the nonlinear realization of the global affine symmetry spontaneously broken to the Poincare one at the Planck scale, with the emergent pseudo-Goldstone boson corresponding to the tensor graviton and the extra gravity components treated as the candidates on the dark matter and dark energy. The model is shown to incorporate an earlier introduced effective field theory of the quartet-metric gravity below the Planck scale and is argued to pave the way towards an underlying theory beyond the Planck scale, with the emergent gravity and spacetime.

Keywords: Modified Gravity, Nonlinear Models, Nonlinear Realizations, Spontaneous Symmetry Breaking, Goldstone Boson, Dark Matter, Dark Energy, Planck scale.

1 Introduction: GR and beyond

General Relativity (GR) is the well-stated contemporary basis of gravity remaining up-to-date in a position to successfully cope with the bulk of the astrophysical an cosmological *manifestations* of gravity. At that, GR (as well as its direct siblings) is well-known to be based upon the *pseudo-Riemannian/metric* paradigm of gravity which is conventionally assumed to be an attribute of gravity. Nevertheless, an underlying *substance* of gravity seems to be still obscured. In this vein, already long ago there was put forward [1, 2] an alternative, *group-theoretic/Nambu-Goldstone (NG)* paradigm wherein gravity has an NG origin being based upon the spontaneous breaking at the Planck scale of the global affine symmetry to the Poincare one, with the emergent NG boson treated as graviton. The latter paradigm proves to be nicely fitted to gravity naturally incorporating the generic signatures of GR: the symmetric second-rank tensor field possessing the self-interactions through a derivative in the ratio to the Planck mass treated as a scale of the spontaneous symmetry breaking (SSB). By means of the group-theoretic techniques of the nonlinear realizations (NRs) and nonlinear models (NMs) for SSB [3]–[6] this allows to ultimately reproduce GR as the metric theory of gravity. Aimed originally merely at reproducing GR such an approach did not, unfortunately, get a proper subsequent development from the theoretical considerations as an alternative to the pseudo-Riemannian/metric GR.

On the other hand, the recent advent of the elusive dark components (dark matter (DM) and dark energy (DE)) in the Universe,[1] the nature of which is completely obscure, causes some (still mainly theoretical) tension within GR. The predominant abundance of such the *ad hock* dark components, though quite legitimate in the GR framework, may be a hint from the Nature at a necessity of going beyond GR, with the elusive dark components being nothing but an integral part of the modified gravity. With such an aim in mind, there was recently proposed as a modification to the metric GR the effective field theory (EFT) of the so-called quartet-metric (QM) GR (for short, QM gravity) [8, 9]. The latter is based originally upon the two physical concepts. First, GR undergoes the Higgs-mode SSB, with the role of the Higgs-like fields for gravity played by the distinct dynamical coordinates given by a scalar quartet. Second, the physical extra gravity fields arising from metric due to SSB serve as the gravitational dark components of the Universe. It was argued that the so constructed EFT of gravity may give rise to a large variety of manifestations beyond GR, to consistently study which remains still a challenge. At that, by the very construction the conventional EFT frameworks inevitably contain a number of the *ad hock* assumptions and parameters implying, conceivably, a vital necessity of further elaborating such a theory, as well as its foundations.

To this end, in the present paper we merge the two above-mentioned routes of the GR modification, so-to-say "in-depth" and "in-width", by consistently constructing an affine-Goldstone NM for GR and beyond. In Sec. 2, we start by presenting NRs for the global affine symmetry. All the considerations here and in what follows are internationally presented in a nutshell adopted to the case at hand. Then, we build the proper NM in an auxiliary affine space, a progenitor of the Minkowski spacetime. In Sec. 3, the constructions are proliferated to an arbitrary manifold, a progenitor of the (pseudo-)Riemannian spacetime. It is shown that NM so constructed supersedes the previously introduced EFT of the QM gravity, with an elaboration and clarification of the latter. For illustration, there are exposed the two extreme cases of such the QM gravity in more detail. In Conclusion, there are summarized the advantages of presenting gravity below the Planck scale as NM in the group-theoretic/NG paradigm vs. EFT in the pseudo-Riemannian/metric paradigm. Finally, we point out a conceivable prospect of going beyond the NM frameworks above the Planck scale towards an underlying theory, with the emergent gravity and spacetime.[2] For completeness, in Appendix there are exposed the group-theoretic generalities concerning SSB, NRs and NMs.

2 Affine realization space

2.1 Affine-to-Poincare SSB

To start, let $R^4 = \{q^a\}$, be an *affine space*, with the coordinates q^a marked by the indices $a, b, \ldots = 0, \ldots, 3$, acted upon by the global affine transformations $(A, \Delta) \in IGL(4, R) \equiv Aff(R^4)$ as $q \to Aq + \Delta$, or in the full notation

$$(A, \Delta) \;:\; q^a \to A^a{}_b q^b + \Delta^a. \tag{1}$$

[1] See, e.g., [7].

[2] For the earlier attempts at applying the group-theoretic/NG approach to modify GR and go beyond the Planck scale, with the emergent gravity and spacetime, cf., [10, 11].

At that, the index 0 is *a priori* nothing but a notation. The NM under construction is based upon the symmetry breaking pattern

$$G \equiv GL(4,R) \to H \equiv SO(1,3), \tag{2}$$

with $SO(1,3) \subset GL(4,R)$ the residual (global) Lorentz subgroup.[3,4] Just after a particular embedding of the unbroken subgroup $SO(1,3)$ into the broken $GL(4,R)$ the affine space gets converted into the Minkowski spacetime, $R^4 \to R_{(1,3)}$, with a foliation onto the time and space: $q^a = (q^0, q^i)$, $i = 1,2,3$, with the affine index 0 assumed to be chosen so as to acquire its conventional time meaning. The respective NM $G/H = GL(4,R)/SO(1,3)$ describes the SSB $G \to H$ in the NG mode, with the appearance of the $d_{G/H} = d_G - d_H = 10$ (pseudo-)Nambu-Goldstone (pNG) bosons associated ultimately with gravitation and the gravitational dark components.

2.2 Quasi-symmetric NR

The construction of the proper NM, call it the *affine-Goldstone* one, fallows directly to the general procedure of NRs (see, Appendix). What is of specifics in the case at hand is that here the (left) coset) representative $\tilde\vartheta$, associated ultimately with the pNG boson, may be parametrized as a 4×4 (local) matrix $\tilde\vartheta^a_{(b)}(q)$ transforming under $A \in GL(4,R)$ as $\tilde\vartheta \to A\tilde\vartheta\Lambda^{-1}(A,\tilde\vartheta)$. or in the full notation

$$A \;:\; \tilde\vartheta^a_{(b)}(q) \to \tilde\vartheta'^a_{(b)}(Aq) = A^a{}_c \tilde\vartheta^c_{(d)}(q) \Lambda^{-1d}{}_b(A, \tilde\vartheta(q)). \tag{3}$$

At that, $\Lambda = \Lambda(A,\tilde\vartheta) \in SO(1,3)$ is to be properly defined to specify a particular NR. In what is shown above and what follows, an affine index in the parentheses indicates that it undergoes transformations just under the residual Lorentz subgroup $SO(1,3)$. Only such indices are allowed to be raised and lowered by meas of the Lorentz-invariant Minkowski symbol η^{ab} (and, respectively, η_{ab}). For the genially affine indices such a procedure is not allowed without the explicit violation of the affine symmetry. With $\tilde\vartheta$ being a group element it has an inverse $\tilde\vartheta^{(a)}_b$ transforming under $A \in GL(4,R)$, in short, as $\tilde\vartheta^{-1} \to \Lambda(A,\tilde\vartheta)\tilde\vartheta^{-1}A^{-1}$.[5]

An arbitrary matrix $\vartheta \in GL(4,R)$ contain *a priori* sixteen components. To fix a coset representative $\tilde\vartheta$ there should be leaved in the matrix just ten independent components, equal to the difference of the dimensions of the affine and Lorentz groups, through imposing six auxiliary conditions. In the case at hand the most natural choice for such a condition is the *quasi-symmetric* one:

$$\tilde\vartheta^a_{(c)}\eta^{cb} = \tilde\vartheta^b_{(c)}\eta^{ca}, \tag{4}$$

or, for short, $\tilde\vartheta\eta = (\tilde\vartheta\eta)^T = \eta\tilde\vartheta^T$, eliminating the (quasi)-anti-symmetric part otherwise present in an arbitrary ϑ. With account for the pseudo-orthogonality property of $\Lambda \in SO(1,3)$:

$$\eta^{ac}\Lambda^d{}_c\eta_{db} = \Lambda^{-1a}{}_b, \tag{5}$$

[3] In fact, we consider the full affine and inhomogeneous Lorentz (Poincare) groups. But remaining unbroken, the inhomogeneous parts of the groups are explicitly omitted.

[4] Note in passing that all the fallowing is technically insensitive to the dimension of spacetime $d \geq 2$, as well as to its signature (p,q), $p+q=d$.

[5] For simplicity, we will use symbol ϑ^{-1} only in the short-hand notation.

or, for short, $\eta\Lambda^T\eta = \Lambda^{-1}$, to preserve the imposed condition under A there should fulfill in short

$$A\tilde{\vartheta}\eta\Lambda^T = \Lambda\tilde{\vartheta}\eta A^T. \tag{6}$$

At $A = \Lambda_0$, with an arbitrary global $\Lambda_0 \in SO(1,3)$, one clearly gets $\Lambda = \Lambda_0$ implying, in particular $\Lambda = I$ at $A = I$. A general solution to (6) may be looked for by perturbations, uniquely at least in a vicinity of unity. The resulting $\Lambda = \Lambda(A, \tilde{\vartheta})$ defines the particular NR, which reduces to the conventional linear (adjoint) representation on the unbroken Lorentz subgroup;

$$\Lambda_0 \ : \ \tilde{\vartheta}(y) \to \Lambda_0\tilde{\vartheta}(y)\Lambda_0^{-1}, \tag{7}$$

or with account for the pseudo-orthogonality of Λ_0 as the symmetric Lorentz tensor:

$$\Lambda_0 \ : \tilde{\vartheta}\eta \to \Lambda_0\tilde{\vartheta}\eta\Lambda_0^T. \tag{8}$$

At last, decomposing $\tilde{\vartheta}$ in the weak-field limit as

$$\tilde{\vartheta}^a_{(c)}\eta^{cb} \simeq \eta^{ab} + \chi^{ab}, \quad \chi^{ab} = \chi^{ba}, \tag{9}$$

preserved by the Lorentz subgroup, one may interpret χ as an affine-tensor NG boson for SSB at hand. Imposing other auxiliary conditions, one can similarly get other particular NRs. In accord with the general theory of NRs, all the latter ones are to be equivalent. To deal directly with a particular restricted NR is quite cumbersome. For this reason, we consider below a more general (yet simpler) NR to which any particular one is explicitly equivalent.

2.3 Local-Lorentz linearized NR

Instead of a (left) coset representative $\tilde{\vartheta}(y)$, let us choose as a field variable for SSB $G \to H$ the (left) coset itself, i.e., the whole subset $\{\vartheta(y)\}$ of the elements of G equivalent up to the (right) multiplication by H and containing $\tilde{\vartheta}$ as a representative: $\{\vartheta = \tilde{\vartheta}h\}$, $h \in H$. To this end, define for the case at hand as a new field variable the 4×4 local matrix $\vartheta^a_\alpha(y)$ (with an inverse ϑ^α_a) transforming under $A \in GL(4,R)$ up to an arbitrary $\Lambda(y) \in SO(1,3)_{\rm loc}$ as

$$(A, \Lambda(y)) \ : \ \vartheta^a_\alpha \to A^a{}_b\vartheta^b_\beta\Lambda^{-1\beta}{}_\alpha(y), \tag{10}$$

or in short $\vartheta \to A\vartheta\Lambda^{-1}(y)$ (respectively, $\vartheta^{-1} \to \Lambda(y)\vartheta^{-1}A^{-1}$). In what is shown above, $SO(1,3)_{\rm loc}$ is an auxiliary local Lorentz group (not a subgroup of $GL(4,R)$ as before). At that, though ϑ^a_α contains formally sixteen components, six of them can be eliminated by means of the auxiliary transformations $\Lambda(y) \in SO(1,3)_{\rm loc}$ leaving as required precisely ten independent components. The left-out components may ultimately be associated with the NG boson corresponding to SSB $GL(4,R) \to SO(1,3)$. Namely, fixing properly a gauge for $SO(1,3)_{\rm loc}$ we can get any particular NR. Thus, transforming $\vartheta \to \tilde{\vartheta} = \vartheta\tilde{\Lambda}^{-1}$, with $\tilde{\Lambda} \in SO(1,3)_{\rm loc}$ satisfying with account for the pseudo-symmetricity of $\tilde{\Lambda}$ to the relation

$$\vartheta^a_\alpha\eta^{\alpha\gamma}\tilde{\Lambda}^\beta_\gamma|_{\beta=b} = \vartheta^b_\beta\eta^{\beta\gamma}\tilde{\Lambda}^\alpha{}_\gamma|_{\alpha=a}, \tag{11}$$

or in short $\vartheta\eta\tilde{\Lambda}^T = (\vartheta\eta\tilde{\Lambda}^T)^T = \tilde{\Lambda}\eta\vartheta^T$, we can achieve that ϑ gets quasi-symmetric, $\tilde{\vartheta}\eta = (\tilde{\vartheta}\eta)^T = \eta\tilde{\vartheta}^T$, recovering thus the quasi-symmetric NR. In the same vein, we can get any other particular NR, all of them (in accord with the general NR theory) being equivalent to each other and to the local-Lorentz one. Thus, in what follows we will use the latter NR, with the symmetry group in the factorized affine-Lorentz form:

$$G \times H_{\rm loc} = GL(4,R) \times SO(1,3)_{\rm loc}. \tag{12}$$

Such a linearized NR containing the *hidden* local symmetry for the SSB pattern (2), may be treated as an essence of gravity, succinctly encoding the bulk of its structure up to the modification caused by the world manifold (see, Sec. 3).[6,7] According to (12), there can be envisaged the three generic types of the (finite dimensional) affine-Lorentz fields: the affine protometric, the affine tensors and Lorentz spinors, to be specified below.

The QM GR, like the metric GR itself and its direct siblings, is originally a theory of gravity based on the *metric paradigm* in the *EFT frameworks*. Now we are going to construct a more advanced theory of gravity (encompassing QM GR) based on the *NG paradigm* in the *NM frameworks*. With the internal and external spaces in the case of the affine symmetry coinciding we construct the proper NM for consistency in the two steps. First, we construct the model in an auxiliary affine space (a progenitor of the Lorentz spacetime) and proliferate it then to an affine manifold and, as a result, to the physical pseudo-Riemannian spacetime.

2.4 Affine-Lorentz fields

Affine protometric To describe the dynamics of the affine NG boson ϑ consider in accord with the general prescription the (slightly modified) so-called Maurer-Cartan form as follows:

$$\Omega_c^{\alpha\beta} = \vartheta^\alpha_d \partial_c \vartheta^d_\gamma \eta^{\gamma\beta}, \tag{13}$$

or in short $\Omega_a = \vartheta^{-1}\partial_a\vartheta\eta$. According to (10), Ω_a is an affine vector under $GL(4,R)$ and, with account for the pseudo-orthogonality of Λ given by Eq. (5), transforms under $SO(1,3)_{\rm loc}$ inhomogeneously as

$$(A, \Lambda(y)) \ : \ \Omega_c \to A^{-1b}{}_c\Big(\Lambda\Omega_b\Lambda^T + \Lambda\partial_b\eta\Lambda^T\Big). \tag{14}$$

Decomposing Ω_c onto the symmetric and anti-symmetric parts

$$\Omega_c^{\pm\alpha\beta} \equiv \frac{1}{2}(\Omega_c^{\alpha\beta} \pm \Omega_c^{\beta\alpha}), \tag{15}$$

or in short $\Omega_c^\pm \equiv (\Omega_c \pm \Omega_c^T)/2$, we can see from (14) that the symmetric part transforms homogeneously under $\Lambda(y)$, while the anti-symmetric one transforms inhomogeneously, with such a decomposition onto symmetric and anti-symmetric parts being specific to the case at hand. Thus we can use these parts separately, respectively, as a Lorentz tensor and a Lorentz connection for describing the fermion fields (see, later on). On the other hand, being a tensor the Lorentz-symmetric part of $\Omega_a^{+\alpha\beta}$ can be used to describe the NG boson ϑ by its own through constricting their local-Lorentz invariant combinations.

Instead, due to a freedom of choosing the field variables in NM we can start directly from the local-Lorentz invariant combinations

$$\theta^{ab} \equiv \vartheta^a_\alpha \eta^{\alpha\beta}\vartheta^b_\beta, \quad \theta^{ab} = \theta^{ba}, \tag{16}$$

[6]The auxiliary local non-compact group $SO(1,3)_{\rm loc}$, in distinction with a putative gauge one, is not equipped with the physical gauge fields, so that in the case at hand there appears no *prior* problems with unitarity.

[7]For the gravity Goldstone field in a Lorentz gauge theory, cf., [12].

or in short $\theta = \vartheta\eta\vartheta^T$, $\theta = \theta^T$, with the inverse[8]

$$\theta_{ab} \equiv \vartheta^\alpha_a \eta_{\alpha\beta} \vartheta^\beta_b, \quad \theta_{ab} = \theta_{ba}, \tag{17}$$

or in short $\theta^{-1} = \vartheta^{-1T}\eta\vartheta^{-1}$, $\theta^{-1} = \theta^{-1T}$. Due to the freedom of changing in NM the field variables, instead of the affine-Lorentz mixed representation ϑ^a_α, corresponding to the affine NG boson, consider as a new field variable such a genuinely-affine tensor θ^{ab} (or its inverse θ_{ab}) corresponding to the point-like correlated symmetric pair of the NG bosons. Being symmetric, $\theta = \theta^T$, this tensor automatically contains the required number, ten, of the independent components irrespective of the particular gauge for $\Lambda(y)$. Call θ^{ab} the *affine protometric*. Under $\Lambda(y) \in SO(1,3)_{\rm loc}$ the protometric is the Lorentz scalar, while under $A \in GL(4,R)$ transforms, in short, as

$$(A, \Lambda(y)) \ : \ \theta \to A\theta A^T, \tag{18}$$

with the symmetry of θ automatically preserved. As a result, the NR for this field becomes the conventional linear representation of the whole affine group (but not only of its unbroken subgroup as would be minimally guaranteed in a general case of NR). At last, instead of the derivative of θ^{ab} we can use the *affine tensor* Γ^c_{ab} defined as[9]

$$\Gamma^c_{ab} \equiv \frac{1}{2}\theta^{cd}(\partial_a\theta_{bd} + \partial_b\theta_{ad} - \partial_d\theta_{ab}), \tag{19}$$

so that

$$\partial_c\theta_{ab} = \theta_{ad}\Gamma^d_{bc} + \theta_{bd}\Gamma^d_{ac}. \tag{20}$$

In the weak-field limit we have

$$\theta^{ab} \simeq \eta^{ab} + 2\chi^{ab}, \tag{21}$$

with the NG boson χ^{ab} from (9). The correlated pair of the NG boson, $\vartheta\eta\vartheta^T$, resulting ultimately in the Riemannian metric may be called the affine *protometic*.[10] Such a field proves to be self-sufficient, while itself being inevitably omnipresent presenting the environment for consistent description of the other fields to be considered below.

Affine tensor fields Consider now an arbitrary affine-tensor field $\Phi^{a,\dots}_{b,\dots}(y)$. The protomtric θ^{ab} and θ_{ab} may serve as a counterpart of the metric appearing in the affine space after SSB. This allows, respectively, to raise and lower the affine-tensor indices, converting the otherwise nonequivalent affine tensor into the equivalent ones. On the other hand, the affine tensor Γ^c_{ab} allows to define a counterpart of the covariant derivative ∇_a allowing to disentangle the differentiation and the operations of rising/lowering the affine indices. To this end, define for some vector fields V_a and U^a such a covariant derivative ∇_a as follows:

$$\begin{aligned} \nabla_a U^c &= \partial_a U^c + \Gamma^c_{ad} U^d, \\ \nabla_a V_b &= \partial_a V_b - \Gamma^d_{ab} V_d, \end{aligned} \tag{22}$$

[8] Not to mix with the affine-violating combination $\eta_{ac}\eta_{bd}\theta^{cd} \neq \theta_{ab}$ or in short $\eta\theta\eta \neq \theta^{-1}$.

[9] Note that the symmetric genially-affine tensor $\Gamma^c_{ab} = \Gamma^c_{ba}$, like $\partial_c\theta_{ab}$, contains the same number, forty, of components as the symmetric affine-Lorentz tensor $\Omega^{+\alpha\beta}_a$.

[10] At that, one more local-Lorentz singlet NG boson correlation $\vartheta^a_\alpha\vartheta^\alpha_b = \delta^a_b$ proves to be ϑ-independent, with all ten independent components of ϑ presented precisely by $\theta = \vartheta\eta\vartheta^T$.

so that $U^c V_c$ is a scalar, with $\nabla_a(U^c V_c) = \partial_a(U^c V_c)$. This allows to proliferate further the action of ∇_a on an arbitrary affine tensor $\Phi^{c_1,\cdots}_{b_1,\cdots}$ through the independent action on each of the indices as shown above. It follows hereof, in particular, that

$$\nabla_a\theta^{bc} = \nabla_a\theta_{bc} = 0, \tag{23}$$

so that $\nabla_a V^b \equiv \nabla_a\theta^{bc}V_c = \theta^{bc}\nabla_a V_c$, etc. Likewise, we can deal with arbitrary affine-tensor fields and their combinations.[11,12]

Lorentz spinor fields While to account for the protometric , $\theta = \vartheta\eta\vartheta^T$, as well as the tensor matter and continuous media, it suffices to consider transformations only under $A \in GL(4,R)$, to account for the fermion matter it is required a different type of transformations. Namely, to this end it is necessary to consider the affine NG boson ϑ itself (in the line with ϑ^{-1}), which is in fact the basic gravity field, accounting thus for transformations under $\Lambda(y) \in SO(1,3)_{\rm loc}$. So, let $\rho_(\Lambda)$ be a finite-dimensional linear representation of the Lorentz group, with a generic fermion field Ψ transforming as[13]

$$\Lambda(y)\ ;\ \Psi \to \rho(\Lambda)\Psi. \tag{24}$$

Now, the Lorentz anti-symmetric part $\Omega^{-\alpha\beta}_a$, to be called the *spin-connection*, allows to introduce a covariant under $SO(1,3)_{\rm loc}$ derivative, generically, as follows:

$$\nabla_a\Psi = \partial_a\Psi + \Omega^{-\alpha\beta}_a\rho(T_{\alpha\beta})\Psi, \tag{25}$$

with $T_{\alpha\beta} = -T_{\beta\alpha}$ being the generators of the Lorentz group, and $\Omega^{-\alpha\beta}_a$ playing the role of the auxiliary (composed) Lorentz gauge fields. It follows, that $\nabla_a\Psi$ transforms homogeneously like Ψ itself:

$$\Lambda(y)\ ;\ \nabla_a\Psi \to \nabla_a\Big(\rho(\Lambda)\Psi\Big) = \rho(\Lambda)\nabla_a\Psi. \tag{26}$$

E.g., for the Dirac bi-spinors ψ let us define conventionally the Lorentz generators as $T_{\alpha\beta} = \sigma_{\alpha\beta} \equiv [\gamma_\alpha, \gamma_\beta]/2$, with the Lorentz γ-matrices defined by

$$\{\gamma^\alpha, \gamma^\beta\} = 2\eta^{\alpha\beta}, \tag{27}$$

At that, the affine γ-matrices, $\gamma^a \equiv \vartheta^a_\alpha\gamma^\alpha$, to be used in constructing the affine-invariant combinations, fulfills in turn the relation

$$\{\gamma^a, \gamma^b\} = 2\theta^{ab}. \tag{28}$$

Likewise, we can define the affine matrix $\sigma_{ab} \equiv \vartheta^\alpha_a\vartheta^\beta_b\sigma_{\alpha\beta}$, etc. Such matrices are to be used in constructing the Lorentz-invariant bi-linear combinations transforming only under the affine group, say, $\bar\psi\gamma_a\psi$ as the affine vector, etc. More generally, we can consider the mixed affine-Lorentz spin-tensors $\chi^{a,\cdots}$ transforming simultaneously both under $GL(4,R)$ and $SO(1,3)_{\rm loc}$.

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[11]Note that a symmetric affine tensor T^{ab}, formally similar to θ^{ab}, behaves quite differently from the latter, so that generally $\nabla_c T^{ab} \neq 0$.

[12]Up to now, we considered only the affine coordinates, but the introduced covariant derivatives allow moreover to extend the former to the arbitrary curvilinear coordinates in the affine space.

[13]Note in passing that while the matter tensors may have sense also at the level of the unbroken $GL(4,R)$, the *finite-dimensional* matter spinors may appear only *after* the SSB $GL(4,R) \to SO(1,3)$.

2.5 Affine protogravity

Now we are equipped enough to construct the *affine-Lorentz* NM $GL(4,R)/SO(1,3)$. First, we do this in the affine auxiliary space and then proliferate the construction onto the physical pseudo-Riemannian spacetime. We restrict the subsequent consideration by the pure NG part, with the inclusion of the tensor and spinor matter being straightforward by means of the techniques presented above. Without explicit violation of the affine symmetry the pure affine-NG action may depend only on θ^{ab} (as well as its inverse) and its derivatives. Allowing for the explicit affine symmetry violation up to the Lorentz one we can also use the Minkowski symbol η_{ab} (or η^{ab}). Instead of the latter we can consider the (Lorentz) tensor

$$\Theta^a{}_b \equiv \theta^{ac}\eta_{cb}, \tag{29}$$

appearance of which implies an explicit violation of the affine symmetry, still with the residual Lorentz one. Thus, the most general protometric affine scalar Lagrangian may be chosen as $L_G = L_G(\partial_c\theta^{ab}, \theta^{ab}, \Theta^a{}_b)$. With the derivatives $\partial_c\theta_{ab}$ expressed through Γ^c_{ab} given by (40), the Lagrangian may generically be partited onto the kinetic and potential contributions as follows:

$$L_G = \frac{1}{2}\kappa_0^2 K(\Gamma^c_{ab}, \theta^{ab}, \Theta^a{}_b) - V(\Theta^a{}_b), \tag{30}$$

where κ_0 of the dimension of mass designates an SSB scale. The most general kinetic term in the second-derivative order is as follows:

$$K = \sum_{p=1}^{5} \varepsilon_p(\Theta) K_p, \tag{31}$$

with ε_p, $p = 1,\ldots,5$, some free dimensionless parameters, generally, dependent on Θ, with the partial kinetic contributions being

$$\begin{aligned} K_1 &= \theta^{ab}\Gamma^c_{ac}\Gamma^d_{bd}, \qquad K_2 = \theta_{ab}\theta^{cd}\theta^{ef}\Gamma^a_{cd}\Gamma^b_{ef},\\ K_3 &= \theta^{ab}\Gamma^c_{ab}\Gamma^d_{cd}. \qquad K_4 = \theta_{ab}\theta^{cd}\theta^{ef}\Gamma^a_{ce}\Gamma^b_{df},\\ K_5 &= \theta^{ab}\Gamma^d_{ac}\Gamma^c_{bd}. \end{aligned} \tag{32}$$

Further, by admitting the second-order derivatives of θ and proliferating the affine symmetry to that under the arbitrary curvilinear coordinate transformations $y^a \to y'^a = y'^a(y)$ in the affine space, we can formally construct from the protometric θ_{ab} a counterpart of the Riemannian curvature tensor, then a Ricci tensor R_{ab} and at last a Ricci scalar $R \equiv \theta^{ab}R_{ab}$.[14] By this token, we can supplement L_G by the term

$$L_g \equiv -\frac{1}{2}\kappa_0^2(1+\varepsilon_0(\Theta))R, \tag{33}$$

with an arbitrary dimensionless coefficient ε_0. And finally, the potential term $V(\Theta)$ is an arbitrary scalar polynomial of $\Theta^a{}_b$, like $\mathrm{tr}(\Theta^2) \equiv \Theta^a{}_b\Theta^b{}_a$, etc., including, generally, a constant and $\det(\Theta^a{}_b)$. Due to the explicit violation of the affine symmetry by terms containing Θ, the GB ϑ becomes, in fact, the *pseudo-Nambu-Goldstone* (pNG) boson. Likewise, we could construct in the auxiliary affine space the most general affine and Lorentz-invariant Lagrangian for the pNG boson supplemented by the matter fields. The construction on the affine space treated above is as such nothing but an auxiliary construction. To describe the real world, the construction is incorporated below into a topological manifold – a progenitor of the metric spacetime.

[14]To generate such the transformations in the affine space it is sufficient, in fact, to consider a closure of the affine and conformal transformations [2, 13].

3 Dynamical world manifold

3.1 General covariance

Let the real world – a set of the primary events (points) – be modeled by a four-dimensional topological manifold M^4 endowed with the arbitrary smooth enough coordinates x^μ, $\mu = 0, \ldots, 3$. (As before, the index 0 is originally nothing but a notation.) Let now $M^4 \leftrightarrow R^4$ be a local one-to-one map of the manifold on the auxiliary affine space given by some invertible transformation functions $y^a = Y^a(x)$, with $x^\mu = x^\mu(y)$. Call by definition the distinct coordinates $y^a = Y^a(x)$ the *quasi-affine* coordinates on M^4.[15] In terms of y^a, let us map all the quantities on R^4 into the respective quantities on M^4. *A priori*, the map given by $Y^a(x)$ may be arbitrary signifying ultimately an incompleteness of the approach. To eliminate this uncertainty we treat $Y^a(x)$ in what fallows as the dynamical variables describing gravity on par with θ_{ab}. Otherwise this means that we interpret gravity as a net result of imposing both the field-theoretic (due to θ_{ab}) and geometrical (due to Y^a) effects.[16] For GC, express further the so obtained structures in terms of the arbitrary observer's coordinates $x^\mu = x^\mu(y)$. Defining for such the coordinate transformations on M^4 the *quasi-affine* tetrad $Y^a_\mu \equiv \partial_\mu Y^a$ and its inverse, $Y^\mu_a \equiv \partial x^\mu / \partial y^a|_{y=Y(x)}$, we can proliferate NM from the affine space to the world manifold in the GC manner. This may be achieved through the direct substitution $y^a \to x^\mu$ followed by the proper substitutions of the basic quantities. The latter ones may be combined into the three groups shown below.

Metric GR The first group of substitutions is as follows:

$$
\begin{aligned}
\theta_{ab} &\to g_{\mu\nu} \equiv Y^a_\mu Y^b_\nu \theta_{ab}, \\
\theta^{ab} &\to g^{\mu\nu} \equiv Y^\mu_a Y^\nu_b \theta^{ab}, \\
R_{ab} &\to R_{\mu\nu} \equiv Y^a_\mu Y^b_\nu R_{ab}, \\
R \equiv \theta^{ab} R_{ab} &\to R \equiv g^{\mu\nu} R_{\mu\nu}, \\
\det(\theta_{ab}) &\to g \equiv \det(g_{\mu\nu}) = \det(Y^a_\mu)^2 \det(\theta_{ab}), \\
\gamma^a &\to \gamma^\mu \equiv Y^\mu_a \gamma^a,
\end{aligned}
\tag{34}
$$

with

$$
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{35}
$$

In what is shown above, $g_{\mu\nu}$ is to be treated as the Riemannian metric with the inverse $g^{\mu\nu}$, while $R_{\mu\nu}$ proves to be the conventional Ricci curvature tensor constructed from $g_{\mu\nu}$. This group is relevant for GR and its direct siblings. It may ultimately be expressed through a local-Lorentz tetrad ϑ^α_μ:

$$
\vartheta^\alpha_a \to \vartheta^\alpha_\mu \equiv Y^a_\mu \vartheta^\alpha_a, \tag{36}
$$

[15] We leave aside the global topology of M^4 and its mapping Y onto R^4. Generally, the reversibility of Y may require some singular subsets on M^4, with the mapping being patch-wise. Note that the spatially flat Universe ($k = 0$), as implied by the Standard Model of Cosmology, is isomorphic as a topological space to R^4, with the required mapping being thus possible on the whole M^4,

[16] This may conceivably be treated as a kind of implementation of the Mach's principle, with the world as a whole influencing its local properties.

so that, say,

$$\begin{aligned} g_{\mu\nu} &= \vartheta^\alpha_\mu \vartheta^\beta_\nu \eta_{\alpha\beta}, \\ \sqrt{-g} &= \det(\vartheta^\alpha_\mu), \\ \gamma^\mu &= \vartheta^\mu_\alpha \gamma^\alpha, \end{aligned} \tag{37}$$

etc.[17] At that, the local-Lorentz tetrad $\vartheta^\alpha_\mu(X)$ in an arbitrary point X may be associated in GR with the so-called locally-inertial tetrad $\tilde{\vartheta}^\alpha_\mu(X)$ through a locally-inertial gauge $\hat{\Lambda}(X)$ given by

$$\hat{\Lambda}^\alpha{}_\beta(X)\vartheta^\beta_\mu(X) = \hat{\vartheta}^\alpha_\mu(X) \equiv \partial \hat{x}^\alpha_X / \partial x^\mu |_{x=X}. \tag{38}$$

In what is shown above, $\hat{x}^\alpha_X$ are the locally-inertial in a vicinity of the point X coordinates, i.e., those wherein $\hat{g}_{\alpha\beta}(x)|_{x\simeq X} \simeq \eta_{\alpha\beta}$ up to the quadratic deviations from X. This may *post factum* justify the appearance of the locally-inertial tetrad $\hat{\vartheta}_\mu$ for fermions in GR. For comparison, the tetrad $Y^a_\mu \equiv \partial_\mu Y^a$ defines the (chart-wise) affine coordinates $y^a = Y^a(x)$, wherein $\zeta_{ab}(y) = \eta_{ab}$ and $\gamma^c_{ab}(y) = 0$. For GR and its direct siblings, the dependence on Y^a gets completely hidden, with such the theories becoming the purely metric ones. The affine coordinates manifests themselves only for the QM GR extensions (without and with an explicit affine symmetry violation) considered below.

Hard/kinetic extension The second group of terms consists of the GC connection-like tensor

$$\Gamma^c_{ab} \to B^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} - \gamma^\lambda_{\mu\nu}, \tag{39}$$

where

$$\Gamma^\lambda_{\mu\nu} \equiv Y^\lambda_c Y^a_\mu Y^b{}_\nu \Gamma^c_{ab} = \frac{1}{2} g^{\lambda\kappa}(\partial_\mu g_{\nu\kappa} + \partial_\nu g_{\mu\kappa} - \partial_\kappa g_{\mu\nu}) \tag{40}$$

proves to be nothing but the Christoffel connection for the Riemannian metric $g_{\mu\nu}$ and

$$\gamma^\lambda_{\mu\nu} \equiv Y^\lambda_a \partial_\mu Y^a_\nu = Y^\lambda_a \partial_\nu Y^a_\mu. \tag{41}$$

The contribution of $B^\lambda_{\mu\nu}$ signifies the hard/kinetic extension to GR and clearly requires both the local-Lorentz and affine tetrads ϑ^α_μ and Y^a_μ.

Soft/potential extension At last, the third group of terms is as follows:

$$\begin{aligned} \eta_{ab} &\to \zeta_{\mu\nu} \equiv Y^a_\mu Y^b{}_\nu \eta_{ab}, \\ \eta^{ab} &\to \zeta^{\mu\nu} \equiv Y^\mu_a Y^\nu_b \eta^{ab}, \\ \det(\eta_{ab}) &\to \zeta \equiv \det(\zeta_{\mu\nu}) = \det(Y^a_\mu)^2 \det(\eta_{ab}), \\ \Theta^a{}_b \equiv \theta^{ac}\eta_{cb} &\to \Theta^\mu{}_\nu = Y^\mu_a Y^b_\nu \Theta^a{}_b = g^{\mu\lambda}\zeta_{\lambda\nu}, \end{aligned} \tag{42}$$

explicitly dependent on the affine tetrad Y^a_μ (and its inverse). Such the terms originate clearly from those in the affine space containing η_{ab} and imply thus an explicit affine symmetry violation.[18,19]

[17] Instead, presenting $g_{\mu\nu}$ in terms of the affine tetrads as $g_{\mu\nu} = Y^a_\mu Y^b{}_\nu \eta_{ab}$ would signify just the Minkowski spacetime in the curvilinear coordinates x^μ.

[18] Note that the term $\gamma^\lambda_{\mu\nu}$ Eq. (41) defined originally through Y^a_μ (and its inverse) may equivalently be presented as the Christoffel connection corresponding to the quasi-Lorentz metric $\zeta_{\mu\nu}$, with $B^\lambda_{mu\nu}$ getting explicitly the GC tensor. Nevertheless, the dependence on η_{ab} in fact drops off and the term $B^\lambda_{\mu\nu}$ does not imply the explicit affine symmetry violation.

[19] Note also that were Y^a not dynamical but some *prior*/"absolute" coordinates the NM would be either incomplete (under restriction only by the first group of terms shown above expressed entirely

3.2 Quartet-metric GR

Full nonlinear EFT At last, the gravity action is given by

$$S = \int_{M^4} L_G \mathcal{M} d^4x, \tag{43}$$

with L_G a GC scalar Lagrangian and $\mathcal{M}$ an induced manifold measure, i .e., a GC scalar density of the proper weight for S to be a GC scalar. At that, due to a freedom of redefining the Lagrangian there is a *prior* freedom of choosing the measure. E.g., the latter may be defined as $\sqrt{-g}$ or as $\sqrt{-\zeta}$, or as a combination of both. By this token, the action for the pure gravity may be expressed through the metric $g_{\mu\nu}$ and the field $\Theta^\mu{}_\nu$ without loss of generality as follows:[20]

$$S = \int L_G(g_{\mu\nu}, \Theta^\mu{}_\nu)\sqrt{-g}\, d^4x. \tag{44}$$

The second-order Lagrangian in the GC form now becomes

$$L_G = L_g + \frac{1}{2}\kappa_P^2 K(g_{\mu\nu}, B^\lambda_{\mu\nu}, \Theta^\mu{}_\nu) - V(\Theta^\mu{}_\nu), \tag{45}$$

with

$$L_g = -\frac{1}{2}\kappa_P^2(1 + \varepsilon_0(\Theta))R, \tag{46}$$

where R is the conventional Ricci scalar constructed from the Christoffel connection $\Gamma^\lambda_{\mu\nu}(g_{\mu\nu})$. Now we have identified the affine SSB scale κ_0 with the reduced Planck mass $\kappa_P = 1/\sqrt{8\pi G_N}$, with G_N the Newton's constant.[21] The kinetic term K looks as before in (31), with the partial contributions as follows:

$$\begin{aligned} K_1 &= g^{\mu\nu} B^\kappa_{\mu\kappa} B^\lambda_{\nu\lambda}, \qquad K_2 = g_{\mu\nu} g^{\kappa\lambda} g^{\rho\sigma} B^\mu_{\kappa\lambda} B^\nu_{\rho\sigma}, \\ K_3 &= g^{\mu\nu} B^\kappa_{\mu\nu} B^\lambda_{\kappa\lambda}, \qquad K_4 = g_{\mu\nu} g^{\kappa\lambda} g^{\rho\sigma} B^\mu_{\kappa\rho} B^\nu_{\lambda\sigma}, \\ K_5 &= g^{\mu\nu} B^\lambda_{\mu\kappa} B^\kappa_{\nu\lambda}. \end{aligned} \tag{47}$$

The potential $V(\Theta)$ remains as before, with $\Theta^a{}_b$ substituted by $\Theta^\mu{}_\nu = g^{\mu\lambda}\zeta_{\lambda\nu}$. E.g., the simplest case is given by $\varepsilon_1 \neq 0$, with the rest of ε's being zero, so that

$$K = \varepsilon_1(\sigma) K_1 = \varepsilon_1(\sigma) g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma, \tag{48}$$

supplemented by a potential $V = V(\sigma)$, where

$$\sigma \equiv \ln(\det(\Theta^\mu{}_\nu))^{-1/2} = \ln\sqrt{-g}/\sqrt{-\zeta} \tag{49}$$

is nothing but a scalar graviton [8]. Likewise, we could reproduce in the affine-Lorentz NM the more elaborate cases for the QM gravity [9].

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through the local-Lorentz tetrad Y^α_μ absorbing Y^a) or non-closed (under addition of the two last groups of terms dependent on the quasi-affine tetrad Y^a_μ).

[20] Note that $\Theta^{\mu\nu} \equiv \Theta^\mu{}_\lambda g^{\lambda\nu} = g^{\mu\kappa}\zeta_{\kappa\lambda}g^{\lambda\nu} = \Theta^{\nu\mu}$ and $\Theta_{\mu\nu} \equiv g_{\mu\lambda}\Theta^\lambda{}_\nu = \zeta_{\mu\nu}$, so that in fact $L_G = L_G(g_{\mu\nu}, \zeta_{\mu\mu})$.

[21] By this token, the Planck scale κ_P (like the maximal velocity c and the minimal action h) acquires the clear-cut physical meaning as a scale of the world transition from a primary phase to the affine-Goldstone one. Simultaneously, the Big Bang in cosmology may naturally be treated as such a phase transition, with the Universe in the affine-Goldstone phase and the spacetime itself emerging jointly at the Planck scale. A conceivable subsequent phase transition from the affine-Goldstone "plasma" phase to the quartet-metric "molecular one with the release of the large amount of free energy would be natural to associate with inflation in such a Universe.

Weak-field limit Let now in the spacetime emerged due to the primary SSB of the world continuum from the affine to Lorentz phase, during the subsequent formation of the Universe there appear the backgrounds $\bar{Y}^a$ and $\bar{g}_{ab}$ supplemented by the small fluctuations, respectively, ζ^a and h_{ab}:

$$\begin{aligned} Y^a &= \bar{Y}^a + \zeta^a, \\ g_{\mu\nu} &= \bar{g}_{\mu\nu} + h_{\mu\nu}. \end{aligned} \tag{50}$$

Choosing the affine coordinates $y^a = \bar{Y}^a(x)$ and putting for simplicity $\bar{g}_{ab} = \eta_{ab}$ we get (operating indices through η_{ab}) in the linear approximation (LA)

$$\begin{aligned} \zeta_{ab} &= \eta_{ab} + \partial_a\zeta_b + \partial_b\zeta^a, \\ \Theta_{ab} &= \eta_{ab} - h_{ab} + \partial_a\zeta_b + \partial_b\zeta_a. \end{aligned} \tag{51}$$

Likewise, the kinetic terms exhibit the same substitution

$$h_{ab} \to h'_{ab} = h_{ab} - (\partial_a\zeta_b + \partial_b\zeta_a). \tag{52}$$

This means that there takes place the Higgs-mode SSB of GR, with transforming the metric gauge components into the physical ones to be treated as the dark components of the Universe in addition to the tensor graviton. All the following under choosing the flat background with $\bar{g}_{ab} = \bar{\zeta}_{ab} = \bar{\Theta}_{ab} = \eta_{ab}$ goes as in QM GR [8, 9]. To illustrate the variety of the arising possibilities consider the two fallowing extreme cases.

(i) *Tensor graviton.* Imposing on $\bar{\varepsilon}_p \equiv \varepsilon_p(\bar{\Theta})$ the constants [8]

$$\begin{aligned} \bar{\varepsilon}_1 = 0, &\qquad \bar{\varepsilon}_2 = -\lambda, \\ \bar{\varepsilon}_3 = \bar{\varepsilon}_t - \lambda, &\qquad \bar{\varepsilon}_4 = \lambda, \\ \bar{\varepsilon}_5 = -\bar{\varepsilon}_t + 3\lambda, & \end{aligned} \tag{53}$$

with $\bar{\varepsilon}_t$ and λ some free parameters, we recover in LA the conventional GR Lagrangian in an obvious notation as follows:

$$L_G = \frac{\kappa_P^2}{8}(\bar{\varepsilon}_0 + \bar{\varepsilon}_t)\Big((\partial_c h'^{ab})^2 - 2(\partial_a h'^{ab})^2 + 2\partial_a h'^{ab}\partial_b h'^c_c - (\partial_a h'^b_b)^2\Big). \tag{54}$$

To reproduce GR precisely we should additionally put $\bar{\varepsilon}_0 + \bar{\varepsilon}_1 = 0$, recovering in LA the Newton's gravity. The weak-field post-Newtonian contributions to L_G impose, generally, some restrictions on the left-out parameters $\bar{\varepsilon}_t$ and λ. The deviations from the relations shown above would imply some additional kinetic contributions beyond GR already in LA being, as could be anticipated, highly suppressed. Moreover, even at the zero deviations from GR in LA the full nonlinear theory may still essentially deviate from GR in the strong-field limit producing some additional restrictions/predictions. At last, the potential V may be chosen so to recover the Fiertz-Pauli term for the massive tensor graviton possessing the descent massless limit. The extreme case presented above corresponds to modification of the genially tensor gravity. In particular, it would be quite instructive to study a modification of such a gravity for the strong fields (e.g., in black holes, etc,) without modifying its LA.

(ii) *Scalar graviton.* Other extreme case with $\bar{\varepsilon}_0 = 0$, $\bar{\varepsilon}_1 \neq 0$ and the rest of $\bar{\varepsilon}_p$ being zero corresponds to addition of the scalar graviton $\sigma = h'^c_c/2 \equiv h'/2$, with

$$K = \bar{\varepsilon}_1 K_1 = \frac{1}{4}\bar{\varepsilon}_1 \partial_a h' \partial^a h'. \tag{55}$$

without modification of the tensor gravity. There is also possible an admixture of a (conceivably problematic) vector graviton, as well as its mixture with the scalar one. Such the additional gravity components, possessing thus in line with the tensor graviton the NG origin, are proposed to be treated ultimately as the dark components (DM, DE, etc.) of the Universe [8, 9].

4 Conclusion: AG/QM gravity and beyond

Altogether, in the paper there is exposed the route of the gravity modification (so to say, in the "width" and "depth") from GR to the affine-Goldstone NM, the latter incorporating EFT of the quartet-metric GR. For studying the *appearance* of gravity it is sufficient to retain at the level of EFT, but for a deeper revealing the *nature* of gravity it is necessary to adhere to the more advanced level of NM. The respective two-faced gravity, being the affine-Goldstone in its nature and the quartet-metric in its appearance, may be termed as the *affine-Goldstone/quartet-metric (AG/QM) gravity*. In conclusion, let us summarize the main advantages provided for such a modified gravity by the NM vs. EFT frameworks.

(*i*) Justifying the set of the gravity fields, $g_{\mu\nu}$ and Y^a, as well as the pattern of symmetries, GC and the global Lorentz symmetry. Refining the types of interactions, with hinting on the possible hierarchy among the three generic group of the spacetime contributions originating from those in the affine space: with the normal affine symmetry and either the enhanced affine symmetry or the explicitly violated one.

(*ii*) Treating gravity on the same group-theoretic footing as other fundamental interactions, with the geometry being merely a conventional "language" of the gravity description. This may, conceivably, facilitate in the future a unified description of all the fundamental interactions below the Planck scale.

(*iii*) Pointing towards the AG/QM gravity as an emergent phenomenon due to a conceivable underlying theory of gravity and spacetime beyond the Planck scale producing the given NM below the Planck scale. This. in principle, implies the two-step transition of the world manifold from a primary metricless phase to that with metric: first to the affine-Goldstone phase and ultimately to the quartet-metric one.

Clearly, the proposed route of the GR modification would ultimately imply abandoning the conventional reductionalism and would require models beyond the modern ones conventionally based on the spacetime. Nevertheless, such a route possesses the deep theoretical foundation and the wide phenomenological prospects, so that its further developing seems to be a worthy challenge.

Appendix. NM generalities

Cosets To begin, the SSB $G \to H$ of a global group G to its subgroup $H \in G$ is described group-theoretically in terms of the so-called *cosets*. A (say, left) coset of a subgroup H in the group G with respect to an element $k \in G$ is defined generically as a subset (not, generally, a subgroup) of G equivalent to the given k up two the (right) multiplication by any $h \in H$, $k \sim kh$. The cosets are either identical or disjoint. Each element of G belongs to one, and only one, coset, with the cosets partitioning the group, i.e., the unification of all the cosets represents the whole group. At that, the coset as a whole is uniquely determined by any its element chosen as a *representative*, say, k. The total set of the (left)coset representatives k constitutes the (left) *coset space* $K = G/H$.

SSB and Goldstone boson Now, if G is a symmetry group of a physical system, let $|I>$ be the system ground state ("vacuum") invariant under G, i.e., $g|I>= |I>$ for any $g \in G$. Let now there takes place SSB $G \to H$ meaning that the invariance of the vacuum lowers up to $H \subset G$, i.e., only $h|I>= |I>$ for any $h \in H$. In this case, for an arbitrary $g \in G$, with the decomposition $g = kh$, $k \in K$ and $h \in H$, there takes place $g|I>= k|I>\equiv |k>\neq |I>$. This corresponds to SSB in the NG mode, with the appearance of a set of the degenerate vacua $|k>$, $k \in K$, the expositions of which correspond to the physical NG bosons. In these terms, the field variable describing the SSB $G \to H$ in the NG mode may conventionally be chosen as a (local) representative $k(x)$, $x \in R^4$. k

Nonlinear realization Further, the result of the action of a group element $g \in G$ on a (left) coset representative $k \in K$, gk, being a group element may as well be decomposed as $gk = k'h$, with a new $k' \in K$ and some $h = h(g, k) \in H$. This implies that the coset representative k transforms under G nonlinearly as

$$g \ : \ k \to k' = gkh^{-1}(g, k).$$

This defines NR of the group G on the (left) coset space $K = G/H$ for SSB $G \to H$.

Nonlinear model And finally, a NM describing GB $k \in K = G/H$ appearing due to SSB $G \to H$ for a pair of the global internal groups $H \subset G$ is a kind of the field theory for the coset (local) representative $k(x) \in K$, $x \in R^4$, determined by the action $S = \int L(k, \psi)d^4x$, with a Lagrangian L invariant under the nonlinearly realized broken symmetry G (in distinction wit EFT built on the conventional linear representations of the symmetry G). At that, a generic matter field ψ transforms under G as a linear representation ρ of the residual group H through $h(g, k)$ as determined above:

$$g \ : \psi \to \psi' = \rho(h(g, k))\psi,$$

with GB k being thus omnipresent (what is, in particular, characteristic of gravity). In the particular cases NM may significantly simplify, with the nonlinearity becoming hidden (cf., e.g., the symmetry pattern considered in the paper).

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