Secondary dips and asymptotics

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Overview

Introduction

- How to detect experimentally the energy region where asymptotics starts to manifest itself?
- Relating appearance of the secondary dips in the differential cross-section of elastic scattering $d\sigma/dt$ with beginning of the asymptotic energy region.
- Differential cross–section $d\sigma/dt$ at large values of $-t$ is most sensitive to the region of low impact parameter values. Such correlation is exploited here to point out the experimental signature of the asymptotic energy region on the grounds of the differential cross–section $d\sigma/dt$ behavior in the region of large transferred momenta.

Absorptive corrections: first dip

- It was shown that developing dip in $d\sigma/dt$ at the ISR in the region of $-t = 1.4\ (GeV/c)^2$ should be associated with increasing role of unitarity with the energy growth, i.e with the absorption at low impact parameter values.
  F. Halzen, Lectures presented at the 1973 Summer Institute on Particle Interactions at Very High Energies, Louvain, Belgium.
- Maximal strength of strong interactions means saturation of an upper bound for the total cross–section. This bound is well known after the names of Froissart and Martin.

Saturation of unitarity

Maximal amplitude in b-space

The above saturation results from saturation of the upper limit for the $\mathrm{Im} f(s,b)$.

- This limit is determined by the unitarity relation for the amplitude $f(s,b)$:

$$\mathrm{Im} f(s,b)[1-\mathrm{Im} f(s,b)] = [\mathrm{Re} f(s,b)]^2 + h_{inel}(s,b), \qquad (1)$$

the inelastic overlap function $h_{inel}(s,b)$ is a nonnegative one (contribution of all intermediate inelastic channels).

$$0 \leq \mathrm{Im} f(s,b) \leq 1 \qquad (2)$$

while a completely different relation takes place for $\mathrm{Re} f(s,b)$

$$-\frac{1}{2}\sqrt{1-4h_{inel}(s,b)} \leq \mathrm{Re} f(s,b) \leq \frac{1}{2}\sqrt{1-4h_{inel}(s,b)}. \qquad (3)$$

Saturation of unitarity

- Saturation of the upper unitarity limit for $\mathrm{Im} f(s,b)$ implies that $\mathrm{Im} f(s,b) \to 1$ at large energies and fixed impact parameters in the region $b < r(s)$ with $r(s)$ rising logarithmically with s, i.e. $r(s) \sim \ln s$. The two other related limits are $\mathrm{Re} f(s,b) \to 0$ and $h_{inel}(s,b) \to 0$ at $b < r(s)$.
- At large energy $\sigma_{tot}(s) \sim \ln^2 s$, $\sigma_{inel}(s) \sim \ln s$ and the ratio of real to imaginary parts of a forward scattering amplitude tends to zero
- The linear logarithmic increase of the $\sigma_{inel}(s)$ is a result of the self-damping of the inelastic channels. Self-damping is a consequence the unitarity requirements. It should also be noted that the upper bound for the inelastic cross–section *excludes* $\ln^2 s$-dependence for $\sigma_{inel}(s)$ at $s \to \infty$ when the ratio $\sigma_{tot}(s)/[(4\pi/t_0)\ln^2(s/s_0)]$ tends to unity at $s \to \infty$.

M. Baker, R. Blankenbecler, Phys. Rev. **128**, *415 (1962).*
T.T. Wu, A. Martin, S.M. Roy, V. Singh, Phys. Rev. D **84**, *025012 (2011).*

Saturation of unitarity and secondary dips

Secondary dips

At large enough energies $\mathrm{Im} f(s,b)$ should have a form of smoothed step function in the impact parameter space. Such form ultimately results in appearance of the secondary dips and bumps in $d\sigma/dt$. Indeed, at the asymptotic energies dominating contribution to the function $\mathrm{Im} F(s,t)$ can be approximated as having an oscillating dependence over $\sqrt{-t}$:

$$\mathrm{Im} F(s,t) \sim r(s) J_1(r(s)\sqrt{-t})/\sqrt{-t}. \quad (4)$$

Zeros of the function $\mathrm{Im} F(s,t)$ are associated with the dips in the differential cross–section of elastic scattering .

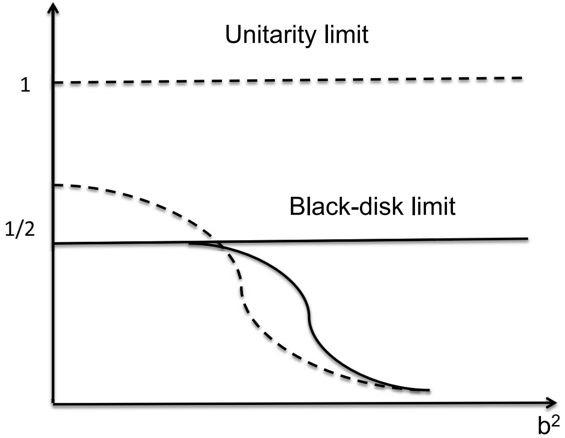

Figure: Schematic representation of the impact parameter dependence of the function Im$f(s, b)$ at the energy values in the region of $\sqrt{s} = 13$ TeV.

A. Alkin, O. Kovalenko and E. Martynov, EPL **102**, *31001 (2013)*

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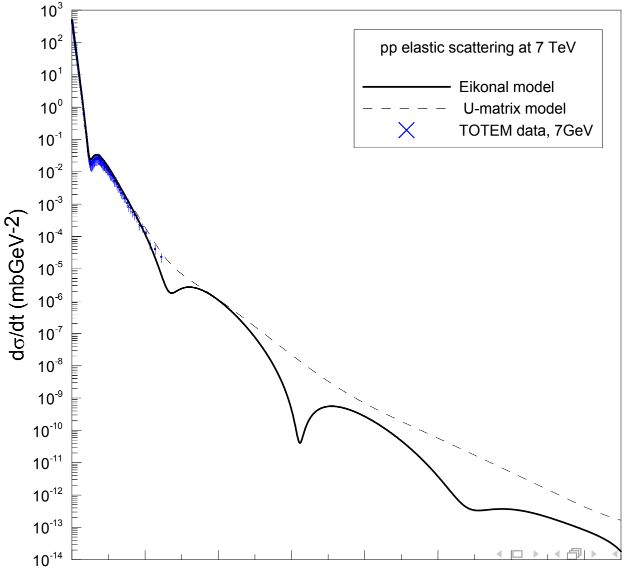
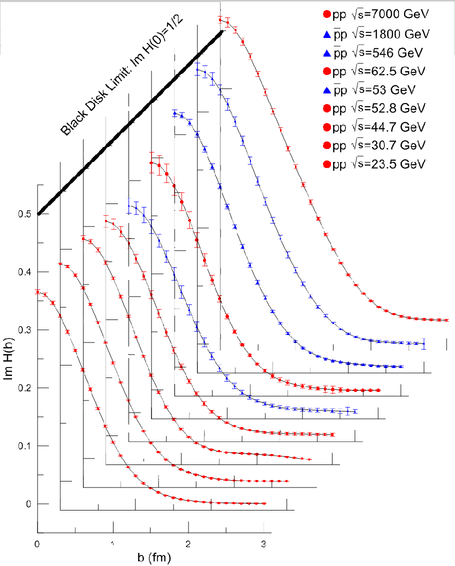


The existing experimental data are not in the asymptotical energy region since the secondary bumps and dips have not been observed up to the energy of $\sqrt{s} = 13$ TeV. Even at this highest available accelerator energy value the differential cross–section has a smooth, without secondary dips and bumps, dependence on the transferred momentum in the region beyond the first dip. The data are also pointing excess of the black-disc limit and not its saturation at the LHC energies.

Simple extrapolation of the energy evolution of the impact parameter picture indicates that asymptotics might be expected in the region of $\sqrt{s} \simeq 40$ TeV.

It should be noted that the data obtained under the cosmic ray studies do not provide an information on the differential cross–section $d\sigma/dt$.

A. Alkin, E. Martynov, O. Kovalenko and S.M. Troshin, Phys. Rev. D **89**, *091501 (2014)*



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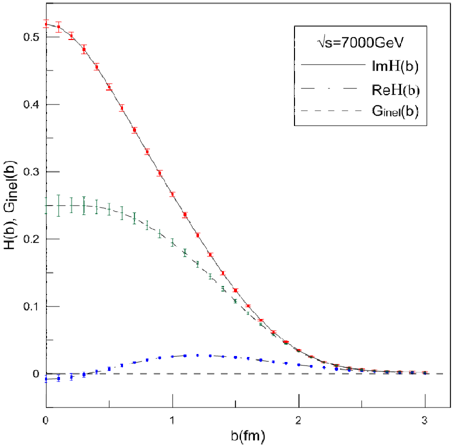
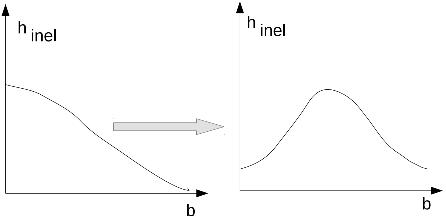

√s=7000GeV
ImH(b)
ReH(b)
Ginel(b)
H(b), Ginel(b)
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
b(fm)


h inel
b
h inel
b

Conclusion

- It may be suggested to conclude on the onset of the asymptotics in a hadron scattering by observation an appearance of the secondary dips and bumps in the $d\sigma/dt$ dependence on $-t$. This is a qualitative prediction, it does not rely on any particular model for the hadron scattering and does not specify, therefore, an explicit value of the energy where the asymptotic region begins just leaving it to the experimental studies. It is based on the impact parameter picture of hadron scattering which is more relevant for study of the asymptotic energy region than considerations of *integrated* over the impact parameter quantities.