# Dualities and inhomogeneous phases in QCD phase diagram

Roman N. Zhokhov

IHEP, IZMIRAN

IHEP

January 29, 2019


© Io Amo La Sicilia

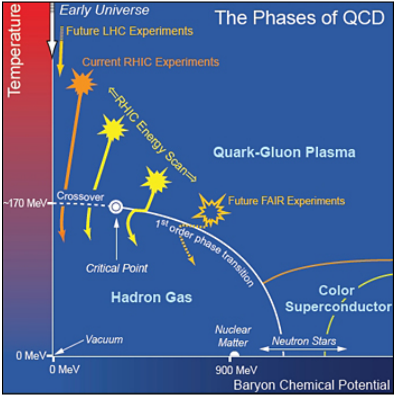
# Group

T.G. Khunjua, MSU and K.G. Klimenko, IHEP

# QCD at finite temperature and nonzero chemical potential

QCD at nonzero temperature and baryon chemical potential plays a fundamental role in many different physical systems. (QCD at extreme conditions)

- neutron stars
- heavy ion collision experiments
- Early Universe



# Methods of dealing with QCD

Methods of dealing with QCD

- First principle calcaltion – lattice Monte Carlo simulations, LQCD

- Effective models

  Nambu–Jona-Lasinio model NJL

**Lattice QCD**

**non-zero baryon chemical potential $\mu_B$**

**sign problem — complex determinant**

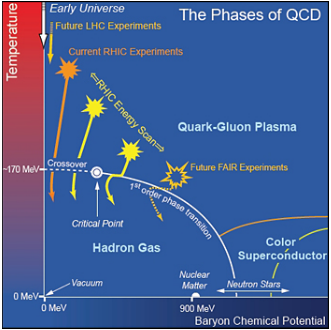
$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

Methods of dealing with QCD

- First principle calcultion – lattice Monte Carlo simulations, LQCD

- Effective models

  Nambu–Jona-Lasinio model NJL



# NJL model

NJL model can be considered as **effective field theory for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

Parameters $G$, $\Lambda$, $m_0$

# NJL model

NJL model can be considered as **effective field theory for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

Parameters $G$, $\Lambda$, $m_0$

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

Relative simplicity allow to consider hot and dense QCD in the framework of NJL model and explore the QCD phase structure (diagram).

# chiral symmetry breaking

**the QCD vacuum has non-trivial structure** due to non-perturbative interactions among quarks and gluons

lattice simulations $\Rightarrow$ **condensation of quark and anti-quark pairs**

$$\langle \bar{q} q \rangle \neq 0, \quad \langle \bar{u} u \rangle = \langle \bar{d} d \rangle \approx (-250 MeV)^3$$

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^\nu \mathrm{i}\partial_\nu q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\Big]$$

$$q \to e^{i\gamma_5 \alpha} q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho - \sigma - \mathrm{i}\gamma^5\pi\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi^2\Big].$$

**Chiral symmetry breaking**

$1/N_c$ expansion, leading order

$$\langle \bar{q}q \rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho - \langle\sigma\rangle\Big]q$$

# Different types of chemical potentials: dense matter with isotopic imbalance

**Baryon chemical potential $\mu_B$**

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

# Different types of chemical potentials: dense matter with isotopic imbalance

**Baryon chemical potential $\mu_B$**

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

**Isotopic chemical potential $\mu_I$**

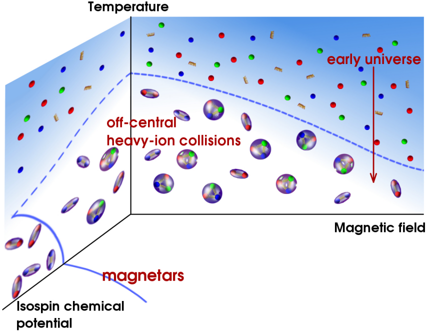
Allow to consider systems with isotopic imbalance.

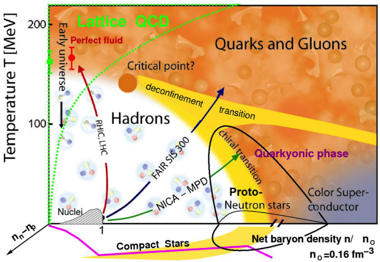
$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

The corresponding term in the Lagrangian is $\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q$

# QCD phase diagram with isotopic imbalance

neutron stars, heavy ion collisions have isotopic imbalance



# Different types of chemical potentials: chiral imbalance

**chiral (axial) chemical potential**

Allow to consider systems with chiral imbalance (difference between between densities of left-handed and right-handed quarks).

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

**chiral (axial) isotopic chemical potential**

Allow to consider systems with chiral isospin imbalance

$$\mu_{I5} = \mu_{u5} - \mu_{d5}$$

so the corresponding density is

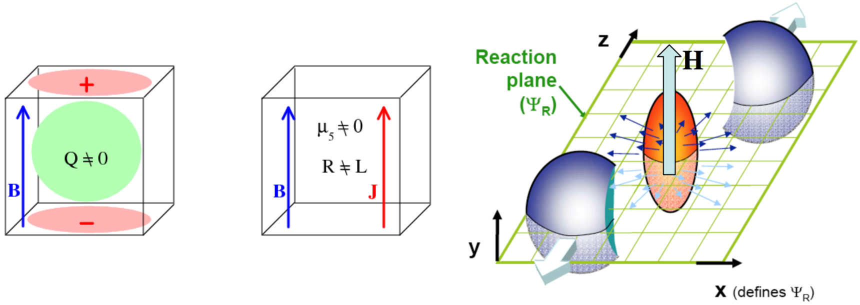
$$n_{I5} = n_{u5} - n_{d5}$$

$$n_{I5} \quad \longleftrightarrow \quad \mu_{I5}$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2} \bar{q} \tau_3 \gamma^0 \gamma^5 q$

If one has all four chemical potential, one can consider different densities $n_{uL}$, $n_{dL}$, $n_{uR}$ and $n_{dR}$

# Chiral magnetic effect



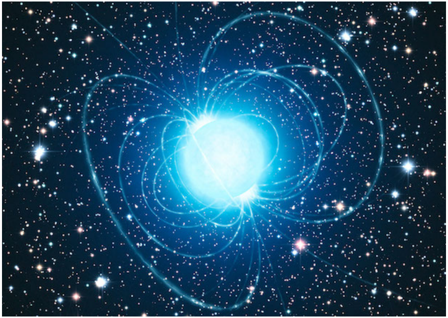

$$\vec{J} = c\mu_5 \vec{B}, \qquad c = \frac{e^2}{2\pi^2}$$


A. Vilenkin, PhysRevD.22.3080,

K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033 [arXiv:0808.3382 [hep-ph]].

# Chiral separation effect

Chiral imbalance could appear in compact stars




$$\vec{J}_5 = c\mu\vec{B}, \qquad c = \frac{e^2}{2\pi^2}$$

there is current and there is $n_5$ and $n_{I5}$

# Model and its Lagrangian

We consider a NJL model, which describes dense quark matter with two massless quark flavors ($u$ and $d$ quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]q +$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

$q$ is the flavor doublet, $q = (q_u, q_d)^T$, where $q_u$ and $q_d$ are four-component Dirac spinors as well as color $N_c$-plets; $\tau_k$ $(k = 1, 2, 3)$ are Pauli matrices.

# Homogeneous case

# Equivalent Lagrangian

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^1 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big].$$

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates $x$,

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0. \quad (1)$$

where $M$ and $\Delta$ are already constant quantities.

# thermodynamic potential

the thermodynamic potential can be obtained in the large $N_c$ limit

$$\Omega(M, \Delta)$$

Projections of the TDP on the $M$ and $\Delta$ axes

No mixed phase $(M \neq 0, \Delta \neq 0)$

it is enough to study **the projections** of the TDP on the $M$ and $\Delta$

**projection of the TDP on the $M$ axis** $F_1(M) \equiv \Omega(M, \Delta = 0)$

**projection of the TDP on the $\Delta$ axis** $F_2(\Delta) \equiv \Omega(M = 0, \Delta)$

## Dualities

The TDP (phase daigram) is invariant

Interchange of condensates

matter content

$$\Omega(C_1, C_2, \mu_1, \mu_2)$$

$$\Omega(C_1, C_2, \mu_1, \mu_2) = \Omega(C_2, C_1, \mu_2, \mu_1)$$

# Dualities of the TDP

The TDP is invariant with respect to the so-called duality transformations (dualities)

**1) The main duality**

$$\mathcal{D}: \; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

$$\nu \longleftrightarrow \nu_5 \text{ and PC} \longleftrightarrow \text{CSB}$$

**2) Duality in the CSB phenomenon**

$$F_1(M) \text{ is invariant under } \mathcal{D}_M \; : \; \nu_5 \leftrightarrow \mu_5$$

**3) Duality in the PC phenomenon**

$$F_2(\Delta) \text{ is invariant under } \mathcal{D}_\Delta \; : \; \nu \leftrightarrow \mu_5$$

PC phenomenon breaks $\mathcal{D}_M$ and CSB phenomenon $\mathcal{D}_\Delta$ duality

# Dualities in different approaches

- **Large N$_c$ orbifold equivalences** connect gauge theories with different gauge groups and **matter content** in the large N$_c$ limit.

  M. Hanada and N. Yamamoto,
  JHEP 1202 (2012) 138, arXiv:1103.5480 [hep-ph],
  PoS LATTICE **2011** (2011), arXiv:1111.3391 [hep-lat]

# Dualities in large $N_c$ orbifold equivalences

two gauge theories with gauge groups $G_1$ and $G_2$ with $\mu_1$ and $\mu_2$

Duality

$$G_1 \longleftrightarrow G_2,\ \mu_1 \longleftrightarrow \mu_2$$

$G_2$ is sign problem free

$G_1$ has sign problem, can not be studied on lattice

# Dualities in large $N_c$ limit of NJL model

$$\Omega(C_1, C_2, \mu_1, \mu_2)$$

Duality

$$C_1 \longleftrightarrow C_2,$$

$$\mu_1 \longleftrightarrow \mu_2$$

QCD with $\mu_1$ —- sign problem free,
and with $\mu_2$ has sign problem, can not be studied on lattice

# Inhomogeneous case

In vacuum the quantities $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate $x$.

in a dense medium the ground state expectation values of bosonic fields might depend on spatial coordinates

CDW ansatz for CSB
the single-plane-wave LOFF ansatz for PC

$$\langle\sigma(x)\rangle = M\cos(2kx^1), \quad \langle\pi_3(x)\rangle = M\sin(2kx^1),$$
$$\langle\pi_1(x)\rangle = \Delta\cos(2k'x^1), \quad \langle\pi_2(x)\rangle = \Delta\sin(2k'x^1)$$

equivalently

$$\langle\pi_\pm(x)\rangle = \Delta e^{\pm 2k'x^1}$$

# Duality

**Duality in homogeneous case has been shown**

$$\mathcal{D}: \;\; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

**In (1+1) case duality was shown as well**

# Duality

**Duality in inhomogeneous case is shown**

$$\mathcal{D}_I : \qquad M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5, \quad k \longleftrightarrow k'. \tag{2}$$

# Duality

It is interesting feature

Duality is not just accidental property

but deep property of the phase structure

not automatic

# Two other dualities

**Two other dualities are valid only in homogeneous case**

**Duality in the CSB phenomenon** $\mathcal{D}_M \,:\, \nu_5 \leftrightarrow \mu_5$

**Duality in the PC phenomenon**

$$\mathcal{D}_\Delta \,:\, \nu \leftrightarrow \mu_5$$

# Physical point

## non-zero current quark masses

# Physical point

$$L = \bar{q}\left[\gamma^{\nu}\mathrm{i}\partial_{\nu} + \mu_i\Gamma^i - m_0\right]q + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\right]$$

current quark mass $m_0 \neq 0$

# Physical point

Anzats is

$$\langle \sigma(x) \rangle = M \cos(2kx^1) - m_0, \quad \langle \pi_3(x) \rangle = M \sin(2kx^1)$$

$$\langle \pi_1(x) \rangle = \Delta \cos(2k'x^1), \quad \langle \pi_2(x) \rangle = \Delta \sin(2k'x^1)$$

Andersen, Adhikari 2016 (1+1) dim
Phys. Rev. D 95, 054020 (2017)

$$\Omega(M, \Delta, k, k') = \frac{M^2 - 2m_0 M \delta_{k,0} + m_0^2 + \Delta^2}{4G} + \mathrm{i} \int \frac{d^4 p}{(2\pi)^4} \ln \overline{D}(p),$$

# Duality and the physical point

**Duality in the physical point is approximate**

in the leading order of large N approximation

**but in inhomogeneous case it is exact**

# The Strength of Duality

# Isospin asymmetric dense matter

**Non-zero baryon density** $\mu_B \neq 0$

**Non-zero isospin density** $\mu_I \neq 0$

## Inhomogeneous phases for dense matter

**homogeneous phases** — **vacuum** $\mu_B = 0$

**Inhomogeneous phases** — $\mu_B \neq 0$

**There have been found a lot of evidence of inhomogeneous phases at non-zero bayon density**

# $(\mu_B/3, T)$ phase diagram

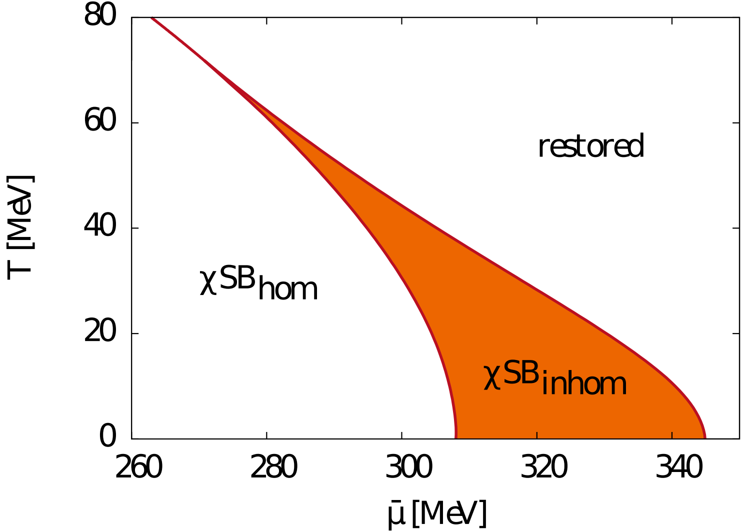



Figure: $(\mu_B/3, T)$ phase diagram with inhomogeneous CSB phase

# Isospin asymmetric dense matter

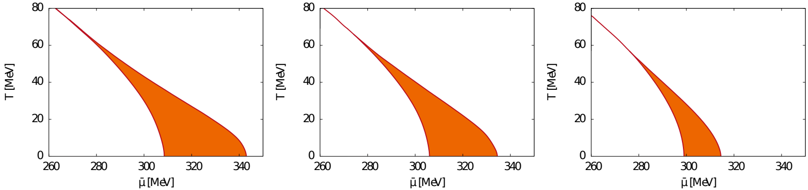
**Non-zero baryon and isospin density** $\mu_B \neq 0$, $\mu_I \neq 0$

**CSB phase — inhomogeneous (CDW anzatz)**

**PC phase — homogeneous**

Buballa, Nowakowski, Carignano, Wambach

# Isospin asymmetric dense matter




$|\mu_I| = 20\,\text{MeV}$ $\quad$ $|\mu_I| = 60\,\text{MeV}$ $\quad$ $|\mu_I| = 120\,\text{MeV}$

Figure: Phase diagram in the $\bar{\mu} - T$ plane for three different values of $\mu_I$. The shaded areas indicate the regions where a CDW-like modulation of the condensates is favored over a homogeneous solution.

Nowakowski, Buballa, Carignano, Wambach arXiv:1506.04260 [hep-ph]

# Physical point

It was all done in the chiral limit

What about physical point

# Physical point

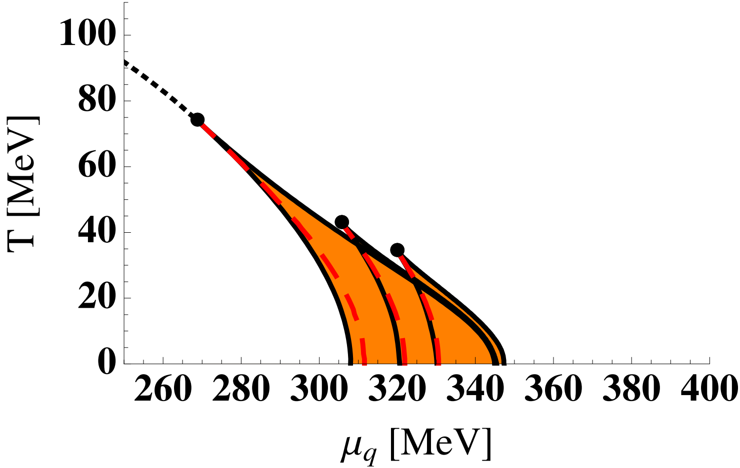



Figure: Same plot for m = 5 MeV and m = 10 MeV.
D. Nickel Phys.Rev.D80:074025,2009

# Isospin asymmetric dense matter

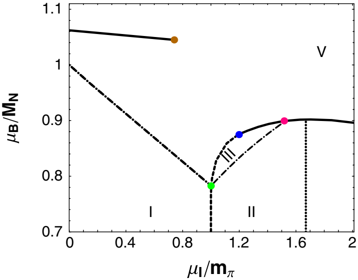
**Non-zero baryon and isospin density** $\mu_B \neq 0$, $\mu_I \neq 0$

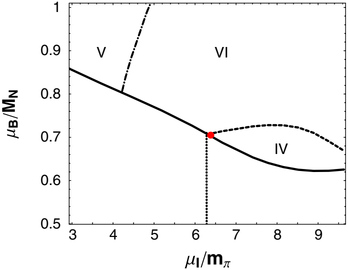
**CSB phase — homogeneous**

**PC phase — inhomogeneous**

Cheng-fu Mu, Lian-yi He, and Yu-xin Liu

# Isospin asymmetric dense matter




$\mu_I < 2m_\pi$ $\qquad$ $\mu_I > 3m_\pi$

Figure: Phase diagram in the $\mu - \mu_I$ plane. IV is ICPC phase; left for $\mu_I < 2m_\pi$ and right for $\mu_I > 3m_\pi$.
Cheng-fu Mu, Lian-yi He, and Yu-xin Liu Phys. Rev. D 82, 056006 2010

**Non-zero baryon and isospin density $\mu_B \neq 0$, $\mu_I \neq 0$**

**CSB phase — inhomogeneous**

**PC phase — inhomogeneous**

**Let us combine the obtained results to get full $(\mu, \mu_I)$ phase diagram**

# Isospin asymmetric dense matter

**It is possible**

**ICSB and ICPC phases do not intersect almost anywhere**

## schematic $(\nu, \mu)$-phase diagram

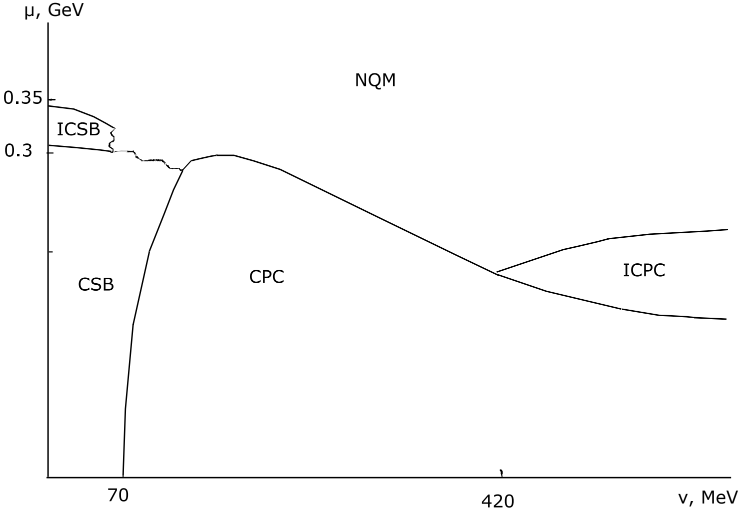



Figure: Combined schematic $(\nu, \mu)$-phase diagram.

to get $(\nu_5, \mu)$-phase diagram, one just need to take the $(\nu, \mu)$-phase diagram and make the following transformations:

(i) exchange axis $\nu$ to the axis $\nu_5$,

(ii) rename the phases ICSB $\leftrightarrow$ ICPC, CSB $\leftrightarrow$ CPC, and NQM phase stays intact here

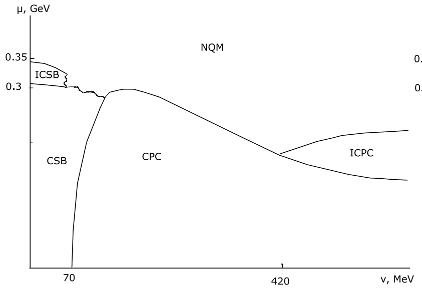



Figure: $(\nu, \mu)$-phase diagram

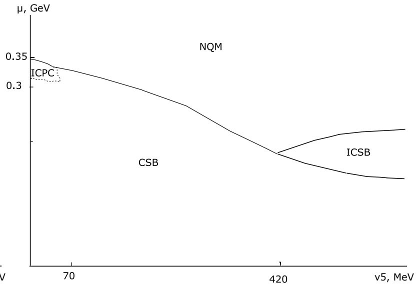



Figure: $(\nu_5, \mu)$-phase diagram

They are dualy conjugated to each other

# Conclusions

- Showed that there is duality in inhomogeneous case

  -Duality is not just **accidental property** but **deep property** of the phase structure

  - Two other dualities is valid only in homogeneous case

  - Duality is approximate in the physical point, exact for inhomogeneous phases

- Full phase diagram $(\mu_B, \mu_I)$

  dense $\mu_B \neq 0$ and isotopically asymmetric $\mu_I \neq 0$ quark matter

- Full phase diagram $(\mu_B, \nu_5)$

  dense $\mu_B \neq 0$ and chirally asymmetric $\mu_{I5} \neq 0$ quark matter

# Conclusions

- To get $(\mu_B, \nu_5)$ phase diagram **only duality was used without calculations**
- Duality is **not just entertaining mathematical property** but an **instrument with very high predictivity power**
- $(\mu_B, \nu_5)$ phase diagram is **quite rich** and contains various **inhomogeneous phases**

# Thanks for the attention