Comment on energy dependence of the slope parameter

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Overview

Abstract

- We discuss energy dependence of the slope parameter in elastic proton scattering. It is shown that unitarity generates energy dependence of the slope parameter in geometrical models consistent with the experimental results including recent LHC data.

Introduction

- $$B(s) = \frac{d}{dt}\ln\frac{d\sigma}{dt}|_{t=o}.$$
  Rate of the slope parameter increase getting larger at the LHC.
- Geometrical radii are energy independent, and determined by the minimal mass of the exchanged quanta responsible for the scattering. The generation of the energy–dependent interaction radius is due to unitarity. Unitarization — the total cross–sections rise has been discovered. Pomeron pole contribution with the intercept $\alpha(0)$ greater than unity. Such contribution would finally violate unitarity and therefore requires unitarization.
- Regge model with linear trajectory $\sim (s/s_0)^{\alpha(t)}$ – diffraction cone shrinkage ab initio, $B(s)$ logarithmically increases with the energy, $B(s) \sim \alpha'(0)\ln(s/s_0)$, $\alpha'(0) \neq 0$, butunitarity requires its double logarithmic asymptotic growth, $B(s) \sim \ln^2(s/s_0)$ if the total cross-section saturates the Froissart-Martin bound — slope of the Pomeron trajectory $\alpha'(0)$ is an energy–dependent "effective" function.

Geometrical models and the slope parameter

- Second reason for unitarization (besides total cross–section increase). Input amplitude itself does not imply growth of slope parameter with energy in geometrical models. Unitarization generates energy dependence of the slope parameter and makes energy dependence of the slope parameter consistent with experimental trend at all energies.
- In geometrical models an input amplitude is an overlap of the matter distributions of the colliding hadrons $D_1 \otimes D_2$ (Chou and Yang). Factorization results also from the tower diagrams calculations in the electrodynamics (Cheng and Wu).
- $B(s)$, is determined by the mean value of the impact parameter squared b^2, (real part is neglected)

$$\langle b^2 \rangle_{tot} = \frac{\int_0^\infty b^3 dbf(s,b)}{\int_0^\infty bdbf(s,b)}. \quad (1)$$

Slope parameter

- Unitarity
$$\mathrm{Im}F(s,t) = H_{el}(s,t) + H_{inel}(s,t),$$
- $$B(s) = B_{el}(s) + B_{inel}(s).$$
- The functions $H_{el,inel}$ can be obtained on the basis of the impact parameter analysis with help of Fourier–Bessel transformation.
- $$B_{el,inel}(s) \sim \frac{\sigma_{el,inel}}{\sigma_{tot}(s)} \langle b^2 \rangle_{el,inel}(s).$$
- $B(s)$ is determined by $\sigma_{el,inel}$ and average values $\langle b^2 \rangle_{el,inel}(s)$. Averaging is going over corresponding overlap functions. Different unitarization schemes provide different asymptotics for $B_{inel}(s)$.

Rational (U-matrix) unitarization

- $$f(s,b) = u(s,b)/[1+u(s,b)], \tag{2}$$
  where u is a non-negative function.
- In the geometrical models the $u(s,b)$ has a factorized form:
  $$u(s,b) = g(s)\omega(b), \tag{3}$$
  $g(s) \sim s^\lambda$ at large values of s and $\omega(b)$ exponentially decreases of at $b \to \infty$. The power dependence on energy guarantees unitarity saturation $f \to 1$ at fixed b and $\sigma_{tot} \sim \ln^2 s$.
- $$B_{el}(s) \sim \ln^2 s \tag{4}$$
- $$B_{inel}(s) \sim \ln s \tag{5}$$
  since $\langle b^2 \rangle_{el,inel}(s) \sim \ln^2 s$ and $\sigma_{el,tot}(s) \sim \ln^2 s$ while $\sigma_{inel}(s) \sim \ln s$ at $s \to \infty$.

Form of $u(s, B)$

- In the framework of the geometrical considerations a typical way to construct the function $\omega(b)$ as it was already noted is to represent it as a convolution of the two matter distributions in transverse plane as it was proposed by Chou and Yang:

$$\omega(b) \sim D_1 \otimes D_2 \equiv \int D_1(\mathbf{b}_1) D_2(\mathbf{b} - \mathbf{b}_1). \tag{6}$$

  This function can also be constructed by taking into account the hadron quark structure.

$$\omega(b) \sim \exp(-\mu b). \tag{7}$$

Shadow unitarization

- There is no way to discriminate elastic and inelastic contributions into $B(s)$ at $s \to \infty$ when only the shadow scattering mode is assumed. In this mode $f \to 1/2$ at $s \to \infty$ and b-fixed. In this case both contributions $B_{el}(s)$ and $B_{inel}(s)$ are proportional to $\ln^2 s$ at $s \to \infty$ and have smilar energy dependencies at finite energies. It is a typical situation with $B(s)$ behavior in the unitarization schemes based on eikonal or continued unitarity. At present one can rely on impact parameter analysis to discriminate different unitarization schemes. Indeed, much higher energies are needed for such a discrimination under that study of the integrated observables.

Modern energies

- At the highest LHC energy the function $u(s, b = 0)$ approaches unity or has a little bit higher value. Below such energies one can expand the scattering amplitude f over the function u according to Eq. (2).

$$B(s) \simeq a + bs^{\lambda}. \quad (8)$$

- Comparison of Eq. (8) with the experimental data (See Fig.1) allows one to get a reasonable value for the χ^2/ndf around 0.8 and the value of a parameter λ to be at $\lambda \simeq 0.1$.

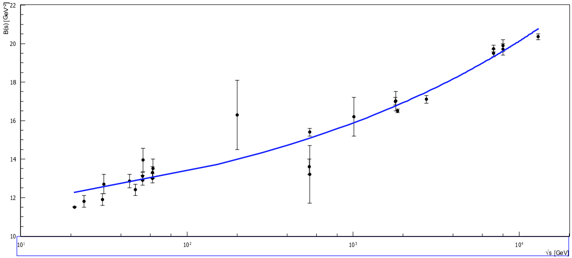

Figure: Energy dependence of slope parameter $B(s)$ in pp and $\bar{p}p$ collisions.

Conclusion

- Adhering to the geometrical models, we should conclude also that the observed total cross-section increase at the accelerator energies is of a preasymptotic nature. Measured recently small value of the real to imaginary parts ratio of the elastic scattering amplitude can be interpreted as a slowdown of total cross-section increase.

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