

Dualities of the $(N_c = 2, 3, \infty)$ QCD phase diagram: chiral imbalance, baryon density



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Russian
Science
Foundation



Фонд развития
теоретической физики
и математики

K.G. Klimenko, IHEP

T.G. Khunjua, University of Georgia

details can be found in

JHEP 06 (2020) 148 arXiv:2003.10562 [hep-ph]
Phys.Rev. D100 (2019) no.3, 034009 arXiv: 1904.07151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

The work is supported by

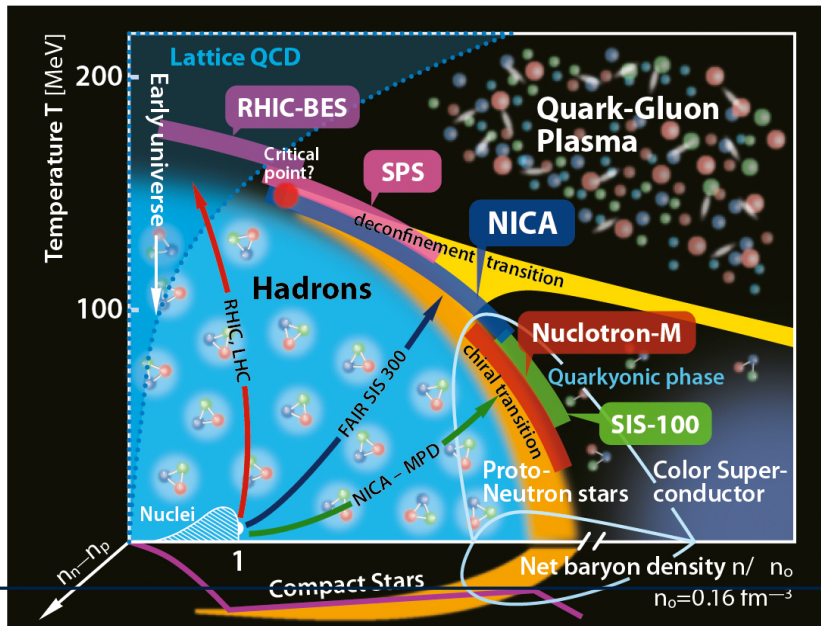
- ▶ Russian Science Foundation (RSF)

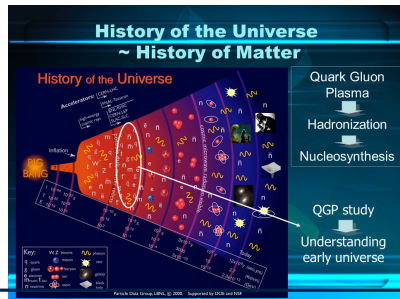
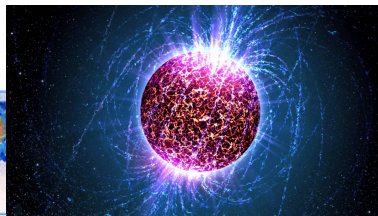
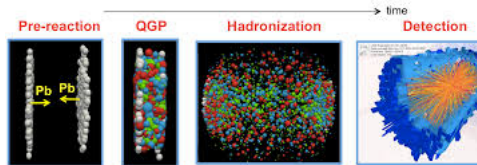
under grant number 19-72-00077

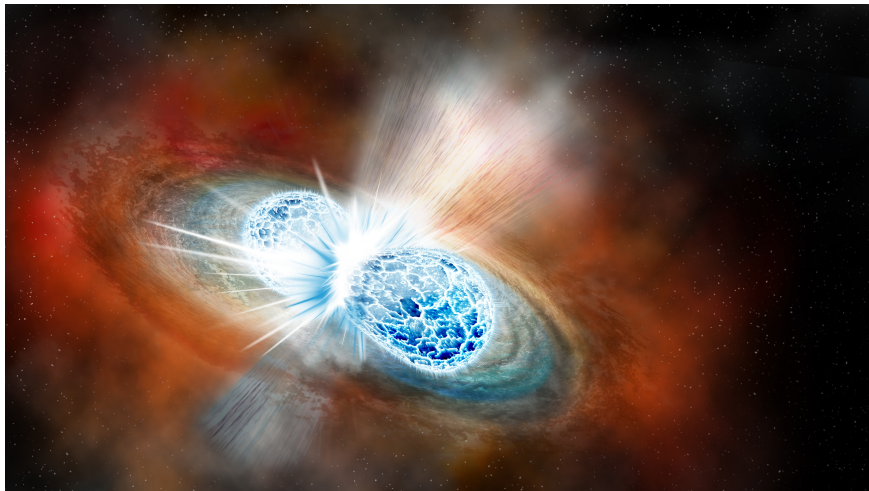


- ▶ Foundation for the Advancement of Theoretical Physics and Mathematics



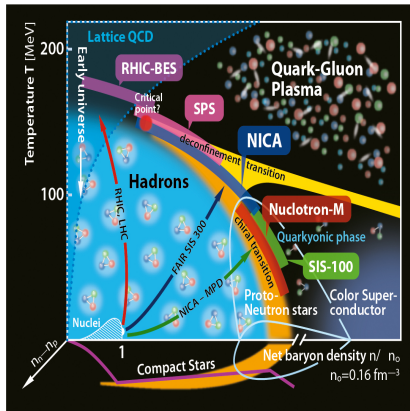




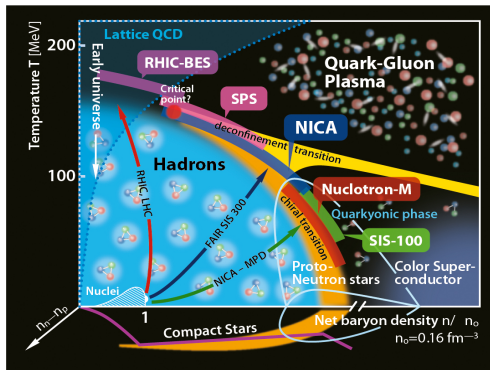


QCD at T and μ
(QCD at extreme conditions)

- heavy ion collisions
- neutron stars
- Early Universe
- Neutron star mergers



- ▶ Chiral symmetry breaking
 - ▶ Confinement
-



Two main phase transitions

- confinement-deconfinement
- chiral symmetry breaking phase—chiral symmetric phase

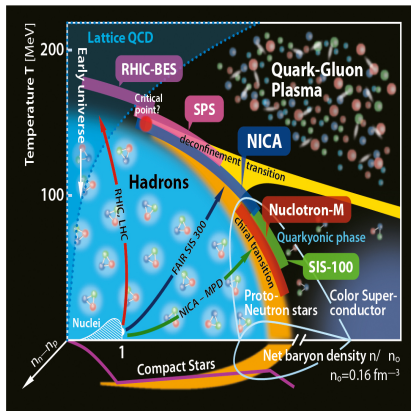
QCD at T and μ

(QCD at extreme conditions)

- ▶ neutron stars
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Methods of dealing with QCD

- ▶ First principle calculation
– lattice QCD
- ▶ Effective models
- ▶ DSE, FRG
- ▶

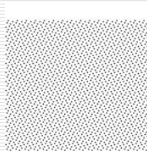




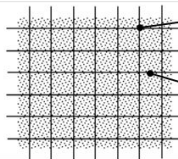
QCD on a space-time lattice

K. G. Wilson 1974

Space-time continuum



Space-time lattice



q_n

quark fields on
lattice sites

$U_{n\mu}$

gluon fields on
lattice links

□ Feynman path integral

■ Action $S_{QCD} = \frac{1}{g_s^2} \sum_P \text{tr}(UUUU) + \sum_f \bar{q}_f (\gamma \cdot U + m_f) q_f$

- Physical quantities as **integral averages**



*Monte Carlo
Evaluation of
the path integral*

$$\langle O(U, \bar{q}, q) \rangle = \frac{1}{Z} \int \prod_{n\mu} dU_{n\mu} \prod_n d\bar{q}_n dq_n O(U, (\bar{q}, q)) e^{-S_{QCD}}$$

It is well known that **at non-zero baryon chemical potential μ_B lattice simulation** is quite challenging due to the **sign problem**
complex determinant

$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

Methods of dealing with QCD

- ▶ perturbative QCD, pQCD, high energy
- ▶ First principle calculation – lattice Monte Carlo simulations, LQCD

- ▶ **Effective models**

Chiral perturbation theory χPT

Nambu–Jona-Lasinio model, NJL

Polyakov-loop extended Nambu–Jona-Lasinio model PNJL

Quark meson model

NJL model can be considered as **effective model for QCD**.

the model is **nonrenormalizable**

Valid up to $E < \Lambda \approx 1 \text{ GeV}$ $\mu, T < 600 \text{ MeV}$

Parameters G, Λ, m_0 chiral limit $m_0 = 0$

dof– **quarks**, no gluons only **four-fermion interaction**

attractive feature — dynamical CSB

the main drawback – lack of confinement (PNJL)

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^\nu i\partial_\nu q + \frac{G}{N_c} \left[(\bar{q}q)^2 + (\bar{q}i\gamma^5 q)^2 \right]$$

$$q \rightarrow e^{i\gamma^5 \alpha} q$$

continuous symmetry

$$\tilde{\mathcal{L}} = \bar{q} \left[\gamma^\rho i\partial_\rho - \sigma - i\gamma^5 \pi \right] q - \frac{N_c}{4G} \left[\sigma^2 + \pi^2 \right].$$

Chiral symmetry breaking

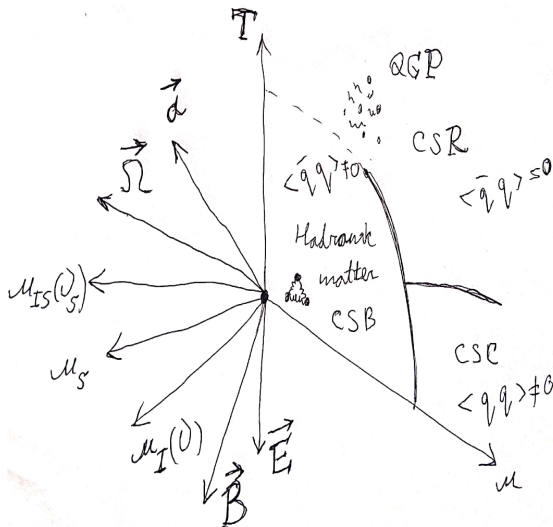
$1/N_c$ expansion, leading order

$$\langle \bar{q}q \rangle \neq 0$$

$$\langle \sigma \rangle \neq 0 \quad \longrightarrow \quad \tilde{\mathcal{L}} = \bar{q} \left[\gamma^\rho i\partial_\rho - \langle \sigma \rangle \right] q$$

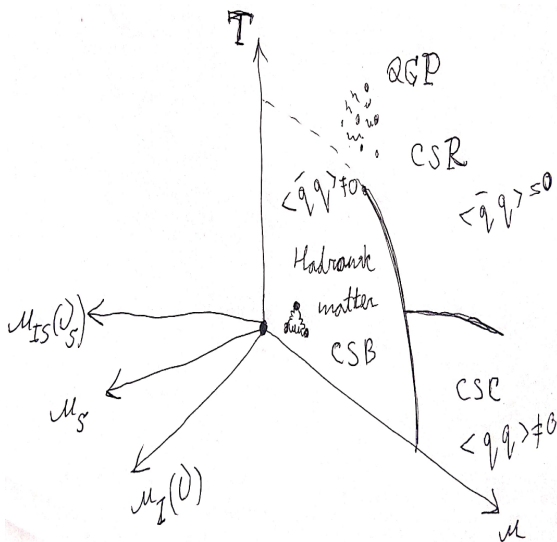
More than just QCD at (μ, T)

- ▶ more chemical potentials μ_i
- ▶ magnetic fields
- ▶ rotation of the system $\vec{\Omega}$
- ▶ acceleration \vec{a}
- ▶ finite size effects (finite volume and boundary conditions)



More than just QCD at (μ, T)

- ▶ **more chemical potentials** μ_i
- ▶ magnetic fields
- ▶ rotation of the system
- ▶ acceleration
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Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3} \bar{q} \gamma^0 q = \mu \bar{q} \gamma^0 q,$$

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3} \bar{q} \gamma^0 q = \mu \bar{q} \gamma^0 q,$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2} \bar{q} \gamma^0 \tau_3 q = \nu (\bar{q} \gamma^0 \tau_3 q)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

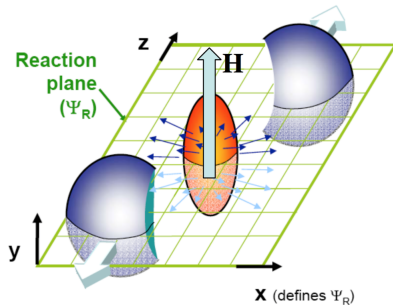
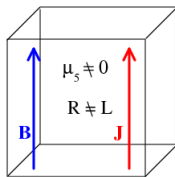
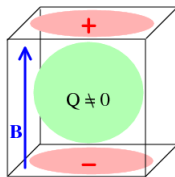
chiral (axial) chemical potential

Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

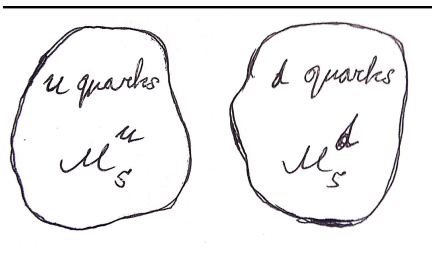
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$



$$\vec{J} \sim \mu_5 \vec{B},$$



$$\mu_5^u \neq \mu_5^d \quad \text{and} \quad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2} \bar{q} \tau_3 \gamma^0 \gamma^5 q = \nu_5 (\bar{q} \tau_3 \gamma^0 \gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \quad n_{I5} \longleftrightarrow \nu_5$$

- Chiral isospin imbalance n_{I5} and hence μ_{I5} can be **generated by** $\vec{E} \parallel \vec{B}$

(for n_5 and μ_5 M. Ruggieri, M. Chernodub, H. Warringa et al)

- in **dense quark matter**
 - Chiral separation effect
(Thanks for the idea to Igor Shovkovy)
 - Chiral vortical effect

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \quad \nu = \frac{\mu_I}{2}, \quad \mu_5, \quad \nu_5 = \frac{\mu_{I5}}{2}$$

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \quad \nu = \frac{\mu_I}{2}, \quad \mu_5, \quad \nu_5 = \frac{\mu_{I5}}{2}$$

Two colour QCD case

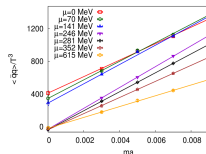
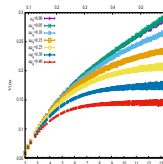
QC_2D

There are similar transitions:

- confinement/deconfinement
- chiral symmetry breaking/restoration

At large μ

- deconfinement
- chiral symmetry is restored



At large T

- deconfinement
- chiral symmetry is restored

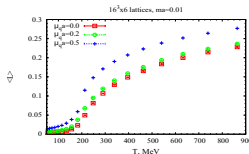


Figure 1: Polyakov loop as a function of T for three values of the baryon chemical potential.

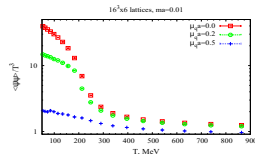


Figure 2: Chiral condensate as a function of T for three values of the baryon chemical potential. The ordinate axis is given on a logarithmic scale.

A lot of quantities coincide up to few dozens percent

SU(2)

SU(3)

Critical temperature

Phys. Lett. B712 (2012) 279-283, JHEP 02 (2005) 033

$$T_c/\sqrt{\sigma}=0.7092(36)$$

$$T_c/\sqrt{\sigma}=0.6462(30)$$

Topological susceptibility

Nucl.Phys.B 715 (2005) 461-482

$$\chi^{\frac{1}{4}}/\sqrt{\sigma}=0.3928(40)$$

$$\chi^{\frac{1}{4}}/\sqrt{\sigma}=0.4001(35)$$

Shear viscosity

JHEP 1509 (2015) 082, Phys.Rev. D 76 (2007) 101701

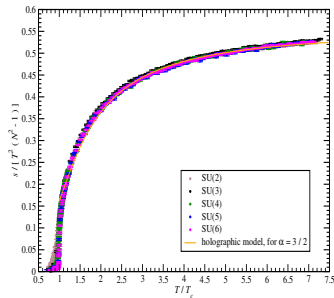
$$\eta/s=0.134(57)$$

$$\eta/s= 0.102(56)$$

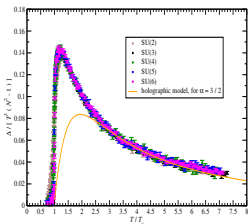
Thermodynamic properties are similar

The **entropy density** per gluon degree of freedom ($N^2 - 1$), in units of T^3 , as a **function of the temperature** (in units of T_c), for the **gauge group** $SU(N_c)$. These are the results obtained from lattice simulations.

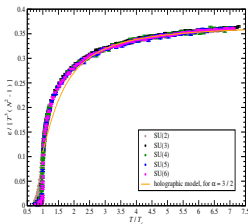
Entropy density



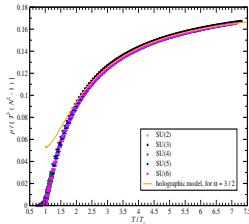
Trace of the energy-momentum tensor

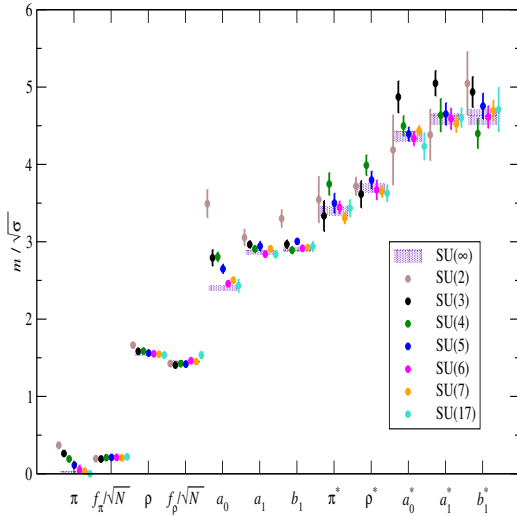


Energy density



Pressure





Mesonic spectrum at different N_c are similar

Physics Reports 526 (2013) 93-163,

Masses (in units of the square root of the string tension σ) of various meson states, and the decay constants of the pion and the rho meson, obtained from a quenched computation in the chiral limit.

Some properties of dense medium are similar

diquark condensate Δ at asymptotically large baryon chemical potential

$$\Delta = b\mu_B g^{-5} \exp\left(-\frac{c}{g}\right)$$

b and c are constants, g is the small gauge coupling

D. T. Son, Phys. Rev. D 59 (1999) 094019

T. Schaefer, F. Wilczek, Phys. Rev. D 60, 114033 (1999), arXiv:hep-ph/9906512

D.T. Son, M.A. Stephanov, arXiv:hep-ph/0011365

Quarks have baryon number one-half $B = 1/2$

Baryons consist of two quarks (diquarks) instead of three.

In effective NJL type models

Mesons (quark-antiquark)		dynamic hadronic
	are	degrees of freedom
Baryons (diquarks)		

This can teach us important lessons about phase transitions of quark-hadron matter at non-zero baryon density

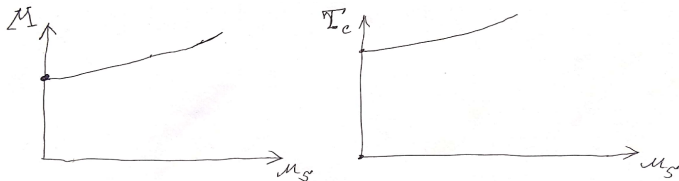
After bosonisation in NJL type models diquarks starts to be dynamical and investigations of $N_c = 2$ NJL type models, where both mesons (quark-antiquark composite fields) and baryons (diquarks) acts as dynamic hadronic degrees of freedom, can teach us important lessons about phase transitions of quark-hadron matter at non-zero baryon density.

A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

lattice results show the **catalysis**

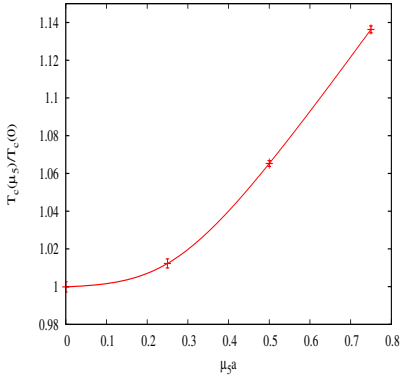
(ITEP lattice group, V. Braguta, A. Kotov, et al)

QCD at non-zero μ_5 

catalysis of CSB by chiral imbalance:

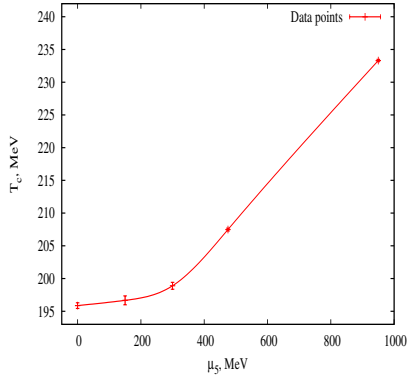
- ▶ increase of $\langle \bar{q}q \rangle$ as μ_5 increases
- ▶ increase of critical temperature T_c of chiral phase transition (crossover) as μ_5 increases

SU(2)



V. Braguta, A. Kotov et al, *JHEP* 1506, 094
(2015), PoS LATTICE 2014, 235 (2015)

SU(3)



V. Braguta, A. Kotov et al, *Phys. Rev. D* 93,
034509 (2016), arXiv:1512.05873 [hep-lat]

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\begin{aligned}\mathcal{L} = & i\bar{q}\gamma^\nu\partial_\nu q + \bar{q}\mathcal{M}\gamma^0 q + \\ & + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + (\bar{q}i\gamma^5\sigma_2\tau_2q^c)(\bar{q}^ci\gamma^5\sigma_2\tau_2q)\right]\end{aligned}$$

q is the flavor doublet, $q = (q_u, q_d)^T$,

q_u and q_d are four-component Dirac spinors.

$q^c = C\bar{q}^T$, $\bar{q}^c = q^TC$ are charge-conjugated spinors.

$$\mathcal{L} = i\bar{q}\gamma^\nu\partial_\nu q + \bar{q}\mathcal{M}\gamma^0 q + \\ + G \left[(q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + (\bar{q}i\gamma^5\sigma_2\tau_2 q^c)(\bar{q}^c i\gamma^5\sigma_2\tau_2 q) \right]$$

quark matter with nonzero

baryon density $n_B = (n_u + n_d)/3 \equiv n_q/3$ (conjugated to μ_B)

isospin density $n_I = (n_u - n_d)/2$ (conjugated to μ_I)

chiral isospin density $n_{I5} = (n_{u5} - n_{d5})/2$ (conjugated to μ_{I5})

$$\bar{q}\mathcal{M}\gamma^0 q \equiv \bar{q} \left[\frac{\mu_B}{2} + \frac{\mu_I}{2}\tau_3 + \frac{\mu_{I5}}{2}\gamma^5\tau_3 + \mu_5\gamma^5 \right] \gamma^0 q$$

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\begin{aligned}\tilde{L} = & \bar{q} \left[i\hat{\partial} + \mathcal{M}\gamma^0 - \sigma - i\gamma^5 \vec{\tau} \vec{\pi} \right] q - \frac{\sigma^2 + \vec{\pi}^2}{4G} - \frac{\Delta^* \Delta}{4H} \\ & - \frac{\Delta}{2} \left[\bar{q} i\gamma^5 \sigma_2 \tau_2 q^c \right] - \frac{\Delta^*}{2} \left[\bar{q}^c i\gamma^5 \sigma_2 \tau_2 q \right]\end{aligned}$$

Equations of motion for bosonic fields

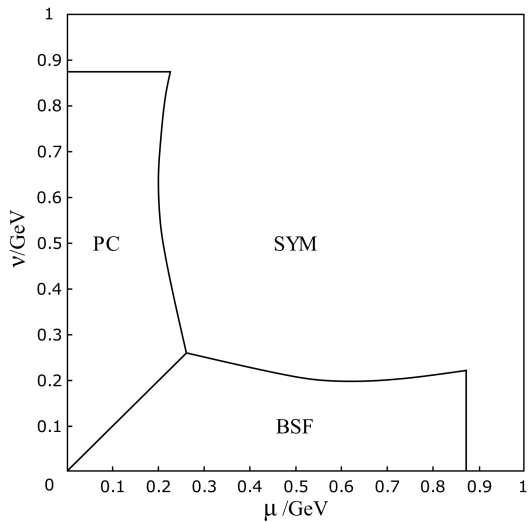
$$\begin{aligned}\sigma(x) &= -2G(\bar{q}q); \quad \pi_a(x) = -2G(\bar{q}i\gamma^5 \tau_a q) \\ \Delta(x) &= -2H \left[\bar{q}^c i\gamma^5 \sigma_2 \tau_2 q \right] = -2H \left[q^T C i\gamma^5 \sigma_2 \tau_2 q \right] \\ \Delta^*(x) &= -2H \left[\bar{q} i\gamma^5 \sigma_2 \tau_2 q^c \right] = -2H \left[\bar{q} i\gamma^5 \sigma_2 \tau_2 C \bar{q}^T \right]\end{aligned}$$

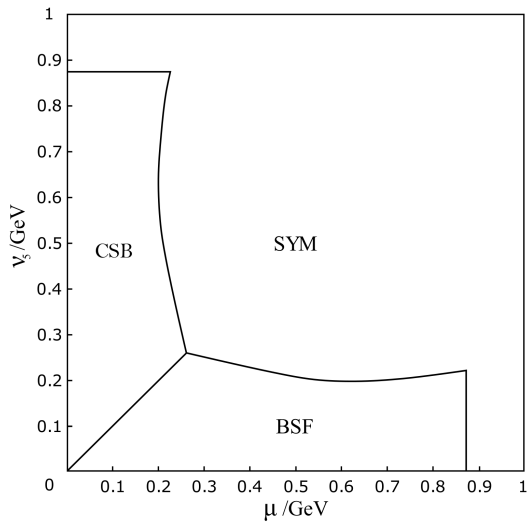
Condansates ansatz $\langle\sigma(x)\rangle$, $\langle\pi_a(x)\rangle$ and $\langle\Delta(x)\rangle$ do not depend on spacetime coordinates

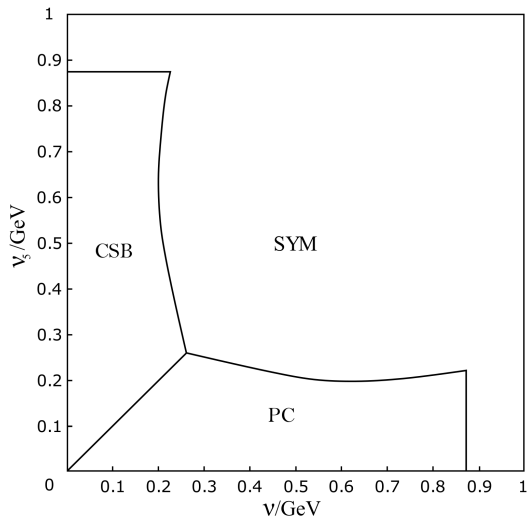
$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \pi, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0,$$

$$\langle\Delta(x)\rangle = \langle\Delta^*(x)\rangle = \Delta$$

where M , π and Δ are already constant quantities.







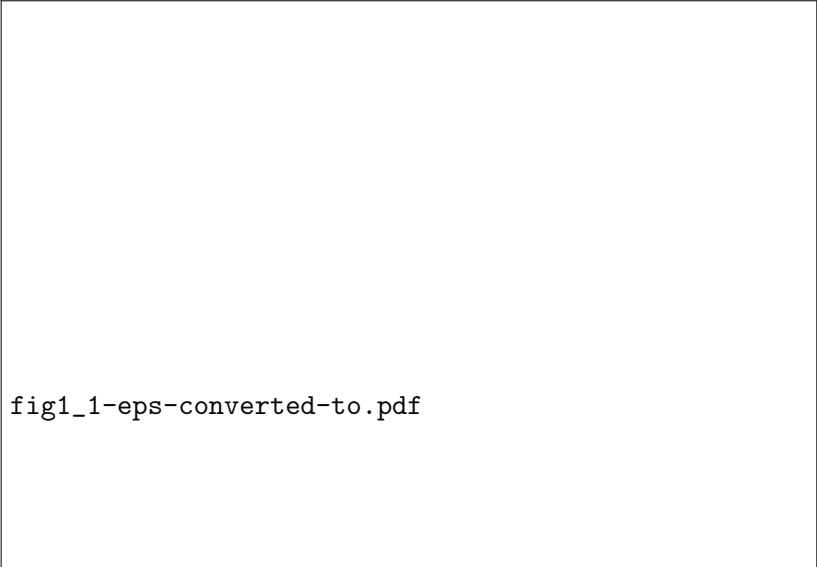


fig1_1-eps-converted-to.pdf

Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

The TDP

$$\Omega(T, \mu, \mu_i, \dots, \langle \bar{q}q \rangle, \dots)$$

The TDP

$$\Omega(T, \mu, \mu_i, \dots, \langle \bar{q}q \rangle, \dots)$$

$$\Omega(T, \mu, \nu, \nu_5, \dots, M, \pi, \dots)$$

The TDP

$$\Omega(T, \mu, \mu_i, \dots, \langle \bar{q}q \rangle, \dots) \qquad \Omega(T, \mu, \nu, \nu_5, \dots, M, \pi, \dots)$$

The TDP (phase daigram) is invariant under
Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

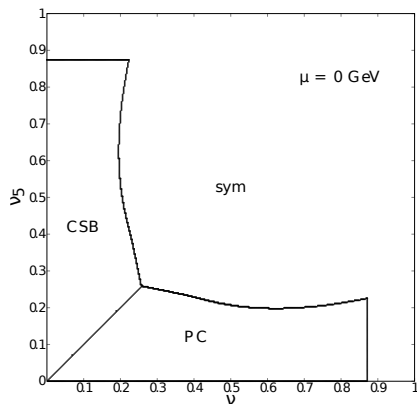


Figure: NJL model results

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

$$\mathcal{D} : M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral
symmetry breaking and pion
condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

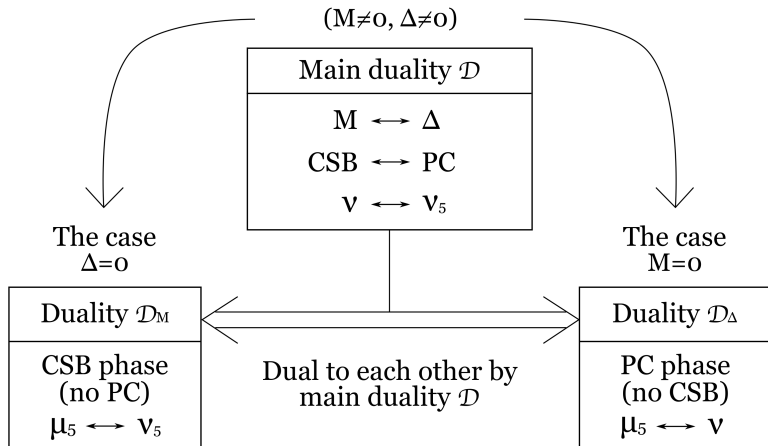
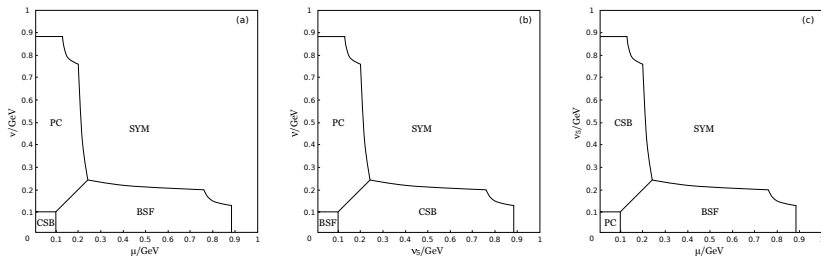


Figure: Dualities



$$(a) \quad \mathcal{D}_1 : \quad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|$$

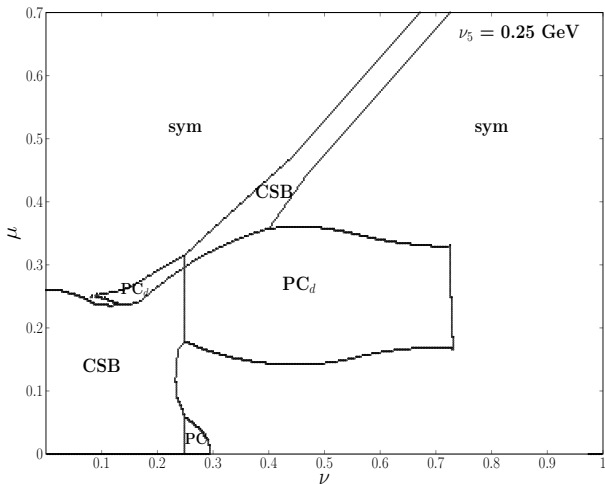
$$(b) \quad \mathcal{D}_3 : \quad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1$$

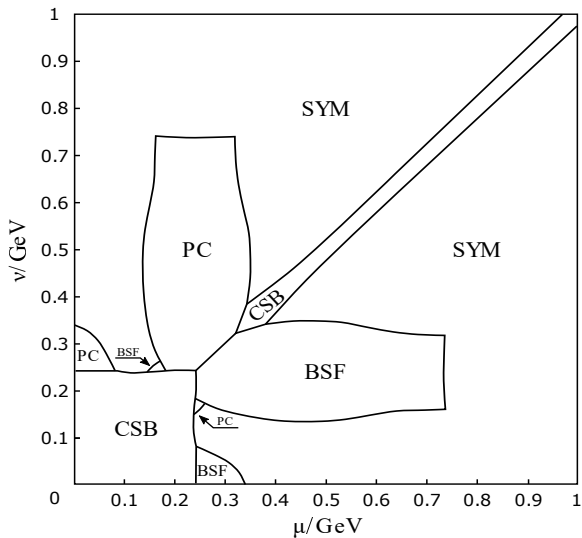
$$(c) \quad \mathcal{D}_2 : \quad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|$$

Uses of Dualities

How (if at all) it can be used

in three color case discussed in Particles 2020, 3(1), 62-79





- ▶ Based on the **duality** one can show that there is **no mixed phase**, i.e. two non-zero condensates simultaneously.

This greatly simplifies the numeric calculations.

- ▶ Phase diagram is **highly symmetric** due to **dualities**

The **whole phase diagram**, including diquark condensation, **in two color case** can be obtained from the results of **three color case** without any diquark condensation.

In the early 1970s Migdal (Sawyer, Scalapino, Kogut, Manassah) suggested the possibility of **pion condensation in a nuclear matter**

A.B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2210 (1971) [Sov. Phys. JETP 36, 1052 (1973)]; A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rept. 192, 179 (1990).
R.F. Sawyer, Phys. Rev. Lett. 29, 382 (1972); J. Kogut, J.T. Manassah, Physics Letters A, 41, 2, 1972, Pages 129-131

(In medium pion mass properties and the RMF models.)
pion condensation with zero momentum (s-wave condensation) is **highly unlikely to be realized** in nature in **matter of neutron star.**

A. Ohnishi D. Jido T. Sekihara, and K. Tsubakihara, Phys. Rev. C80, 038202 (2009) . . .

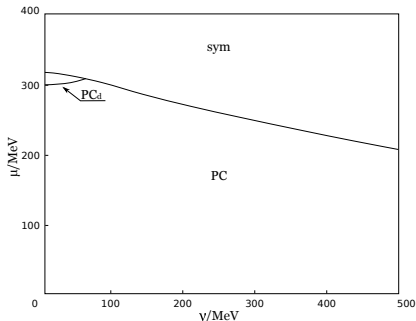


Figure: (ν, μ) phase diagram in NJL model in the chiral limit.

PC phenomenon maybe
could be realized in **dense
baryonic matter (non-zero
baryon density)**

K. G. Klimenko, D. Ebert
J.Phys. G32 (2006) 599-608;
Eur.Phys.J.C46:771-776,(2006)

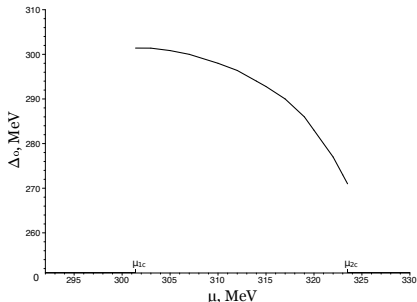
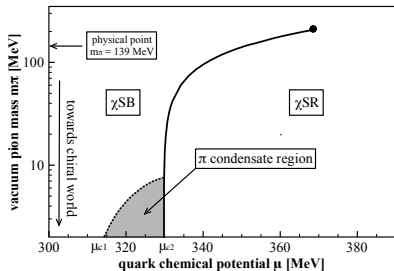


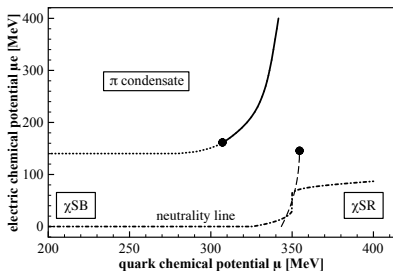
Figure: Pion condensate in dense quark matter in NJL model.

PC phenomenon is realized
in **dense baryonic matter**
even in **charge neutral and**
 β -equilibrated case

K. G. Klimenko, D. Ebert
J.Phys. G32 (2006) 599-608;
Eur.Phys.J.C46:771-776,(2006)



(μ, m_0) phase portrait.



(μ, μ_e) phase portrait.

No PC condensation in the neutral case at the physical point

(H. Abuki, R. Anglani, M. Ruggieri etc.
Phys. Rev. D **79** (2009) 034032.

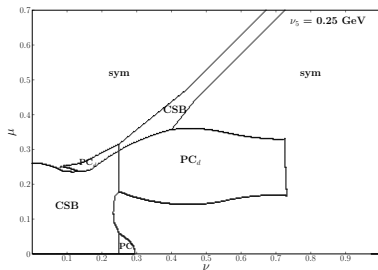
There are a number of **external parameters** such as **chiral imbalance** that can generate **PC** in dense quark matter.

See small review

Symmetry 2019, 11(6), 778

arXiv:1912.08635 [hep-ph]

Special Issue "Nambu-Jona-Lasinio model and its applications" of symmetry



(Thanks to Tomohiro Inagaki)

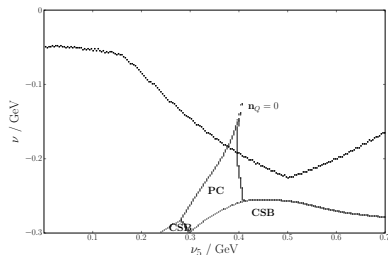
Charge neutrality and β -equilibrium can destroy the generation of PC

So it is interesting to see if chiral imbalance can generate PC in dense quark matter even in this case

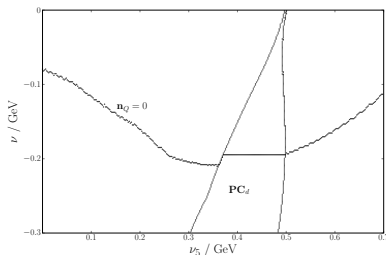
- ▶ Charge neutrality and β -equilibrium in neutron stars
- ▶ There are constraints in HIC ($n_Q = 0.4n_B$)
- ▶ Or β -equilibrium in neutron star mergers

Hot QCD Collaboration arXiv:1812.08235 [hep-lat]

Mark Alford Phys. Rev. C 98, 065806 (2018); arXiv:1803.00662 [nucl-th]



(ν_5, ν) phase portrait at $\mu = 450$ MeV and $\mu_5 = 0$.



(ν_5, ν) phase portrait at $\mu = 500$ MeV and $\mu_5 = 150$ MeV.

Chiral imbalance generates the charged pion condensation in dense electric neutral matter.

There have been discussed several **mechanism of generation of chiral imbalance in neutron stars**.

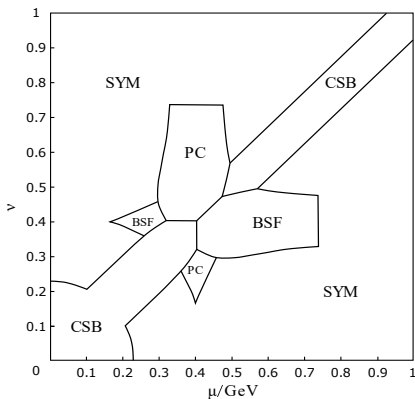
It is interesting in light of new and expected data on masses and radii to extend the studies and

- find the **EOS** in the presence of **chiral imbalance**

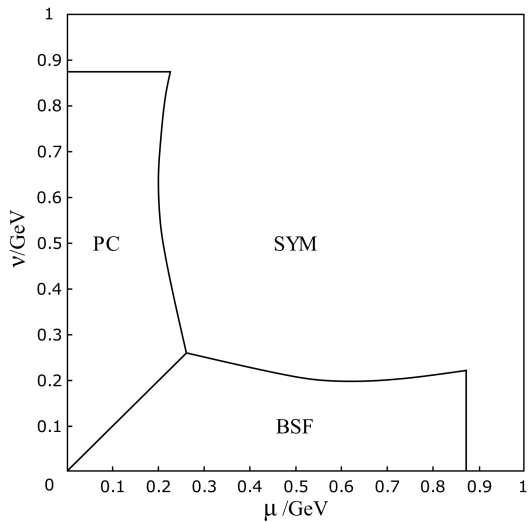
- and explore **the M - R relation for neutron star with chiral imbalance**

(Consideration of phase structure of dense electric neutral baryonic matter with β -equilibrium is a first step in that direction.)

- ▶ PC_d phase has been predicted without possibility of diquark condensation
- ▶ Diquark condensation can take over the PC_d phase
- ▶ In two colour case diquark condensation is in a sense even stronger than in three colour case and starts from $\mu > 0$



PC_d phase is unaffected by BSF phase in two color case.
 Maybe one can infer that it is the case also for 3 color QCD



$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}_a^{\mu\nu}.$$

$$\mathcal{L}_{\text{NJL}} = \sum_{f=u,d} \bar{q}_f \left[i\gamma^\nu \partial_\nu - m_f \right] q_f + \frac{G}{N_c} \left[(\bar{q}q)^2 + (\bar{q}i\gamma^5 \vec{\tau} q)^2 \right]$$

m_f is current quark masses

In the chiral limit $m_f = 0$ the Duality \mathcal{D} is exact

$$m_f : \quad \frac{m_u + m_d}{2} \approx 3.5 \text{ MeV}$$

In our case typical values of $\mu, \nu, \dots, T, \dots \sim 10 - 100 \text{ s MeV}$, for example, 200-400 MeV

One can work in the chiral limit $m_f \rightarrow 0$

$$m_f = 0 \quad \rightarrow \quad m_\pi = 0$$

physical m_f a few MeV \rightarrow physical $m_\pi \sim 140 \text{ MeV}$

Duality between CSB and PC is **approximate** in
physical point

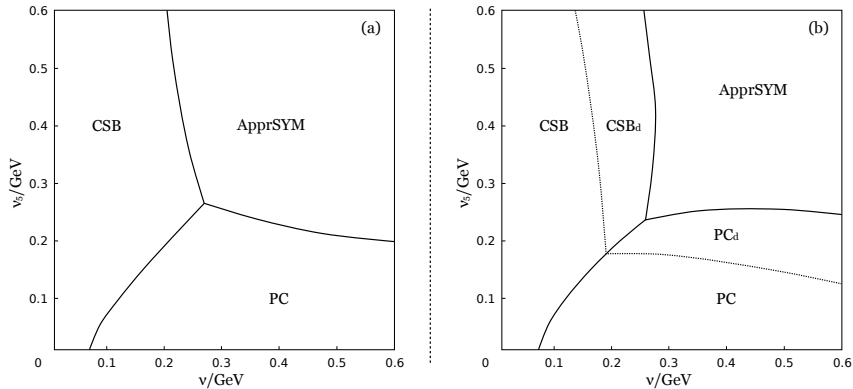


Figure: (ν, ν_5) phase diagram

Dualities on the lattice

$$(\mu_B, \mu_I, \mu_{I5}, \mu_5)$$

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

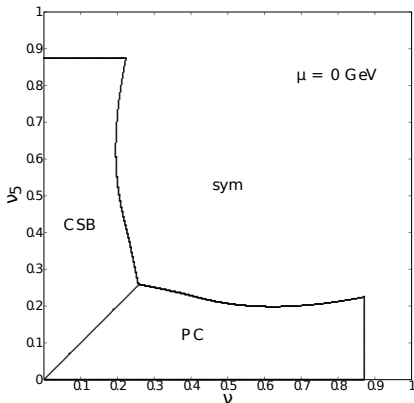
$\mu_B \neq 0$ impossible on lattice but if
 $\mu_B = 0$

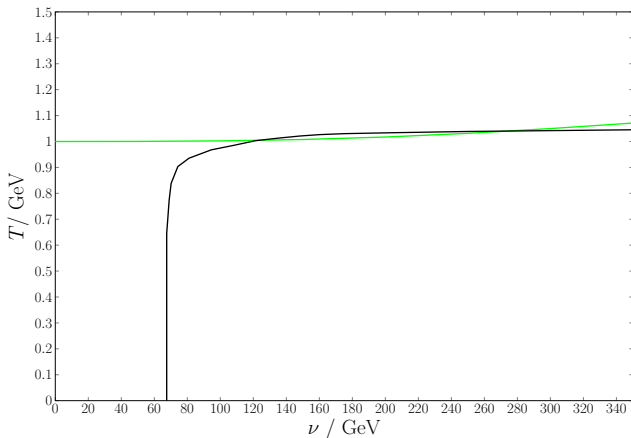
► **QCD at μ_5** — (μ_5, T)

V. Braguta, A. Kotov et al, ITEP
 lattice group

► **QCD at μ_I** — (μ_I, T)

G. Endrodi, B. Brandt et al,
 Emmy Noether junior research
 group, Goethe-University Frankfurt,
 Institute for Theoretical Physics ()





T_c^M as a function of μ_5 (green line) and $T_c^\Delta(\nu)$ (black)

- ▶ (μ_B, μ_I, ν_5) -phase diagram of two color QCD was studied
PC in dense matter with chiral imbalance
in in dense electric neutral matter in β -equilibrium
- ▶ It was shown that there exist dualities
- ▶ Richer structure of Dualities in the two colour case
- ▶ There have been shown how dualities can be used
- ▶ the generation of *PC in dense matter by chiral imbalance* is not obstructed by diquark condensation