Киральная асимметрия в горячей и плотной кварковой материи: Сравнение подходов модели НЙЛ и решеточной КХД

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At normal (Earth) conditions, protons and neutrons form atomic nuclei, and the latter, together with their orbital electrons, form the ordinary matter of our environment.



If the matter is subjected to extreme compression, eventually all chemical and nuclear bonds are broken, and the matter is squeezed from the molecular scale to the sub-particle scale with a density higher than 0.15 baryon per fm^3 .

Introduction

Experimental creation of such dense matter is a very hard problem but such conditions can take place inside compact stars due to compression by gravity into a stable and extremely dense state.



Introduction

Due to technology advances, modern accelerators of elementary particles are now able to collide not only single high energy protons, but also heavy ions consisting of many coupled protons and neutrons.



Introduction

The fundamental theory of the matter in such extreme conditions is quantum chromodynamics (QCD) which is a gauge field theory associated with SU(3) group, where gauge bosons (gluons) play the role of interaction carriers of quarks.

$$\mathcal{L}_{QCD} = \bar{\psi}(i \ \mathcal{D}_a T_a - m)\psi - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu,a}$$

- Covariant derivative, gluon field tensor $D_a^{\mu} = \partial^{\mu} + igA_a^{\mu}$ $F_a^{\mu\nu} = \partial^{\mu}A_a^{\nu} - \partial^{\nu}A_a^{\mu} - gf_{abc}A_b^{\mu}A_c^{\nu}$
- Color matrices and structure constants $[T_a^{(F)},T_b^{(F)}]=if_{abc}T_c^{(F)},\quad (T_a^{(A)})_{bc}=-if_{abc}$

Prominent feature of the QCD

• Chiral symmetry

$$\begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5\tau \frac{\theta}{2}} \begin{pmatrix} u \\ d \end{pmatrix},$$

In the low-energy regime there is a spontaneous chiral symmetry breaking in consequence of which quarks acquire a mass $m_q \approx 300 MeV$

- Asymptotic freedom. In high-energy regime (small distance) the coupling constant is tiny, whereas as energy decrease (distance increase) the coupling constant become large.
- **Renormazability**. All orders of pertrubation series could be made finite due to regularization procedure.

Phase diagram of the QCD



Introduction

The main method of QCD analysis is the perturbative technique on the basis of coupling constant.



Lattice QCD



Effective models

At this moment, effective models are the best tool for investigating of dense quark matter and one of the most widely used effective model is the Nambu–Jona-Lazinio (NJL) model. It was originally formulated in 1961 to describe nucleon mass creation via spontaneous breaking of chiral symmetry in analogy to classical superconductivity and was based on nucleons, pions, and scalar σ -mesons. Later, it was reformulated for quarks, where it was shown that the light quarks acquire mass as a result of the spontaneous breaking of chiral symmetry. The Lagrangian of NJL model has the following form:

$$L = \sum_{k=1}^{N} \bar{\psi}_k i \hat{\partial} \psi_k + \frac{G}{2N} \left[\left(\sum_{k=1}^{N} \bar{\psi}_k \psi_k \right)^2 + \left(\sum_{k=1}^{N} \bar{\psi}_k i \gamma_5 \psi_k \right)^2 \right].$$

The specific feature of the model is a four-fermion interaction which leads to spontaneous chiral symmetry breaking.

Original NJL-model Chiral Symmetry

The Lagrangian of NJL model has the following form:

$$L = \sum_{k=1}^{N} \bar{\psi}_k (i\hat{\partial} - m_0)\psi_k + \frac{G}{2N} \left[\left(\sum_{k=1}^{N} \bar{\psi}_k \psi_k \right)^2 + \left(\sum_{k=1}^{N} \bar{\psi}_k i \gamma_5 \psi_k \right)^2 \right].$$

Usually 1/N-approximation is using which is in some sense analogue of the mean-field approximation at $N \to \infty$. The model, as QCD Lagrangian, is also chiral symmetric (at $m_0 = 0$):

$$\psi_k \to e^{i\theta\gamma_5}\psi_k; \qquad (k=1,...,N).$$

Original NJL-model Bosonization

For convenience we can introduce auxiliary boson fields:

$$\sigma_1 = \frac{G}{N}(\bar{\psi}\psi); \qquad \sigma_2 = \frac{G}{N}(\bar{\psi}i\gamma_5\psi).$$

We can do so, due to Hubbard-Stratonovich identity:

$$\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\sigma_{1}\mathcal{D}\sigma_{2}\exp{i\int L_{\sigma}(x)\mathrm{d}x^{4}} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\exp{i\int L_{\psi}(x)\mathrm{d}x^{4}}$$

So the initial Lagrangian takes the following form:

$$L_{\sigma} = \bar{\psi}i\hat{\partial}\psi - \bar{\psi}(\sigma_1 + i\sigma_2\gamma_5)\psi - \frac{N}{2G}(\sigma_1^2 + \sigma_2^2)$$

Here and therein we omit index k for simplicity.

Original NJL-model Bosonization

Thus chiral symmetry in boson fields takes a form:

$$\begin{split} \psi &\to e^{i\gamma_5}\psi_k \\ \sigma &\to \cos\sigma_1 - \sin\sigma_2 \\ \sigma_2 &\to \sin\sigma_1 + \cos\sigma_2 \end{split}$$

The generation functional of the Green's function has the following form:

$$e^{iW[\bar{J},J]} = \int \mathcal{D}\sigma_1 \mathcal{D}\sigma_2 \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{i\int d^4x [L_\sigma(\bar{\psi},\psi,\sigma_1,\sigma_2) + J\bar{\psi} + \bar{J}\psi]}$$

Let's make Legendre transformation:

$$W[\bar{J},J] = \Gamma[\sigma_{1,2},\bar{\psi},\psi] + \int \mathrm{d}^4 x \Big(J(x)\bar{\psi}(x) + \bar{J}(x)\psi(x) \Big)$$

Original NJL-model Effective action

Thus the expression for the effective action has the following form:

$$e^{i\Gamma[\sigma_{1,2}\bar{\psi}\psi]} = \int \mathcal{D}\sigma_1 \mathcal{D}\sigma_2 \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{i\int d^4x L_\sigma(\bar{\psi},\psi,\sigma_1,\sigma_2)}$$

Let's calculate path integral over fermion fields:

$$e^{i\Gamma[\sigma_{1,2}]} = \int \mathcal{D}\sigma_1 \mathcal{D}\sigma_2 e^{i\frac{N}{2G}\int d^4x(\sigma_1^2 + \sigma_2^2)} i\det(i\hat{\partial} - \sigma_1 - i\gamma_5\sigma_2).$$

Implying that $N \gg 1$ we will use mean-field approximation $\sigma(x) \simeq \langle \sigma(x) \rangle$. Also, for simplicity we imply that the fields σ spatially homogeneous i.e. $\langle \sigma(x) \rangle = \sigma = \text{const.}$

Original NJL-model Effective potential

Using well known expression $\det A = \exp({\rm Tr}\ln A)$ one can calculate remaining path integrals:

$$\frac{1}{N}i\Gamma[\sigma_{1,2}] = -\frac{\Omega}{2G}(\sigma_1^2 + \sigma_2^2) - i\operatorname{Tr}\ln(i\hat{\partial} - \sigma_1 - i\gamma_5\sigma_2), \quad \text{where}$$

 Ω is a volume of the space. Finally, it is well known that in mean-field approximation:

$$\Gamma[\sigma_{1,2}] = -V_{\text{eff}}(\sigma_{1,2})\Omega, \quad \text{so}$$

$$\frac{1}{N}V_{\text{eff}}(\Sigma) = \frac{\Sigma^2}{2G} + 2i\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\ln(\Sigma^2 - p^2), \quad \text{where}$$

$$\Sigma = \sqrt{\sigma_1^2 + \sigma_1^2}$$

Original NJL-model Effective potential

Making a Wick rotation $(p_0 \to ip_0)$ and introduce Lorentz invariant cut-off parameter Λ $(p^2 \leq \Lambda^2)$ one can obtain:

$$V_{\text{eff}}(\Sigma) = \frac{N\Sigma^2}{2G} - \frac{N}{16\pi^2} \left\{ \Lambda^4 \ln\left(1 + \frac{\Sigma^2}{\Lambda^2}\right) + \Lambda^2 \Sigma^2 - \Sigma^4 \ln\left(1 + \frac{\Lambda^2}{\Sigma^2}\right) \right\}$$

The ground state of the system corresponds to the global minimum point (GMP) of the $V_{\rm eff}$:

$$\frac{\partial V_{\text{eff}}(\Sigma)}{\partial \Sigma} = 0 = \frac{N\Sigma}{4\pi^2} \left\{ \frac{4\pi^2}{G} - \Lambda^2 + \Sigma^2 \ln\left(1 + \frac{\Lambda^2}{\Sigma^2}\right) \right\}$$

So it is obvious that if $G < G_c = \frac{4\pi^2}{\Lambda^2}$ there is no non-trivial solutions besides $\Sigma = 0$. But in the case of $G > G_c$ there is non-trivial solution.

Thereby, chiral symmetry is spontaneously broken at some value of coupling constant G due to four-fermion interaction.

Original NJL-model Phase portrait

As we interested in dense quark matter with non-zero temperature one should use QFT with non-zero temperature. Thermodynamical potential has the form:

$$\Omega(T,\mu) = -\frac{T}{V}\mathcal{Z} = \frac{T}{V}\ln\operatorname{Tr}\exp\Big(-\frac{1}{T}\int\mathrm{d}^{3}x\big(\mathcal{H}-\mu\psi^{\dagger}\psi\big)\Big),$$

where one should make a Wick rotation and use Matsubara technique:

$$p^0 = i\omega_n = (2n+1)\pi T, \qquad n \in \mathbb{Z}$$

Since the NJL model is a non-renormalizable theory we have to use fitting parameters for the quantitative investigation of the system. We use the following, widely used parameters:

 $m_0 = 5,5 \,\mathrm{MeV};$ $G = 15.03 \,\mathrm{GeV}^{-2};$ $\Lambda = 0.65 \,\mathrm{GeV}.$

In this case at $\mu = \nu = \nu_5 = 0$ one gets for constituent quark mass the value M = 309 MeV.

Original NJL-model Phase portrait

Phase diagram with the gap



Original NJL-model Phase portrait

Phase diagram with the gap



Dense quark matter with chiral and isotopical imbalance Based on the paper: EPJ C79 (2019) no.2, 151 (arXiv:1812.00772) Khunjua, Klimenko, Zhokhov.

Introduction Models with four-fermion interactions

Isospin asymmetry is the well-known property of dense quark matter, which exists in the compact stars and is produced in heavy ion collisions. On the other hand, the chiral imbalance between left- and right- handed quarks is another highly anticipated phenomenon that could occur in the dense quark matter.

To investigate dense quark under these conditions we use Nambu–Jona-Lasinio (NJL) model and take into account:

- Baryon μ_B chemical potential to investigate non-zero density
- Isospin μ_I chemical potential to investigate non-zero isotopic imbalance
- Chiral isospin μ_{I5} chemical potential to investigate chiral isotopic imbalance
- Non-zero bare quark mass $(m_0 \neq 0)$ to promote real threshold to pion condensation phase
- Non-zero temperature $(T \neq 0)$ in order to make our investigation applicable to hot dense quark matter and compare our NJL-model analysis with the known lattice results

Lagrangian of the model

$$\mathcal{L} = \bar{q} \Big[\gamma^{\nu} \mathrm{i} \partial_{\nu} - m_0 + \frac{\mu_B}{3} \gamma^0 + \frac{\mu_I}{2} \tau_3 \gamma^0 + \frac{\mu_{I5}}{2} \tau_3 \gamma^0 \gamma^5 \Big] q + \frac{G}{N_c} \Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 \vec{\tau}q)^2 \Big]$$

Definitions

- q is the flavor doublet $q = (q_u, q_d)^T$
- q_u and q_d are four-component Dirac spinors as well as color N_c -plets^a
- τ_k (k = 1, 2, 3) are Pauli matrices
- m_0 is the diagonal matrix in flavor space with bare quark masses (from the following $m_u = m_d = m_0$)

 a The summation over flavor, color, and spinor indices is implied

Lagrangian of the model

$$\mathcal{L} = \bar{q} \Big[\gamma^{\nu} i \partial_{\nu} - m_0 + \frac{\mu_B}{3} \gamma^0 + \frac{\mu_I}{2} \tau_3 \gamma^0 + \frac{\mu_{I5}}{2} \tau_3 \gamma^0 \gamma^5 \Big] q + \frac{G}{N_c} \Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5 \vec{\tau}q)^2 \Big]$$

Notations

- μ_B is a baryon number chemical potential
- μ_I is taken into account to promote non-zero imbalance between u and d quarks
- μ_{I5} is stands to promote chiral isospin imbalance between $u_{L(R)}$ and $d_{L(R)}$
- $\bullet~G$ is coupling constant

The new notations of chemical potentials

$$\mu \equiv \frac{\mu_B}{3}; \, \nu \equiv \frac{\mu_I}{2}; \, \nu_5 \equiv \frac{\mu_{I5}}{2}$$

Calculation of the TDP

After all possible analytical calculations, we have the following form for the TDP:

$$\begin{split} \Omega(M,\Delta) = & \frac{(M-m_0)^2 + \Delta^2}{4G} - \sum_{i=1}^4 \int_0^\Lambda \frac{p^2 dp}{2\pi^2} |\eta_i| - \\ & T \sum_{i=1}^4 \int_0^\Lambda \frac{p^2 dp}{2\pi^2} \Big\{ \ln(1 + e^{-\frac{1}{T}(|\eta_i| - \mu)}) + \ln(1 + e^{-\frac{1}{T}(|\eta_i| + \mu)}) \Big\}, \end{split}$$

where η_i are the roots of the following polynomial:

$$(\eta^4 - 2a\eta^2 - b\eta + c) (\eta^4 - 2a\eta^2 + b\eta + c) = 0, a = M^2 + \Delta^2 + |\vec{p}|^2 + \nu^2 + \nu_5^2; b = 8|\vec{p}|\nu\nu_5; c = a^2 - 4|\vec{p}|^2 (\nu^2 + \nu_5^2) - 4M^2\nu^2 - 4\Delta^2\nu_5^2 - 4\nu^2\nu_5^2.$$

$$M = \langle \sigma(x) \rangle + m_0, \quad \Delta = \langle \pi_1(x) \rangle, \quad \langle \pi_2(x) \rangle = 0, \quad \langle \pi_3(x) \rangle = 0.$$

Fitting parameters

Since the NJL model is a non-renormalizable theory we have to use fitting parameters for the quantitative investigation of the system. We use the following, widely used parameters:

 $m_0 = 5,5 \,\mathrm{MeV};$ $G = 15.03 \,\mathrm{GeV}^{-2};$ $\Lambda = 0.65 \,\mathrm{GeV}.$

In this case at $\mu = \nu = \nu_5 = 0$ one gets for constituent quark mass the value M = 309 MeV.

Phases

To define the ground state of the system one should find the coordinates (M_0, Δ_0) of the global minimum point (GMP) of the TDP. We also interested in the quark number density: $n_q = -\frac{\partial \Omega(M_0, \Delta_0)}{\partial \mu}$. We have found the following phases in the system:

- $M = 0; \Delta = 0; n_q = 0$ symmetrical phase (it could be realized only in chiral limit $m_0 = 0$)
- $M \neq 0; \Delta = 0; n_q = 0$ chiral symmetry breaking phase (CSB)
- $M \neq 0$; $\Delta \neq 0$; $n_q = 0$ pion condensation phase with zero quark density (**PC**) (M = 0 in chiral lim.)
- $M \neq 0$; $\Delta = 0$; $n_q \neq 0$ chiral symmetry breaking phase with non-zero quark density (**CSB**_d)
- $M \neq 0; \Delta \neq 0; n_q \neq 0$ pion condensation phase with non-zero quark density (**PC**_d)
- $M \approx m_0$; $\Delta = 0$; $n_q \neq 0$ partially restored (CSB) phase with non-zero quark density (CSB_{dr})

Phase portraits of the model

(ν, ν_5) -phase portraits



(ν, ν_5) -phase portraits



 $\mu = 200 \text{ MeV}$

(ν, ν_5) -phase portraits



Certain dual symmetry, that we have observed in the chiral limit, is broken explicitly. Nevertheless duality is still relatively instructive even at the physical point. On the other hand, the results become more physically adequate due to the threshold $\nu^c = m_\pi/2 \approx 70 MeV$ to the PC phase.

Gaps and Density

Slices of the (ν, ν_5) phase portrait at $\mu = 150$ MeV



One can easily see that the quark matter in PC_d -phase has baryon density approximately equal to the density of the ordinary nuclear matter. Duality is still relatively instructive feature to investigate phase portrait.

Slices of the (ν, ν_5) phase portrait at $\mu = 200$ MeV



One can easily see that the quark matter in PC_d -phase has baryon density approximately equal to the density of the ordinary nuclear matter. Duality is still relatively instructive feature to investigate phase portrait.

ν_5 does promote PC_d-phase

 $\nu_5 = 0 \text{ MeV}$



It is evident from the figures that PC_d phase exist in the very small region of the phase portrait.





One can see from that non-zero isospin chiral potential ν_5 does promote the PC_d phase in a wide range of the parameters.





As one could expect, the system restores broken symmetries under non-zero temperature. Nevertheless, it is easy to see that PC_d phase still occupies wide range of parameters in the phase portrait

Critical temperature T_c and CEP existance comparison with other investigations

Non-perturbative investigations

- K. Fukushima, M. Ruggieri, and R. Gatto. *Phys. Rev.*, D81:114031, 2010. (PNJL-model – T_c decrease)
- M. N. Chernodub and A. S. Nedelin. *Phys. Rev.*, D83:105008, 2011. (Linear σ -model – T_c decrease)
- R. Gatto and M. Ruggieri. Phys. Rev., D85:054013, 2012. (PNJL-model – T_c decrease)
- L. Yu, H. Liu, and M. Huang. Phys. Rev., D90(7):074009, 2014. (NJL-model - T_c decrease)
- S.-S. Xu, Z.-F. Cui, B. Wang, Y.-M. Shi, Y.-C. Yang, and H.-S. Zong. *Phys. Rev.*, D91(5):056003, 2015. (DS approach - T_c increase)
- B. Wang, Y.-L. Wang, Z.-F. Cui, and H.-S. Zong. *Phys. Rev.*, D91(3):034017, 2015. (DS approach - T_c increase)
 - Z.-F. Cui, I. C. Cloet, Y. Lu, C. D. et al. Phys. Rev., D94:071503, 2016. (PNJL-model – T_c increase)

Lattice simulations

A. Yamamoto. *Phys. Rev.*, D84:114504, 2011. (Lattice – T_c increase)

 V. V. Braguta, V. A. Goy, E. M. Ilgenfritz, A. Yu. Kotov, A. V. Molochkov, M. Muller-Preussker, and B. Petersson.
JHEP, 06:094, 2015. (Two color lattice - T_c increase)

V. V. Braguta, E. M. Ilgenfritz, A. Yu. Kotov, B. Petersson, and S. A. Skinderev.
Phys. Rev., D93(3):034509, 2016. (Lattice - T_c increase)

- Don't predict critical end-point (CEP) at all;
- 2 T_c increase;
- Heavy π -meson, $m_{\pi} \approx 400 \text{ MeV}$

The source of the ambiguity

$$\begin{split} \Omega(M,\Delta) = & \frac{(M-m_0)^2 + \Delta^2}{4G} - \sum_{i=1}^4 \int_0^\Lambda \frac{p^2 dp}{2\pi^2} |\eta_i| - \\ & T \sum_{i=1}^4 \int_0^{\Lambda/\infty} \frac{p^2 dp}{2\pi^2} \Big\{ \ln(1 + e^{-\frac{1}{T}(|\eta_i| - \mu)}) + \ln(1 + e^{-\frac{1}{T}(|\eta_i| + \mu)}) \Big\}, \end{split}$$

- R. L. S. Farias, D. C. Duarte, G. Krein, and R. O. Ramos. *Phys. Rev.*, D94(7):074011, 2016. (NJL-model – compare the RS)
- L. Yu, H. Liu, and M. Huang. Phys. Rev., D94(1):014026, 2016. (NJL-model – compare the RS)
- M. Ruggieri and G. X. Peng. J. Phys., G43(12):125101, 2016. (QM-model – compare the RS)

M. Frasca.

Eur. Phys. J., C78(9):790, 2018. (non-local NJL-model – T_c increase)

(ν, T) -phase portraits



Qualitatively comparable with the first principle lattice simulation.

 (ν_5, T) -phase portraits at $\nu = 0$ MeV



We have recently shown that introduction of the chiral chemical potential μ_5 into consideration (with the following term in the Lagrangian: $\frac{\mu_5}{2}\bar{q}\gamma^0\gamma^5 q$) leads to an additional dual-symmetry between $\mu_{I5} \leftrightarrow \mu_5$ in the region where $\Delta = 0$. In other words, in the NJL model (1) we can certainly consider μ_{I5} as a μ_5 (only in the pure CSB phase). So we can compare our results with the known lattice calculations with μ_5 [1512.05873] (Braguta et al.).

(ν_5, T) -phase portraits at $\nu = 0$ MeV



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Conclusions

Conclusions

- We investigate dual symmetry feature at the physical point under non-zero temperature
- We have shown that PC_d is robust under both non-zero quark mass and non-zero temperature
- Using duality symmetry we calculate the T_c curve.