Dualities in dense baryonic (quark) matter with chiral and isospin imbalance

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QCD Lagrangian

The QCD Lagrangian obtained from the gauge principle reads

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=u,d,s} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}_a^{\mu\nu}.$$

f- quark flavor, the quark field q_f consists of a color triplet (subscripts r, g, and b standing for “red,” “green,” and “blue”),

$$q_f = \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix},$$

The covariant derivative is

$$D_\mu \equiv (\partial_\mu - ie\mathcal{A}_\mu), \quad \mathcal{A}_\mu = \mathcal{A}_\mu^a \lambda^a$$

field strength tensor

$$\mathcal{G}_{\mu\nu,a} = \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + g f_{abc} \mathcal{A}_{\mu,b} \mathcal{A}_{\nu,c},$$

quark masses

The six quark flavors are commonly divided into the three light quarks u, d, and s and the three heavy flavors c, b, and t,

$$\begin{pmatrix} m_u = 0.005\,\text{GeV} \\ m_d = 0.009\,\text{GeV} \\ m_s = 0.175\,\text{GeV} \end{pmatrix} \ll 1\,\text{GeV} \leq$$

The quark masses can be estimated on lattice or in QCD sum rule method (not absolute values but the relations between masses can be established in ChPT)
2012 Review of Particle Physics. J. Beringer et al., Phys. Rev. D86, 010001 (2012).

main features of QCD: chiral symmetry breaking

Unlike the QED , **the QCD vacuum has non-trivial structure** due to non-perturbative interactions among quarks and gluons

GOR relation and lattice simulations $\Rightarrow$ **condensation of quark and anti-quark pairs**

$$\langle \bar{q}q \rangle \neq 0, \quad \langle \bar{u}u \rangle = \langle \bar{d}d \rangle \approx (-250MeV)^3$$

quite analogous to the ground state of the BCS superconductor (condensation of the electron Cooper pairs $\langle \bar{\psi}_\uparrow \psi_\downarrow \rangle \neq 0$)

$\langle \bar{q}q \rangle \neq 0$ suggests the existence of the "dynamical mass"

$$\langle \bar{q}q \rangle = -i \lim_{x \to y+0} tr S_F(x,y), \quad S_F(p) = \frac{A(p)}{\gamma p - B(p)}$$

If $B(p) = 0 \Rightarrow \langle \bar{q}q \rangle = 0$ due to $tr\gamma^\mu = 0$ (in chiral limit in PT)

NJL and ladder QCD approach gives $B(p) = M \Rightarrow$ **CSB**

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\right]$$

$$q \to e^{i\gamma_5\alpha}q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\left[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\right]q - \frac{N_c}{4G}\left[\sigma^2 + \pi^2\right].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\left[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\right]q$$

Methods of dealing with QCD

Methods of dealing with QCD

- perturbative QCD (pQCD), high energy
- First principle calcaltion – lattice Monte Carlo simulations, LQCD
- Effective models

  Chiral pertubation theory χPT
  Nambu–Jona-Lasinio model NJL
  Polyakov-loop extended Nambu–Jona-Lasinio model PNJL
  Quark meson model
- $1/N$ expansion (large number of colors) G.t'Hooft.
  the predictions of $\frac{1}{N_c}$ expansions for QCD are mostly of a qualitative nature
- Holographic methods, Gauge/gravity or gauge/string duality AdS/CFT conjecture
- low dimensional models that mimics QCD

lattice QCD at non-zero baryon chemical potential μ_B

It is well known that **at non-zero baryon chemical potential** μ_B **lattice simulation** is quite challenging due to the **sign problem** complex determinant

$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

(1+1)- dimensional Gross-Neveu model

(1+1)-dimensional Gross-Neveu (GN) model possesses a lot of common features with QCD

- renormalizability
- asymptotic freedom
- sponteneous chiral symmetry breaking in vacuum
- dimensional transmutation
- have the similar $\mu_B - T$ phase diagrams

Relative simplicity, renormalizability $\rightarrow$ NJL_2 model can be used as a laboratory for the qualitative simulation of specific properties of QCD at arbitrary energies

(3+1)-dimensional NJL model

NJL model can be considered as **effective field theory for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

Parameters G, Λ, m_0

chiral limit $m_0 = 0$

in many cases chiral limit is a very good approximation

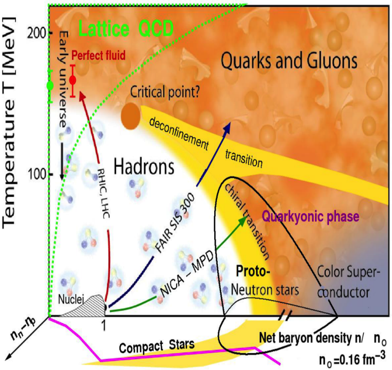
dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

Relative simplicity allow to consider hot and dense QCD in the framework of NJL model and explore the QCD phase structure (diagram).

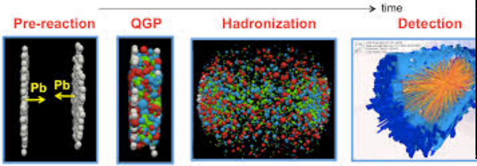
QCD at finite temperature and nonzero chemical potential

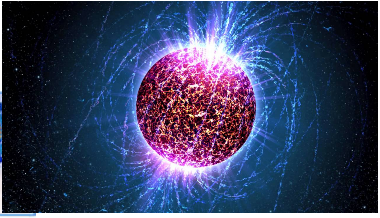
QCD at nonzero temperature and baryon chemical potential plays a fundamental role in many different physical systems. (QCD at extreme conditions)

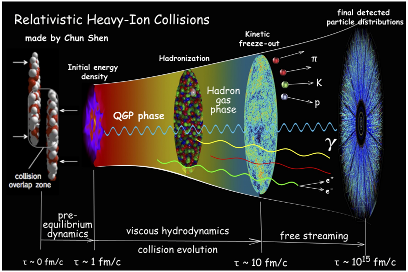
- neutron stars
- heavy ion collision experiments
- Early Universe

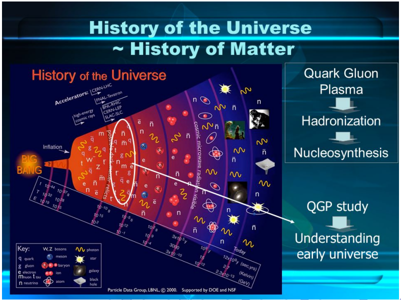


QCD at extreme conditions

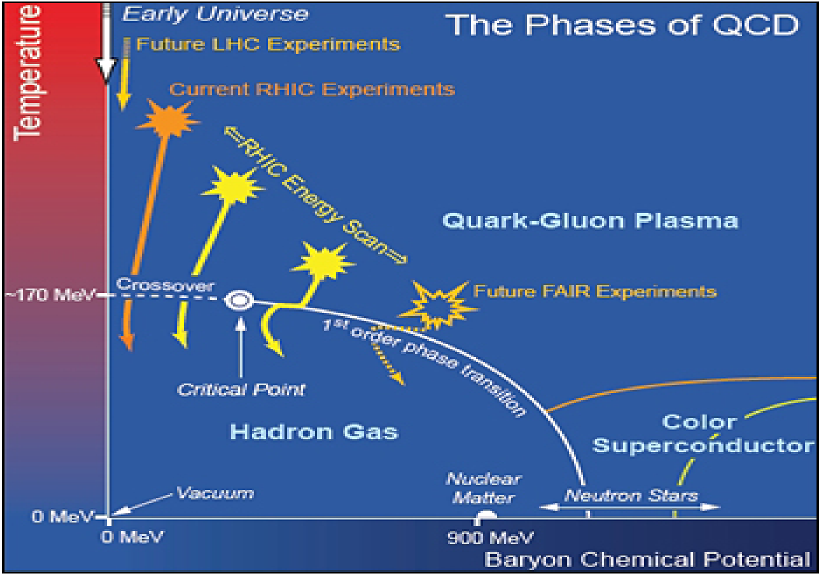








QCD phase portrait



Very brief history and motivation

There has been a lot of activity in this area
pion condensation in NJL_4
K. G. Klimenko, D. Ebert J.Phys. G32 (2006) 599-608
arXiv:hep-ph/0507007
K. G. Klimenko, D. Ebert
Eur.Phys.J.C46:771-776,(2006) arXiv:hep-ph/0510222

also in **(1+1)- dimensional case**, NJL_2
K. G. Klimenko, D. Ebert, PhysRevD.80.125013 arXiv:0902.1861 [hep-ph]

$$\Downarrow$$

pion condensation in dense matter **predicted without certainty**

physical quark mass – no pion condensation in dense medium
H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, M. Ruggieri
Phys.Rev.D79:034032,2009 arXiv:0809.2658 [hep-ph]

Very brief history and motivation

There could be **parameters that generate pion condensation** in dense matter

-Finite volume effects
D. Ebert, T.G. Khunjua, K.G. Klimenko, V.Ch. Zhukovsky, arXiv:1106.2928 [hep-ph] Int. J. Mod. Phys. A 27, 1250162 (2012)

-Inhomogeneous pion condensate
N. V. Gubina, K. G. Klimenko, S. G. Kurbanov, V. Ch. Zhukovsky, PhysRevD.86.085011 arXiv:1206.2519 [hep-ph]

This is all obtained in **(1+1)- dimensional case**, NJL_2

-Pion Condensation by Rotation in a Magnetic field
Y. Liu, I. Zahed Phys. Rev. Lett. 120, 032001 (2018) arXiv:1711.08354 [hep-ph]

Recalling previous results

There has been found **another parameter** that **generate pion condensation in dense matter**

Chiral imbalance

Difference between average number of left-handed and right-handed quarks

The results have been obtained in the framework of

NJL_2 model

Phys. Rev. D 95, 105010 (2017) arXiv:1704.01477 [hep-ph]
Phys. Rev. D 94, 116016 (2016) arXiv:1608.07688 [hep-ph]

NJL_4 model

Phys. Rev. D 97, 054036 (2018) arXiv:1710.09706 [hep-ph]

Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.
compact stars, heay ion collisions

The corresponding term in the Lagrangian is

$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q$, where μ -quark chemical potential $(n_B = 3n_q)$

Isotopic chemical potential μ_I

Allow to consider systems with isotopic imbalance.

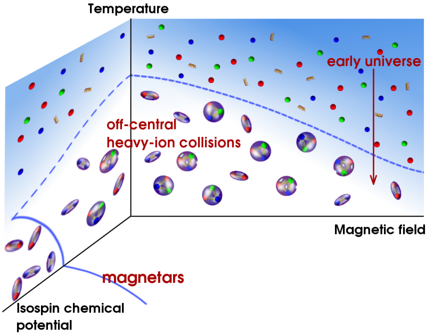
$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

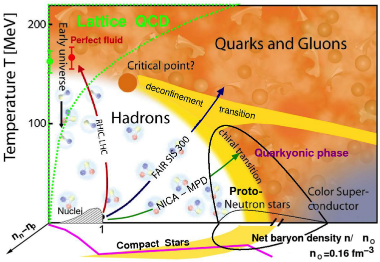
The corresponding term in the Lagrangian is $\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q$

Dense matter with isotopic imbalance in neutron stars, HIC

QCD phase diagram with isotopic imbalance

neutron stars, heavy ion collisions have isotopic imbalance





Different types of chemical potentials: chiral imbalance

chiral (axial) chemical potential

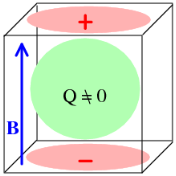
Allow to consider systems with chiral imbalance (difference between between densities of left-handed and right-handed quarks).

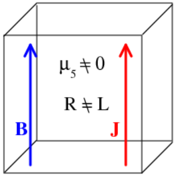
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

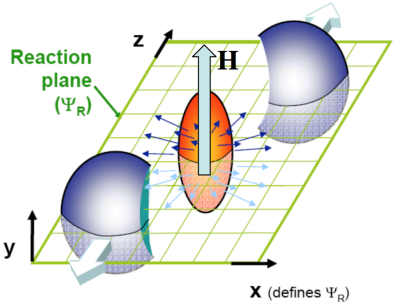
The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

Chiral magnetic effect





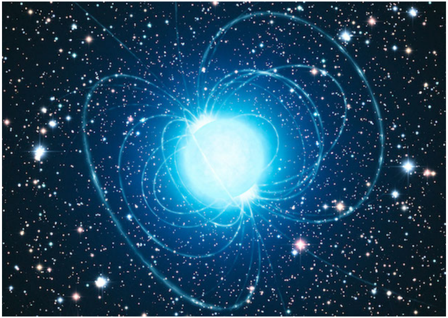


$$\vec{J} = c\mu_5 \vec{B}, \qquad c = \frac{e}{2\pi^2}$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033 [arXiv:0808.3382 [hep-ph]].

Chiral separation effect

Chiral imbalance could appear in compact stars



$$\vec{J}_5 = c\mu\vec{B}, \qquad c = \frac{e}{2\pi^2}$$

there is current and there is n_5

Different types of chemical potentials: chiral imbalance

chiral (axial) isotopic chemical potential

Allow to consider systems with chiral isospin imbalance (mismatch between differences of densities of left-handed and right-handed quarks for u and d quarks)

$$\mu_{I5} = \mu_{u5} - \mu_{d5} = \mu_{uR} - \mu_{uL} + \mu_{dL} - \mu_{dR}$$

so the corresponding density is

$$n_{I5} = n_{u5} - n_{d5} = n_{uR} - n_{uL} + n_{dL} - n_{dR}$$

$$n_{I5} \quad \longleftrightarrow \quad \mu_{I5}$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q$

If one has all four chemical potential, one can consider different densities n_{uL}, n_{dL}, n_{uR} and n_{dR}

Model and its Lagrangian

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]q +$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets; τ_k $(k = 1, 2, 3)$ are Pauli matrices.

Equivalent Lagrangian

To find the thermodynamic potential of the system, we use a semi-bosonized version of the Lagrangian, which contains composite bosonic fields $\sigma(x)$ and $\pi_a(x)$ $(a = 1, 2, 3)$

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^1 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q$$

$$-\frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big]. \qquad (1)$$

For bosonic fields one has

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q). \qquad (2)$$

Condansates ansatz

We will use the following ansat: $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates x,

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0. \tag{3}$$

where M and Δ are already constant quantities.

thermodynamic potential

For the thermodynamic potential in the large N_c limit one can obtain

$$\Omega(M,\Delta) \;=\frac{M^2+\Delta^2}{4G}+\mathrm{i}\int\frac{d^4p}{(2\pi)^4}\ln Det\overline{D}(p),$$

where

$$Det\overline{D}(p)=\left(\eta^4-2a_+\eta^2+b_+\eta+c_+\right)\left(\eta^4-2a_-\eta^2+b_-\eta+c_-\right)\equiv$$
$$\equiv P_+(p_0)P_-(p_0),$$

where $\eta=p_0+\mu$, $|\vec{p}|=\sqrt{p_1^2+p_2^2+p_3^2}$ and

a_+, b_+, c_+ can be found analytically

a_-, b_-, c_- can be obtained from a_+, b_+, c_+ just by changing $\nu\to-\nu$ and $\mu_5\to-\mu_5$.

$$\nu=\frac{\mu_I}{2}\qquad \nu_5=\frac{\mu_{I5}}{2}$$

Projections of the TDP on the M and Δ axes

The roots η_i can be found analytically in the general case

No mixed phase $(M \neq 0, \Delta \neq 0)$

it is enough to study **the projections** of the TDP on the M and Δ axes

projection of the TDP on the M axis $F_1(M) \equiv \Omega(M, \Delta = 0)$

projection of the TDP on the Δ axis $F_2(\Delta) \equiv \Omega(M = 0, \Delta)$

Dualities of the TDP

The TDP is invariant with respect to the so-called duality transformations (dualities)

1) The main duality

$$\mathcal{D}: \; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

$$\nu \longleftrightarrow \nu_5 \text{ and PC} \longleftrightarrow \text{CSB}$$

2) Duality in the CSB phenomenon

$$F_1(M) \text{ is invariant under } \mathcal{D}_M \;:\; \nu_5 \leftrightarrow \mu_5$$

3) Duality in the PC phenomenon

$$F_2(\Delta) \text{ is invariant under } \mathcal{D}_\Delta \;:\; \nu \leftrightarrow \mu_5$$

PC phenomenon breaks $\mathcal{D}_M$ and CSB phenomenon $\mathcal{D}_\Delta$ duality

Dualities in different approaches

- Similar **dualities** between chiral and superconducting condensates have been obtained in **(1+1) and (2+1)-dimensional models**

  D. Ebert, T.G. Khunjua, K.G. Klimenko, V.Ch. Zhukovsky, Phys. Rev. D 90, 045021 (2014), Phys. Rev. D 93, 105022 (2016)

- **Large N$_c$ orbifold equivalences** connect gauge theories with different gauge groups and **matter content** in the large N$_c$ limit.

  M. Hanada and N. Yamamoto, JHEP 1202 (2012) 138, arXiv:1103.5480 [hep-ph], PoS LATTICE **2011** (2011), arXiv:1111.3391 [hep-lat]

Phase structure of the (3+1)-dim NJL model in the case of (μ, μ_I, μ_{I5})

(ν, ν_5) phase portrait at different μ of NJL_4

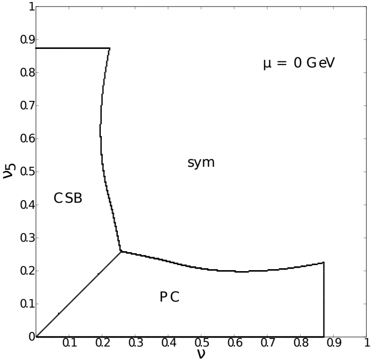


Figure: (ν, ν_5) phase diagram at $\mu = 0$ GeV

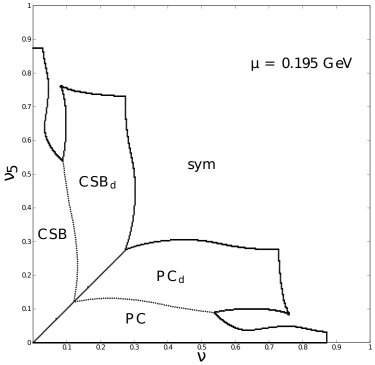


Figure: (ν, ν_5) phase diagram at $\mu = 0.195$ GeV

Consideration of the case with μ_B, μ_I, μ_{I5} and μ_5 chemical potentials in (3+1)-dimensional NJL model

The influence on **the phase structure** of **chemical potentials** μ_B**,** μ_I**,** μ_{I5} **and** μ_5 have been considered in the framework of NJL_4 model

First let us start with consideration of the case $\mu_{I5} = 0$

The influence of **chiral imbalance** in the form of **chiral isospin** μ_{I5} **chemical potential** have on PC condensation has been studied above

How **chiral imbalance** in the form of **chiral** μ_5 **chemical potential** influence PC condensation

Chiral imbalance in the form of chiral μ_5 chemical potential. (ν, μ_5) phase diagram at different μ

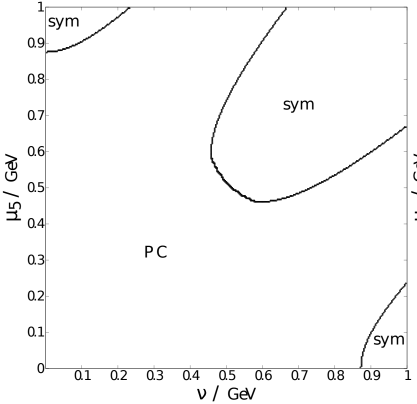


Figure: (ν, μ_5) phase diagram at $\mu = 0.1$ GeV. CSB is the chiral symmetry breaking phase, PC is charged pion condensation

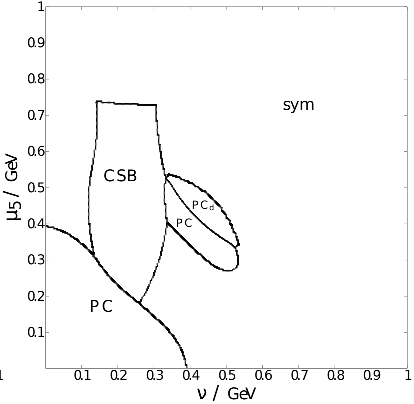


Figure: (ν, μ_5) phase diagram at $\mu = 0.23$ GeV. All the notations are the same

Chiral imbalance in the form of chiral μ_5 chemical potential. Comparison of non-zero μ_{I5} and μ_5 cases

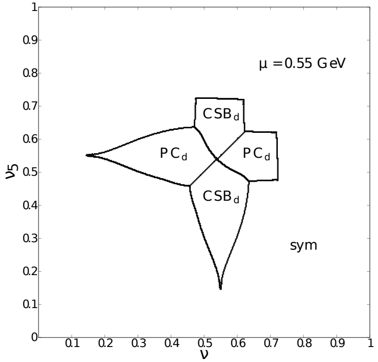


Figure: (ν, ν_5) phase diagram at $\mu = 0.55$. All the notations are the same

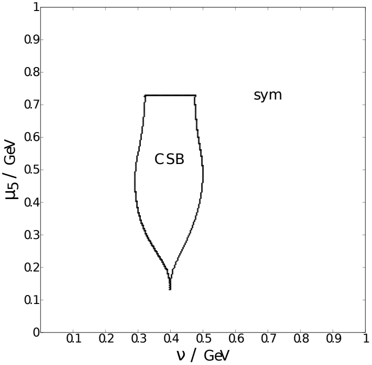


Figure: (ν, μ_5) phase diagram at $\mu = 0.4$ GeV. All the notations are the same

Consideration of the case $\mu_I = 0$

Now let us discuss the **case of** $\mu_I = 0$

Study the **influence of** μ**,** μ_{I5} **and** μ_5 **on the phase diagram**

and present the **(**μ, μ_{I5}, μ_5**)- phase diagram** of the model

In this case one can use **duality** $\mathcal{D}$ **between CSB and PC** to get **phase diagrams** from **already** depicted ones

Chiral imbalance at zero isospin μ_I chemical potential. (μ_5, ν_5) phase diagram at different μ

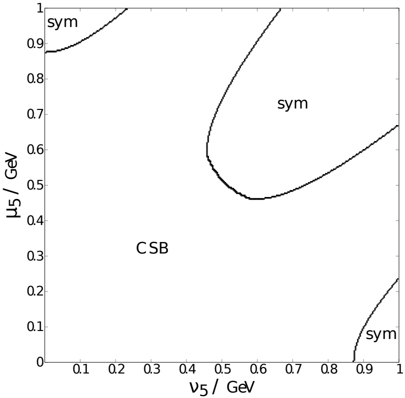


Figure: (μ_5, ν_5) phase diagram at $\mu = 0.1$ GeV. All the notations are the same

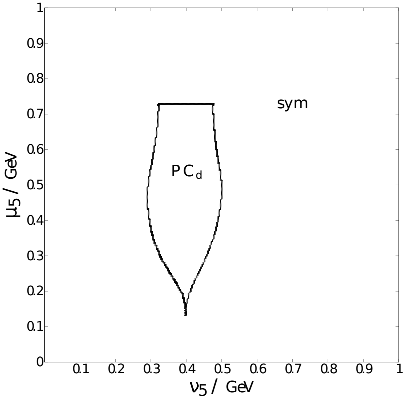


Figure: (μ_5, ν_5) phase diagram at $\mu = 0.4$ GeV. All the notations are the same

Consideration of the general case μ, μ_I, μ_{I5} and μ_5

Now let us discuss **the general case** μ**,** μ_I**,** μ_{I5} **and** μ_5

First, let us consider **how** μ_5 **influence the behavior of (**ν**,** ν_5**) phase diagram with respect to the changes of** μ

and present **(**ν**,** ν_5**)- phase diagrams** at different μ_5 and μ

Then we will present a couple of (ν, μ_5) and (μ_5, ν_5)- phase diagrams

Consideration of the general case μ, μ_I, μ_{I5} and μ_5

In the case all μ, μ_I, μ_{I5} and μ_5 are non-zero
TDP is even function over each of the variables M, Δ and μ
but it is **not true for μ_I, μ_{I5} and μ_5 chemical potentials**

TDP is invariant

$$M \to -M,\ \Delta \to -\Delta,\ \mu \to -\mu$$

is not invariant

$$\mu_I \to -\mu_I,\ \nu_5 \to -\nu_5,\ \mu_5 \to -\mu_5$$

there are symmetries

$$\nu \to -\nu,\ \nu_5 \to -\nu_5 \qquad \nu \to -\nu,\ \mu_5 \to -\mu_5$$

It is enough to explore either **all positive but** μ_5 or **all positive but** ν

$(\nu,\ \nu_5)$- phase diagrams at different μ_5 and μ

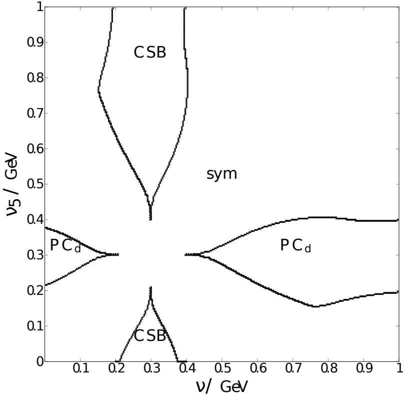


Figure: (ν, ν_5) phase diagram at $\mu_5 = 0.3$ GeV and $\mu = 0.3$ GeV. All the notations are the same

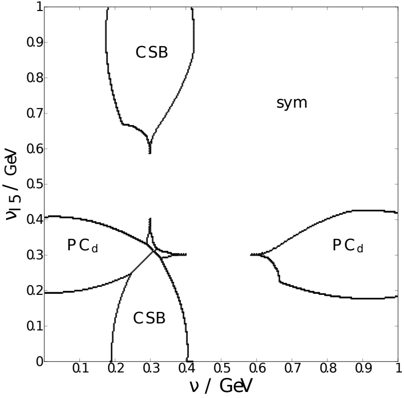


Figure: (ν, ν_5) phase diagram at $\mu_5 = 0.5$ GeV and $\mu = 0.3$ GeV. All the notations are the same

$(\nu,\ \nu_5)$- phase diagrams at different μ_5 and μ. the case μ_5

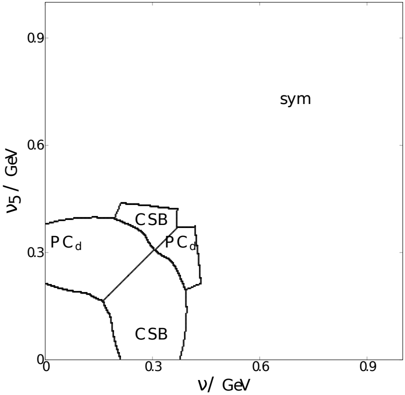


Figure: (ν, ν_5) phase diagram at $\mu_5 = -0.3$ GeV and $\mu = 0.3$ GeV. All the notations are the same

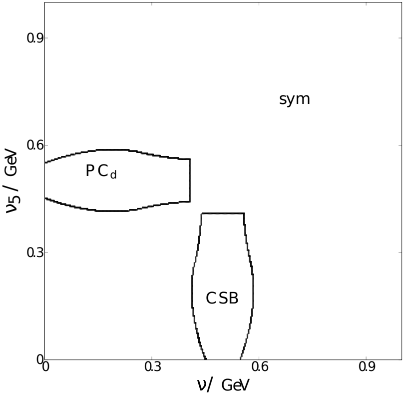


Figure: (ν, ν_5) phase diagram at $\mu_5 = -0.3$ GeV and $\mu = 0.5$ GeV. All the notations are the same

Conclusions

We studied chirally ($\mu_5 \neq 0$) and isotopically ($\mu_I \neq 0$) and chirally isotopically ($\mu_{I5} \neq 0$) asymmetric dense ($\mu_B \neq 0$) quark matter in the framework of (3+1)-dim NJL model.

- there exists **three dualities**.
  First duality is **between the chiral symmetry breaking and the charged pion condensation** phenomena, **two new dualities** hold only for chiral symmetry breaking and charged pion condensation phenomena **separately**.
- According to **duality** for CSB, chiral isospin μ_{I5} chemical potential **influence the CSB phenomenon the exactly same way** as chiral μ_5 chemical potential.
  All the conclusions of **catalysis of dynamical CSB** by μ_5 **holds in exactly the same way** for μ_{I5}.

Conclusions

- According to **duality** for PC chiral μ_5 chemical potential **influence the PC phenomenon exactly the same way** as isospin μ_I chemical potential.
- If $\mu_5 \neq 0$ chiral isospin μ_{I5} chemical potential **generates the PC$_d$ phase** even if isospin $\mu_I = 0$.
- Only non-zero chiral μ_5 chemical potential in the absence of chiral isospin μ_{I5} chemical potential **does generate PC$_d$ phase but not so powerfully as μ_{I5} does**.

Thanks for the attention