Процессы рождения пар фотонов и мюонов на коллайдерах LHC и CLIC, индуцированные фотонами, в моделях с дополнительными размерностями

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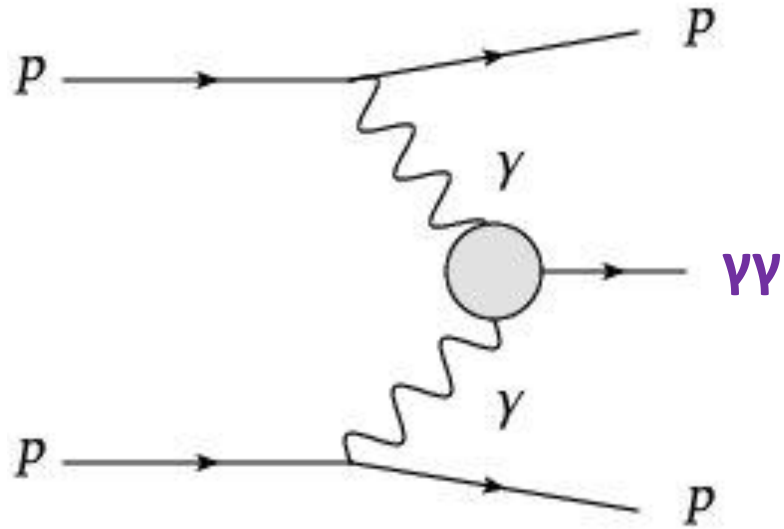
Направлено в печать (часть II)

Семинар ОТФ, 14 января 2020 года

План рассказа (часть I)

- Эксклюзивное рождение пары фотонов на БАК, индуцированное фотонами.
- Модель Рандалл-Сундрума (РС) с одной дополнительной пространственной размерностью и ее обобщение.
- Сечения рассеяния для процесса $pp \to p\gamma\gamma p \to p'\gamma\gamma p'$ при энергии 14 ТэВ в модели РС с малой кривизной пространства-времени.
- Ограничения на 5-мерную гравитационную фундаментальную константу.

Photon-induced diphoton production at the LHC



Schematic diagram for the reaction
pp → p γγ p → p' γγ p'

The goal is to estimate BSM effects in such a process which proceeds via γγ → γγ scattering

Light-by-light scattering ($\gamma\gamma \to \gamma\gamma$) is a quantum mechanical process forbidden in the classical theory of electrodynamics.

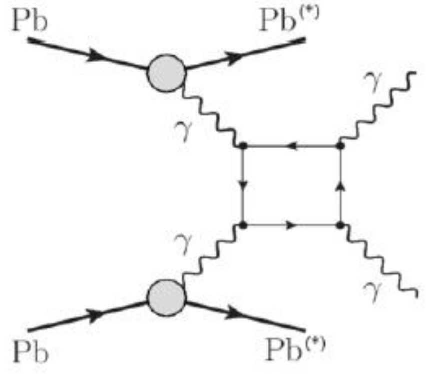
In QED this scattering proceeds via virtual one-loop diagrams Involving fermions which is an $O(\alpha^4 \approx 3 \cdot 10^{-9})$ process, making it challenging to test it experimentally.

Elastic light-by-light scattering remains unobserved: even ultra-intense laser experiments are not yet powerful enough to probe this phenomenon.

$\gamma\gamma \to \gamma\gamma$ process can be measured in heavy ion collision events, with impact parameters lager than twice radius of nuclei, in which strong interaction does not play a role. For lead nucleus (Z=82) EM field is up to 10^{25} V m^{-1}.

Experimental studies of photon-induced reactions at LHC

First observation of $\gamma\gamma$-induced diphoton production – in high energy ultraperipheral heavy ions collisions (ATLAS Collaboration, Nature Phys. 13, 852 (2017)).

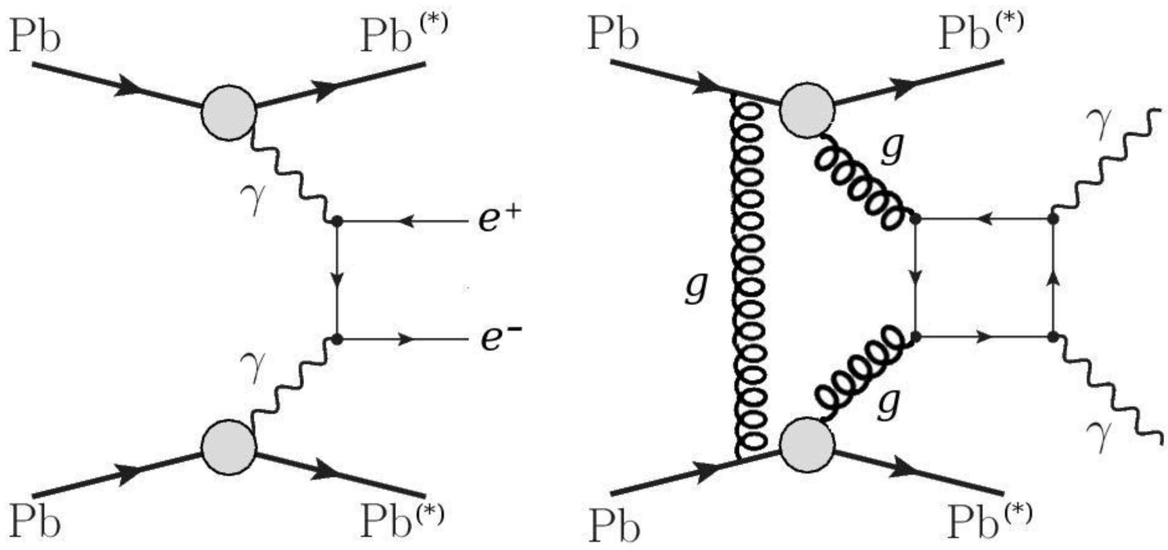


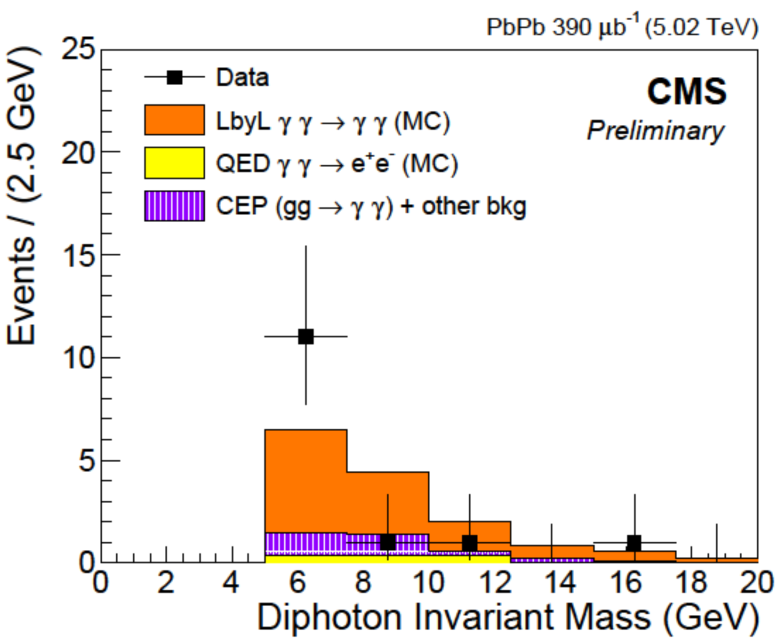
480 fb^{-1}
5 TeV c.m.s. N-N energy
$E_T > 3$ GeV
$|\eta| < 2.4$

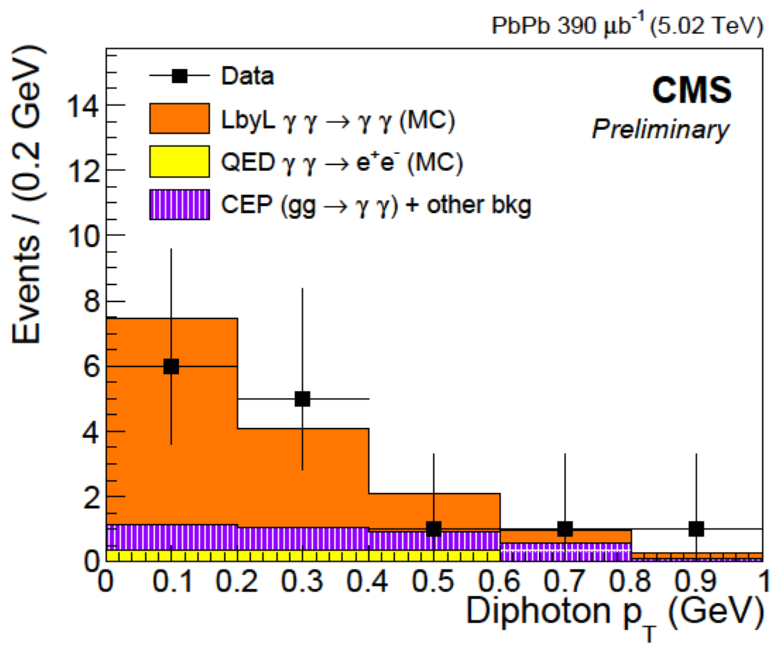
Recently, CMS Collaboration reported the same process (Nucl. Phys. A 982, 791 (2019)).

Dominant backgrounds are:

- QED exclusive electron-positron production with e+(e-) misidentified as photons;
- gluon central exclusive production (CEP) of a pair of photons.







Diphoton invariant mass distribution (left)
Diphoton p_t distributions (right)

pp → p′ γγ p′

Forward detectors at the LHC can detect intact outgoing protons in interval

$$\xi_{\min} < \xi < \xi_{\max}$$

where ξ is momentum fraction loss of the proton

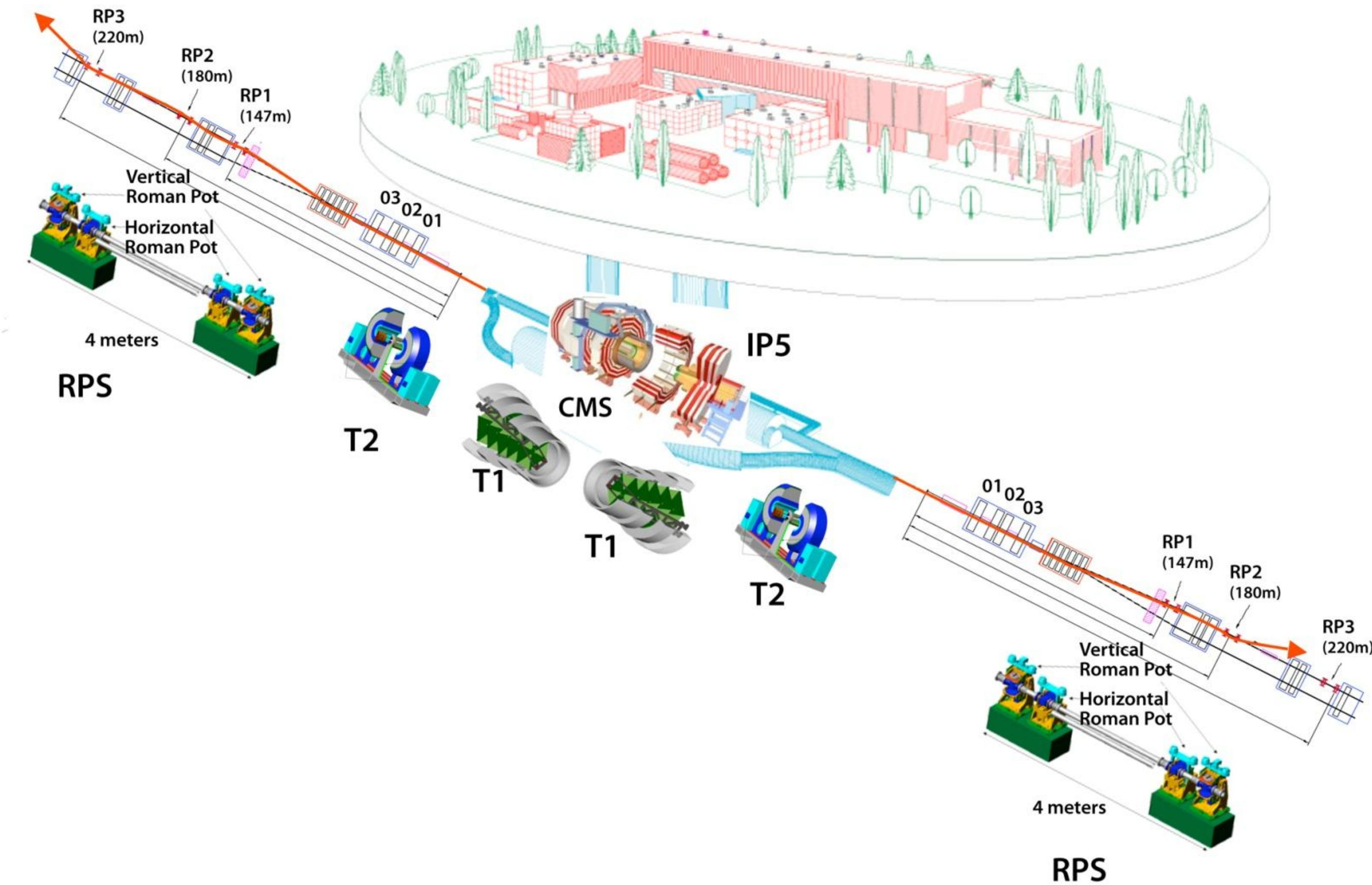
Acceptance range:

ATLAS Forward Physics Collaboration (AFP)

CMS-TOTEM Precision Proton Spectrometer (CT-PPS)

$$\left.\begin{array}{l}\text{AFP}\\ \text{CT-PPS}\end{array}\right\} \quad 0.015 < \xi < 0.15$$

CT-PPS


RP3
(220m)
RP2
(180m)
RP1
(147m)
Vertical
Roman Pot
Horizontal
Roman Pot
03 02 01
4 meters
RPS
T2
T1
CMS
IP5
T1
T2
01 02 03
RP1
(147m)
RP2
(180m)
RP3
(220m)
Vertical
Roman Pot
Horizontal
Roman Pot
4 meters
RPS

Extra (5-th) dimension of the space-time

Маргарита:
– …более всего меня поражает, где все это помещается.
Коровьев:
– Самое несложное из всего! Тем, кто хорошо знаком с пятым измерением, ничего не стоит раздвинуть помещение до желательных пределов. Скажу вам более, уважаемая госпожа, до черт знает каких пределов!

М.А. Булгаков. Мастер и Маргарита. Глава 22. При свечах.

Randall-Sundrum scenario

(Randall & Sundrum, 1999)

Background metric (y is extra coordinate)

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

warp factor

Periodicity: *$(x, y \pm 2\pi r_c) = (x, y)$*

Z_2-symmetry: *$(x, y) = (x, -y)$*

→ orbifold S^1/Z_2 $0 \le y \le \pi r_c$

Two fixed points: $y=0$ and $y=\pi r_c$

→ two (1+3)-dimensional branes

5-dimensional action $S = S_g + S_1 + S_2$

$$S_g = \int d^4x \int dy \sqrt{G} \left(2M_5^3 R^{(5)} - \Lambda\right)$$ **(gravity term)**

$$S_{1(2)} = \int d^4x \sqrt{g_{1(2)}} \left(L_{1(2)} - \Lambda_{1(2)}\right)$$ **(brane terms)**

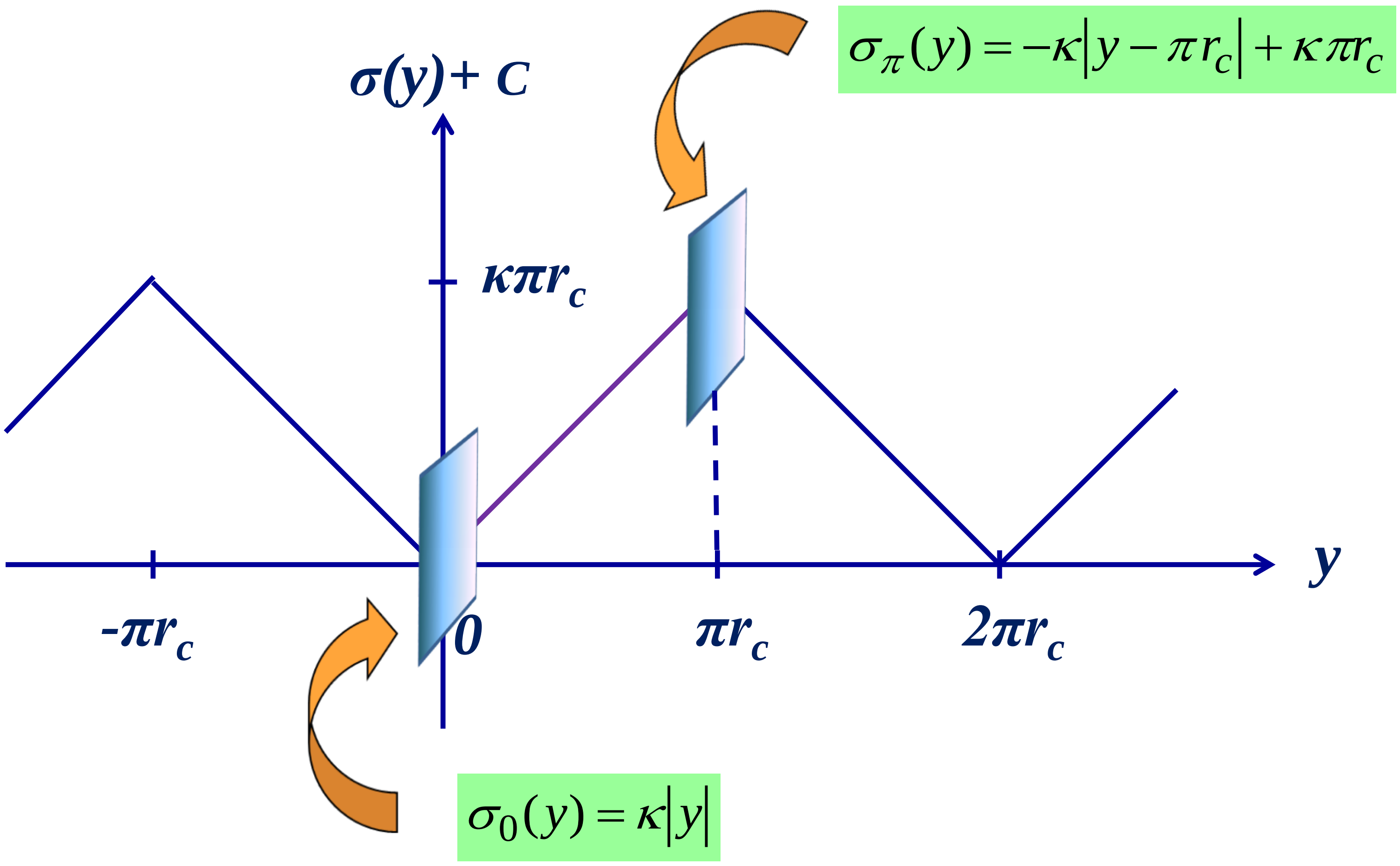
Einstein-Hilbert's equations:

$$\sigma'^2(y) = -\frac{\Lambda}{24M_5^3}$$

$$\sigma''(y) = \frac{1}{12M_5^3}[\Lambda_1 \delta(y) + \Lambda_2 \delta(\pi r_c - y)]$$

Generalized RS-like solution

Two equivalent solutions related to different branes



Generalized solution: $$\sigma(y)=\frac{1}{2}\left[\sigma_0(y)+\sigma_\pi(y)\right]-C$$

with fine tuning

$$\Lambda=-24M_5^3\kappa^2,\quad \Lambda_1=-\Lambda_2=12M_5^3\kappa$$

1-st derivative of σ(y): $$\sigma'(y)=\frac{\kappa}{2}\left[\varepsilon(y)-\varepsilon(y-\pi r_c)\right]$$

2-nd derivative of σ(y): $$\sigma''(y)=\kappa\left[\delta(y)-\delta(y-\pi r_c)\right]$$

$$\sigma(y+2\pi r_c)=\sigma(y)$$ **(periodicity)**

$$\sigma(-y)=\sigma(y)$$ **(Z_2 symmetry)**

Hierarchy relation

$$M_{\mathrm{Pl}}^2 = \frac{M_5^3}{\kappa}\exp(2C)$$

Interaction Lagrangian
(*massive gravitons only*)

$$L(x) = -\frac{1}{\Lambda_\pi}\sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}(x) T_{\alpha\beta}(x)\,\eta^{\mu\alpha}\eta^{\nu\beta}$$

Masses of KK gravitons
(*x_n are zeros of $J_1(x)$*)

$$m_n = x_n M_{\mathrm{Pl}} \exp(-\pi\kappa r_c)\left(\frac{\kappa}{M_5}\right)^{3/2}$$

Masses of KK gravitons m_n and coupling Λ_π depend on constant C via M_5 and κ

→ **Different values of C result in quite diverse physical models**

Two interesting physical scenarios

I. C = 0 $\sigma(0)=0, \quad \sigma(\pi r_c)=\kappa\pi r_c$

→ $M_{\text{Pl}}^2 \simeq \dfrac{M_5^3}{\kappa}$ **that requires** $M_5 \sim \kappa \sim M_{\text{Pl}}$

Masses of KK resonances $m_n \simeq x_n \kappa \exp(-\kappa\pi r_c)$

→ **RS1 model** ***(Randall & Sundrum, 1999)***

Graviton spectrum - heavy resonances, with the lightest one above 1 TeV

II. C = κπr$_c$

$$\sigma(0) = -\kappa\pi r_c, \quad \sigma(\pi r) = 0$$

$$M_{\text{Pl}}^2 \cong \frac{M_5^3}{\kappa}\exp(2\pi\kappa r_c)$$

$\kappa << M_5$ $\kappa r_c \approx 9.5$ for $M_5 = 1\,\text{TeV}, \kappa = 100\,\text{MeV}$

Masses of KK resonances $m_n \cong x_n \kappa$

RSSC model: scenario with small curvature of 5-dimensional space-time

For small κ, graviton spectrum is similar to that of the ADD model

(Giudice, 2005 Petrov & A.K., 2005)

Virtual Gravitons at the LHC

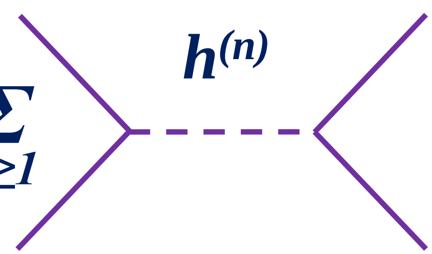
Light-by-light collision at LHC is mediated by KK graviton exchanges in *s*-, *t*- and *u*-channels

Processes: $pp \to \gamma\gamma\,(l^+l^-,\, 2\,jets) + X$

Sub-processes: $\gamma\gamma,\, q\bar{q},\; gg \to h^{(n)} \to \gamma\gamma,\; f\bar{f}$

KK gravitons

***s*-channel:** $\sum\limits_{n\geq 1}$



Matrix element of sub-process

$$\mathrm{M} = \mathrm{A} \times \mathrm{S}$$

Tensor part of graviton propagator

where $$\mathrm{A} = \mathrm{T}^{\mathrm{in}}_{\mu\nu} \mathrm{P}^{\mu\nu\alpha\beta} \mathrm{T}^{\mathrm{f}}_{\alpha\beta}$$

Energy-momentum tensors

$$S(s) = \frac{1}{\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{s - m_n^2 + i m_n \Gamma_n}$$ **(process independent)**

Graviton widths $$\Gamma_n = \eta\, m_n^3 / \Lambda_\pi^2, \;\; \eta \cong 0.1$$

Equivalent Photon Approximation (EPA)

Field of fast charged particle is treated as a beam of quasi-real photons with a small virtuality

(Fermi, 1924; Weizsäcker, 1934; Williams, 1935)

Spectrum of photon emitted by proton

(Q^2 is photon virtuality, $E_\gamma = \xi E$ is its energy)

$$\frac{dN}{dE_\gamma dQ^2} = \frac{\alpha}{\pi}\frac{1}{E_\gamma Q^2}\left[\left(1-\frac{E_\gamma}{E}\right)\left(1-\frac{Q^2_{\min}}{Q^2}\right)F_E + \frac{E_\gamma^2}{2E^2}F_M\right]$$

(Budnev et al.,1975)

$$Q^2_{\min} = \frac{m_p^2 E_\gamma^2}{E(E-E_\gamma)} \qquad F_E = \frac{4m_p^2 G_E^2 + Q^2 G_M^2}{4m_p^2 + Q^2} \qquad F_M = G_M^2$$

$$G_E^2 = \frac{G_M^2}{\mu_p^2} = \left(1+\frac{Q^2}{Q_0^2}\right)^{-4} \quad Q_0^2 = 0.71\,\mathrm{GeV}^2$$

$$\mu_p^2 = 7.78$$

square of proton magnetic moment

Effective γγ-luminosity

$$\frac{dL_{\gamma\gamma}}{dW} = \int_{Q^2_{\min}}^{Q^2_{\max}} dQ_1^2 \int_{Q^2_{\min}}^{Q^2_{\max}} dQ_2^2 \int_{y_{\min}}^{y_{\max}} dy \frac{W}{2y} f_1\left(\frac{W^2}{4y}, Q_1^2\right) f_2\left(y, Q_2^2\right)$$

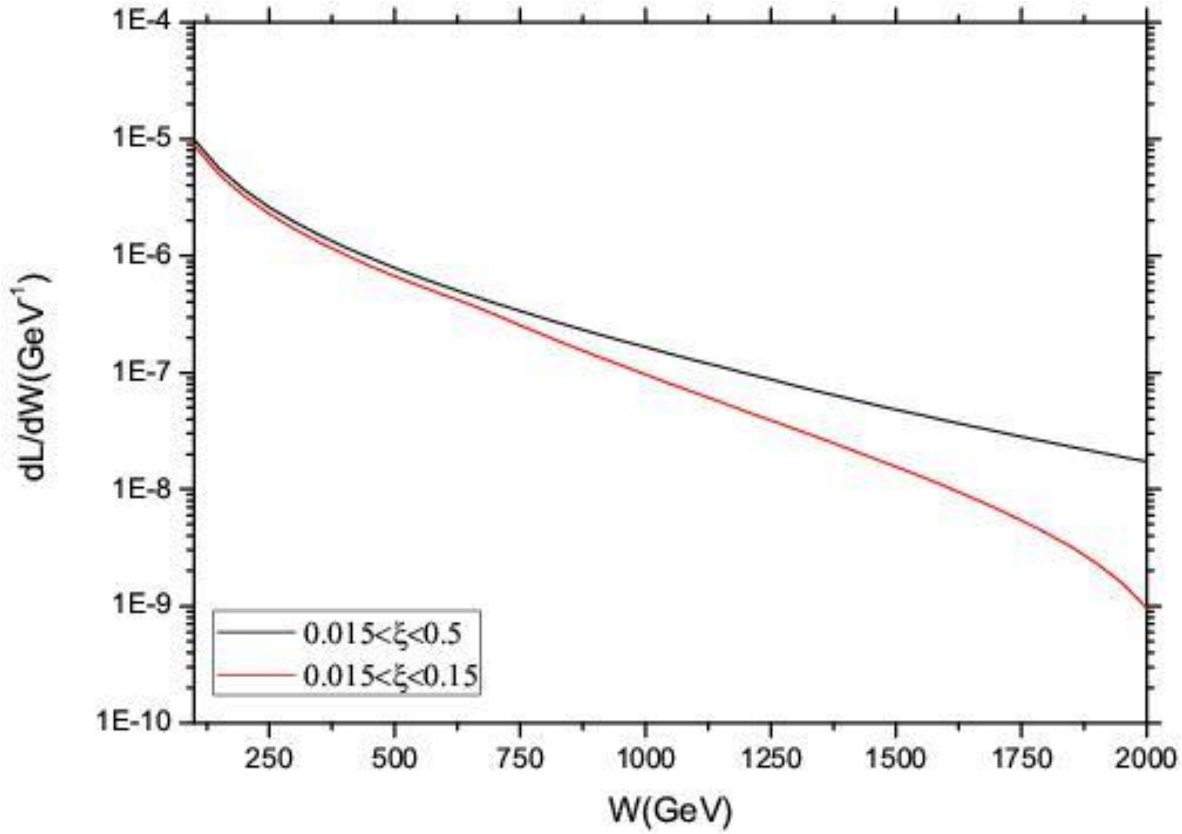
PDFs

$Q^2_{max} = 2$ GeV^2

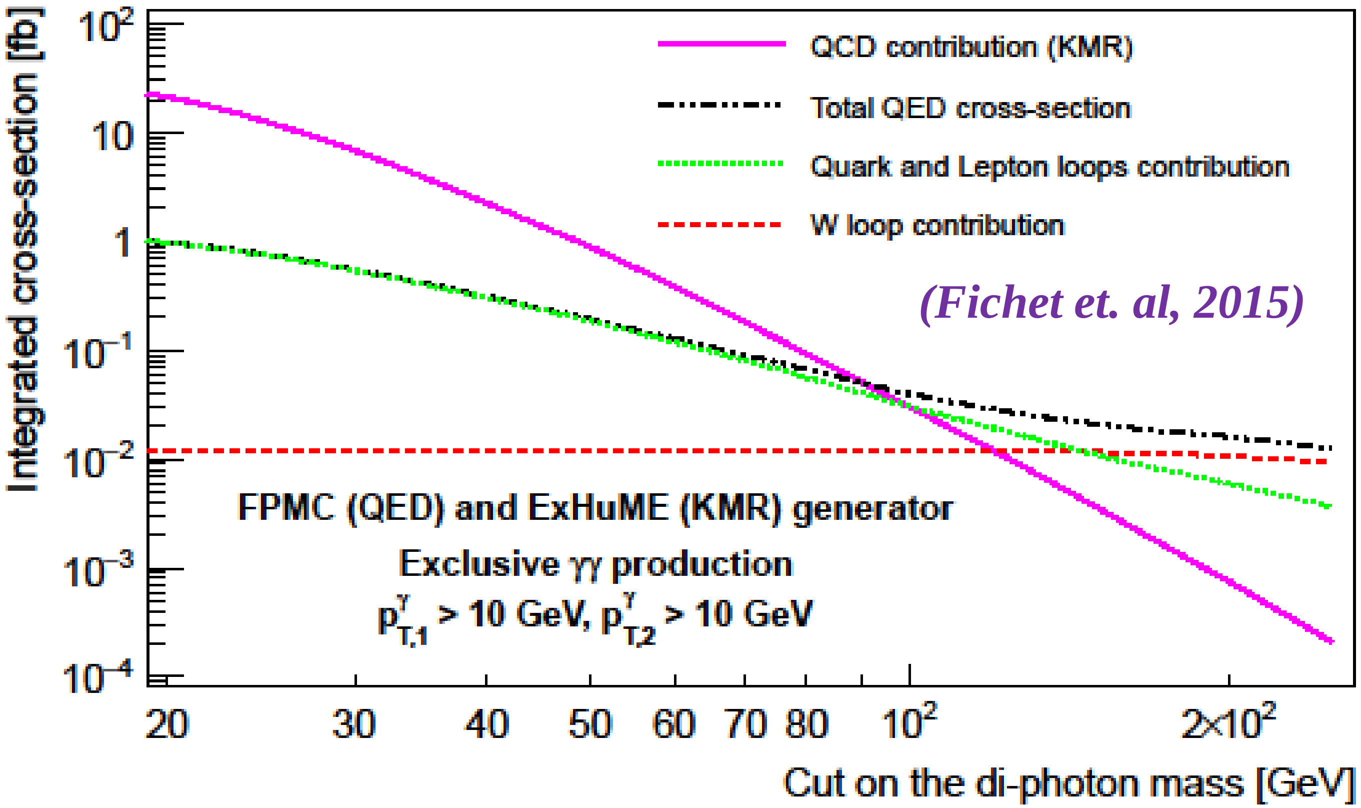
Cross section for the process pp → pγγp → pXp

$$d\sigma = \int \frac{dL_{\gamma\gamma}}{dW} d\sigma_{\gamma\gamma\to X}(W) dW$$

$W = 2E(\xi_1\xi_2)^{1/2}$ is invariant energy of two photons



Effective γγ-luminosity (in GeV^{-1}) as a function of invariant mass of two photons and forward detector acceptance ξ

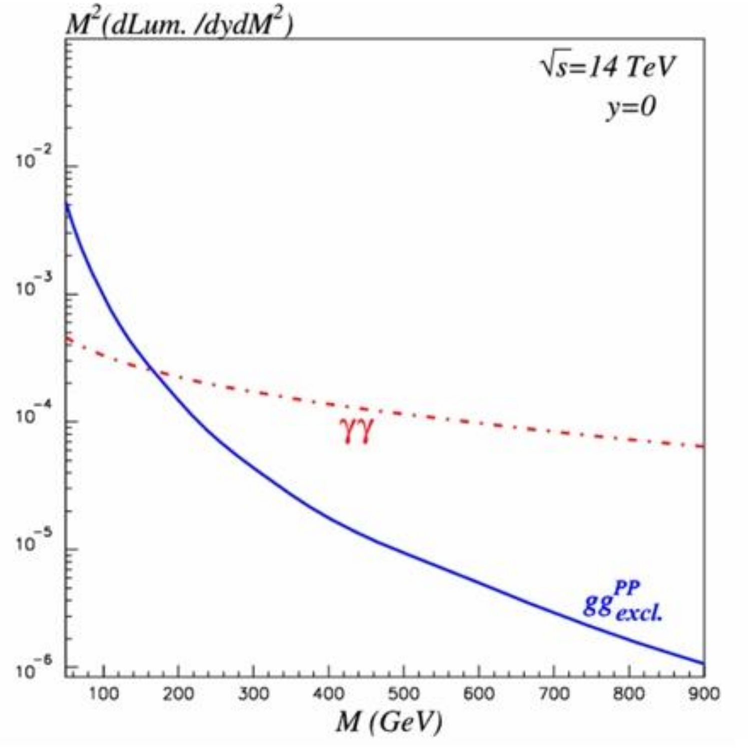


Integrated cross sections of different exclusive diphoton processes with intact protons at the 13 TeV LHC as a function of minimum diphoton mass

γγ process vs. gluon induced process

Exclusive event requires no extra gluon radiation into final state

→ **Sudakov suppression in QCD cross section leads to enhancement in γγ for $M_{\gamma\gamma}$ > 150-200 GeV**



From T, P invariance and Bose statistics:

$$\left|M\right|^2 = 2\left|M_{++++}\right|^2 + 2\left|M_{++--}\right|^2 + 2\left|M_{+-+-}\right|^2 + 2\left|M_{+--+}\right|^2 + 8\left|M_{+++-}\right|^2$$

Sum of electroweak, KK and interference terms:

$$\left|M\right|^2 = \left|M_{\text{ew}}\right|^2 + \left|M_{KK}\right|^2 + \left|M_{\text{int}}\right|^2$$

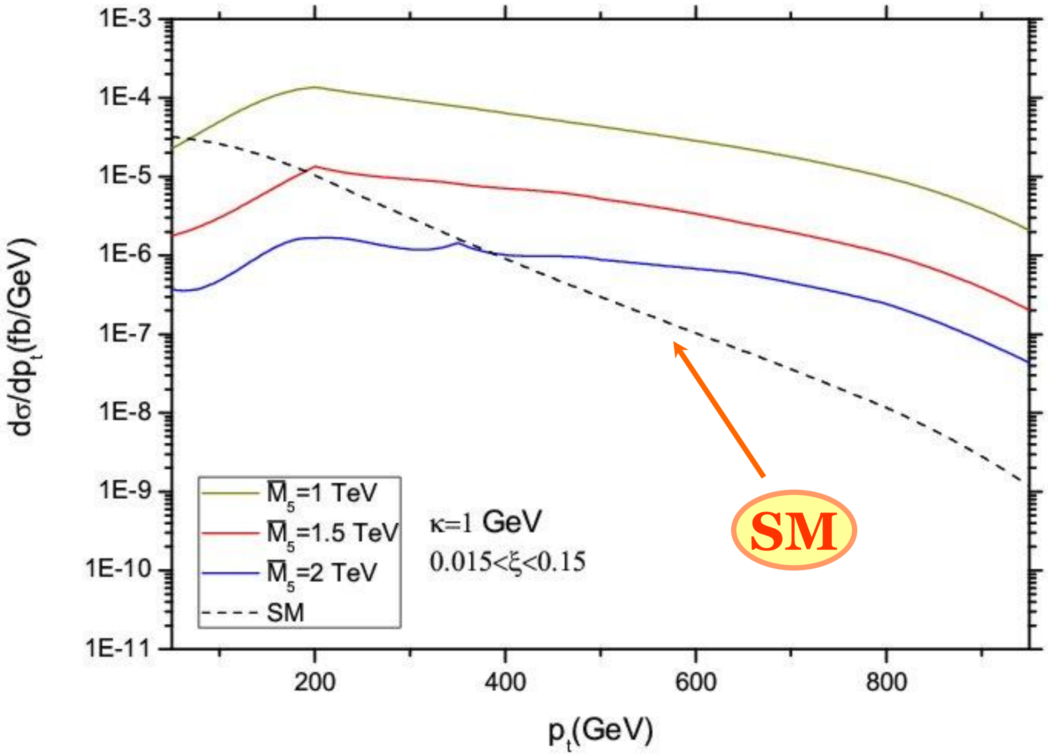
$$\begin{aligned}\left|M_{KK}\right|^2 = \frac{1}{8}\Big\{ &|S(\hat{s})|^2 (\hat{t}^4 + \hat{u}^4) + |S(\hat{t})|^2 (\hat{s}^4 + \hat{u}^4) + |S(\hat{u})|^2 (\hat{s}^4 + \hat{t}^4) \Big\} \\ &+ \frac{1}{8}\Big\{ \left|S^*(\hat{s})\, S(\hat{t}) + S(\hat{s})\, S^*(\hat{t})\right| \hat{u}^4 + \left|S^*(\hat{s})\, S(\hat{u}) + S(\hat{s})\, S^*(\hat{u})\right| \hat{t}^4 \Big\} \\ &+ \frac{1}{8}\Big\{ \left|S^*(\hat{t})\, S(\hat{u}) + S(\hat{t}) S^*(\hat{u})\right| \hat{s}^4 \Big\}\end{aligned}$$

$\hat{s}, \hat{t}, \hat{u}$ are Mandelstam variables of subprocess $\gamma\gamma \to \gamma\gamma$

Differential and total cross sections

Cuts on transverse momenta and rapidities of final photons:

$p_t > 30$ GeV, $|\eta| < 2.5$



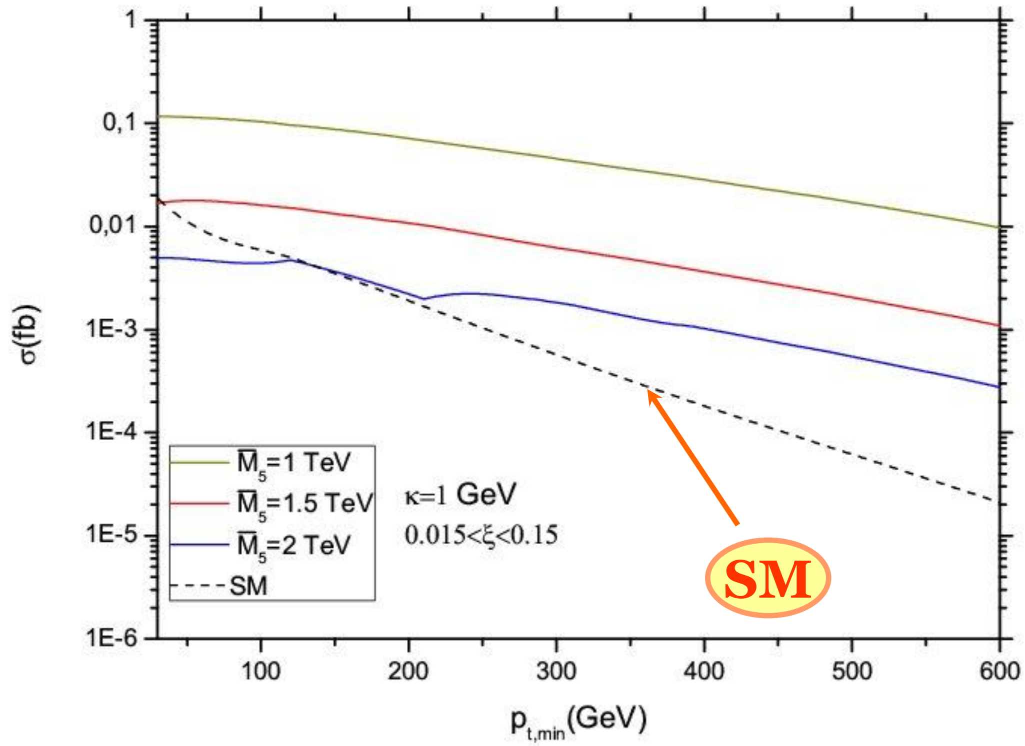
No noticeable dependence on parameter κ

Differential cross section (in fb/GeV) for the process pp → p γγ p → p' γγ p' as a function of transverse momenta of final photons p_t

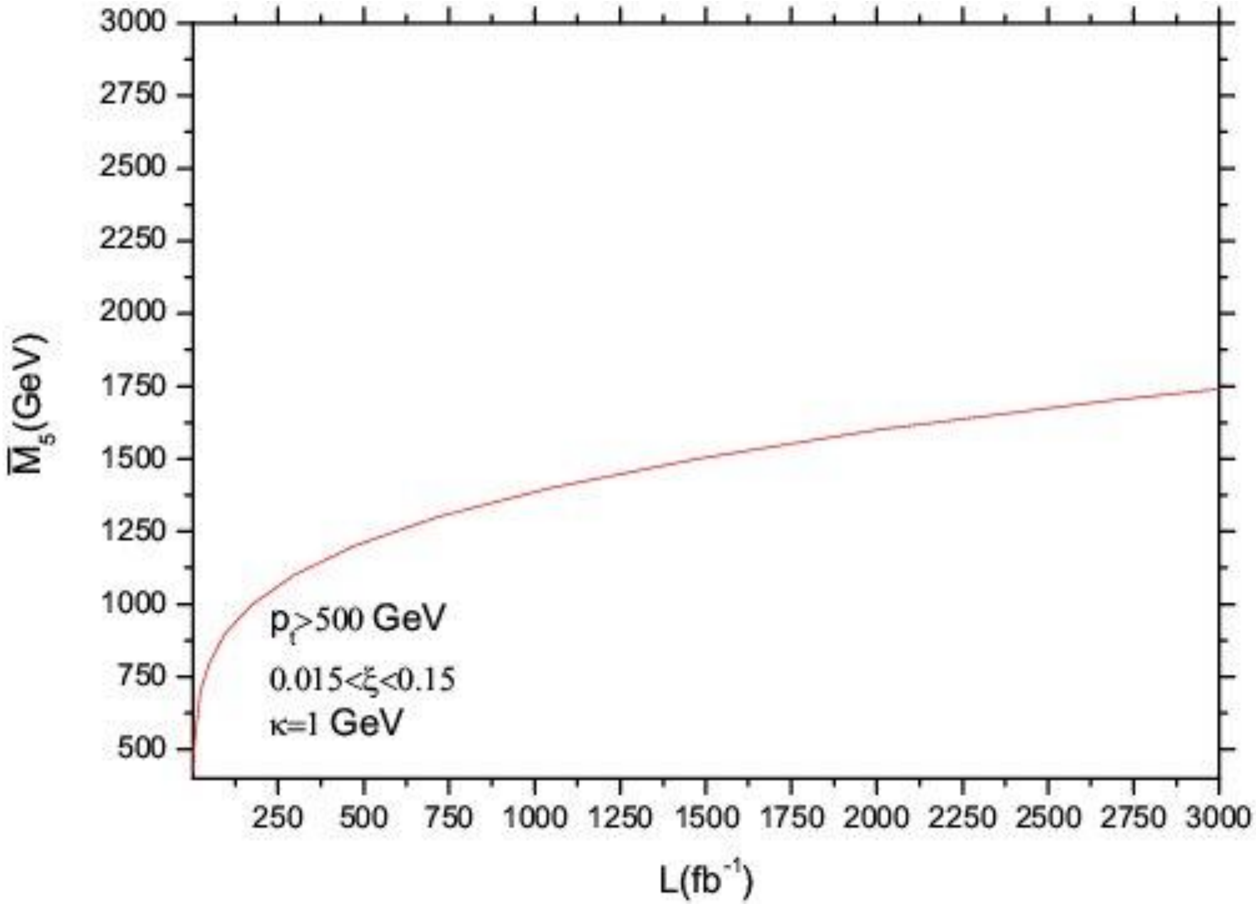
Total cross section as a function of minimal transverse momentum of final photons $p_{t,min}$

$$\sigma(p_t > p_{t,\min}) = \int\limits_{p_{t,\min}} \frac{d\sigma}{dp_t} dp_t$$

$0.015 < \xi < 0.15$ $\kappa = 1$ GeV



Total cross section for the process $pp \to p\,\gamma\gamma\,p \to p'\,\gamma\gamma\,p'$ as a function $p_{t,min}$ for different values of M_5

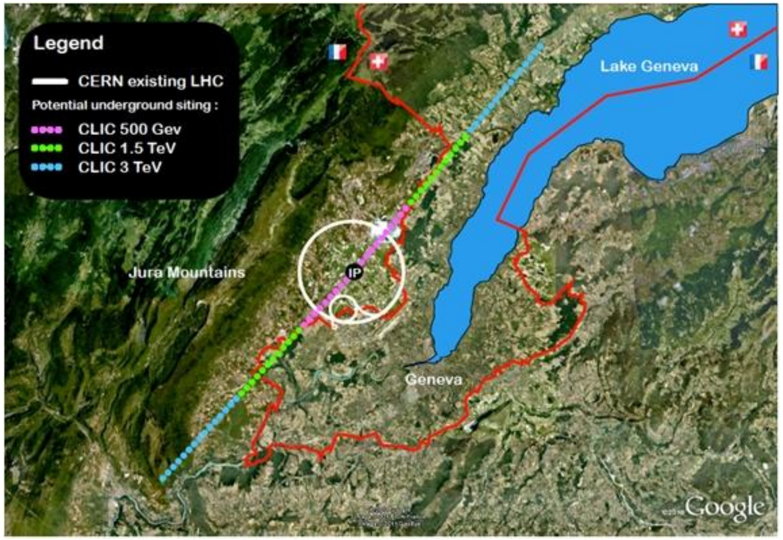


95% C.L. search limit for 5-dimensional Planck scale M_5 as a function of integrated LHC luminosity L with cuts p_t > 500 GeV, |η|< 2.5 imposed

План рассказа (часть II)

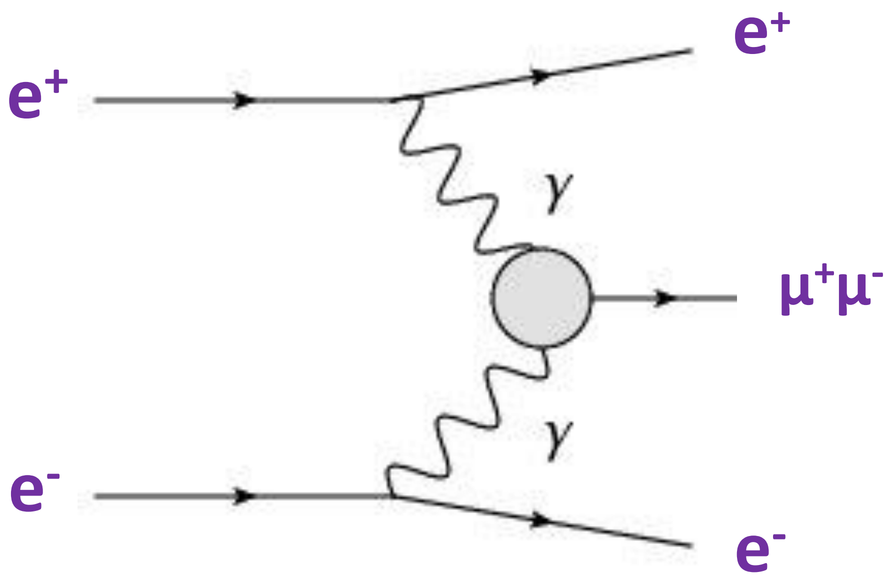
- Рождение пары мюонов на проектируемом коллайдере CLIC, индуцированное фотонами.
- Дифференциальное и полное сечения для процесса $e^+e^- \to e^+\gamma\gamma e^- \to e'^+\mu^+\mu^-e'^-$ в ряде моделей с дополнительными размерностями.
- Ограничения на многомерные гравитационные константы – аналоги массы Планка.
- Общее заключение

Compact Linear Collider (CLIC)



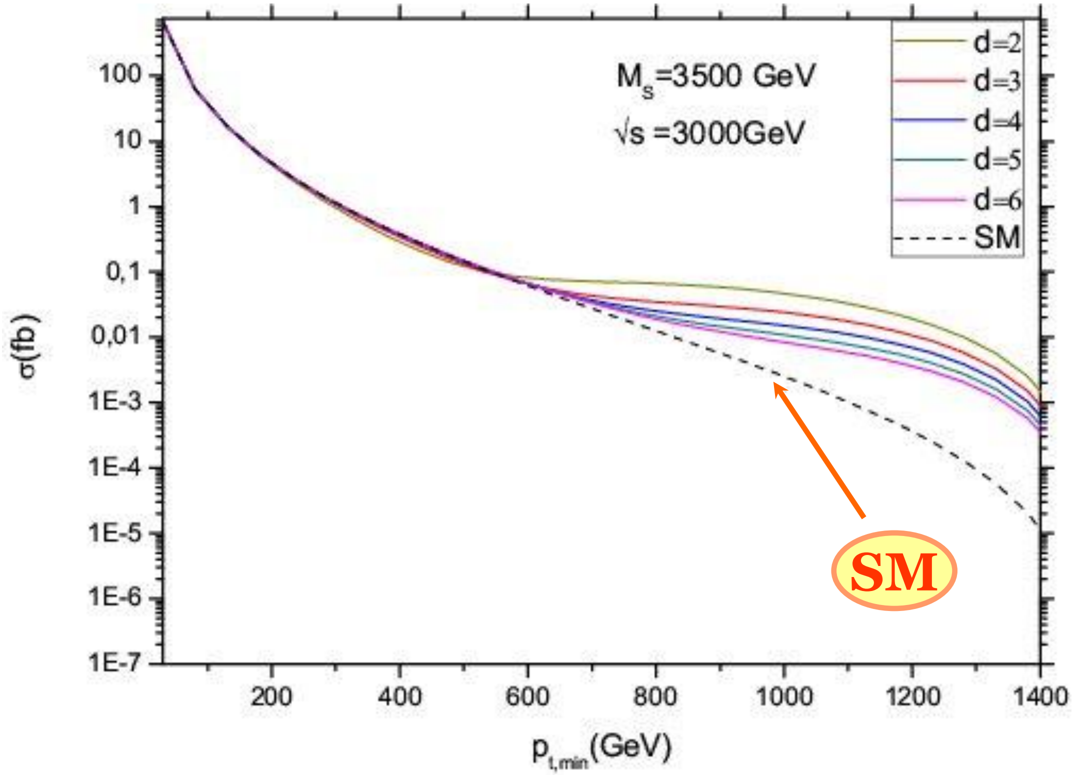
	Beam energy	Integrated luminosity
1-st stage	190 GeV	1000 fb^{-1}
2-nd stage	750 GeV	2500 fb^{-1}
3-rd stage	1.5 TeV	5000 fb^{-1}

Photon-induced dimuon production at the CLIC



Schematic diagram for the reaction

$e+e- \to e+\ \gamma\gamma\ e- \to e'+\ \mu^+\mu^-\ e'-$

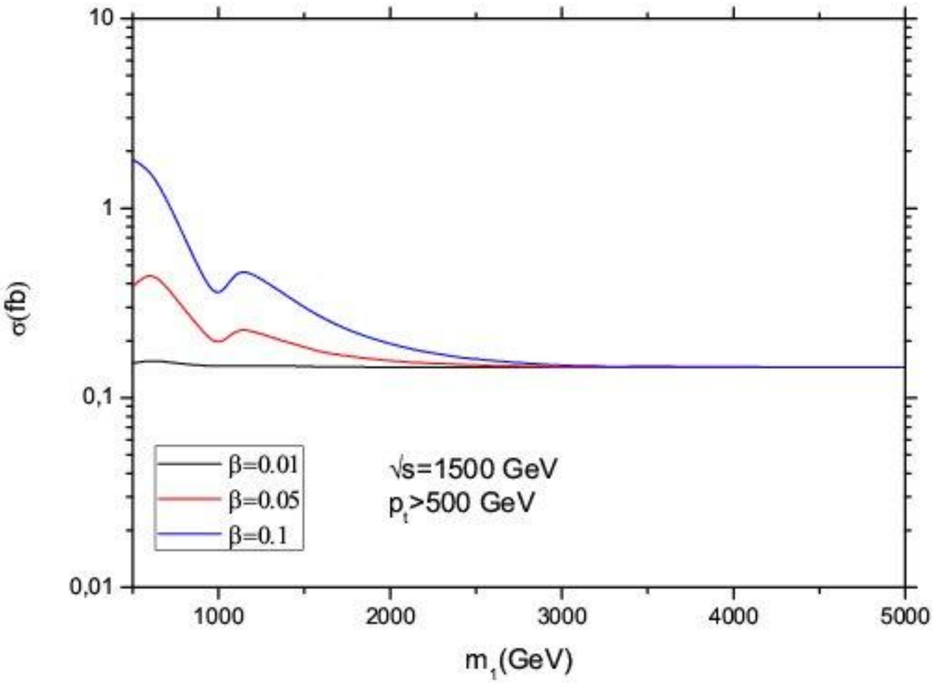


d = number of EDs

$$\sum_n \frac{1}{s-m_n^2} \rightarrow \int^{M_s} \frac{dm m^{d-1}}{s-m^2}$$

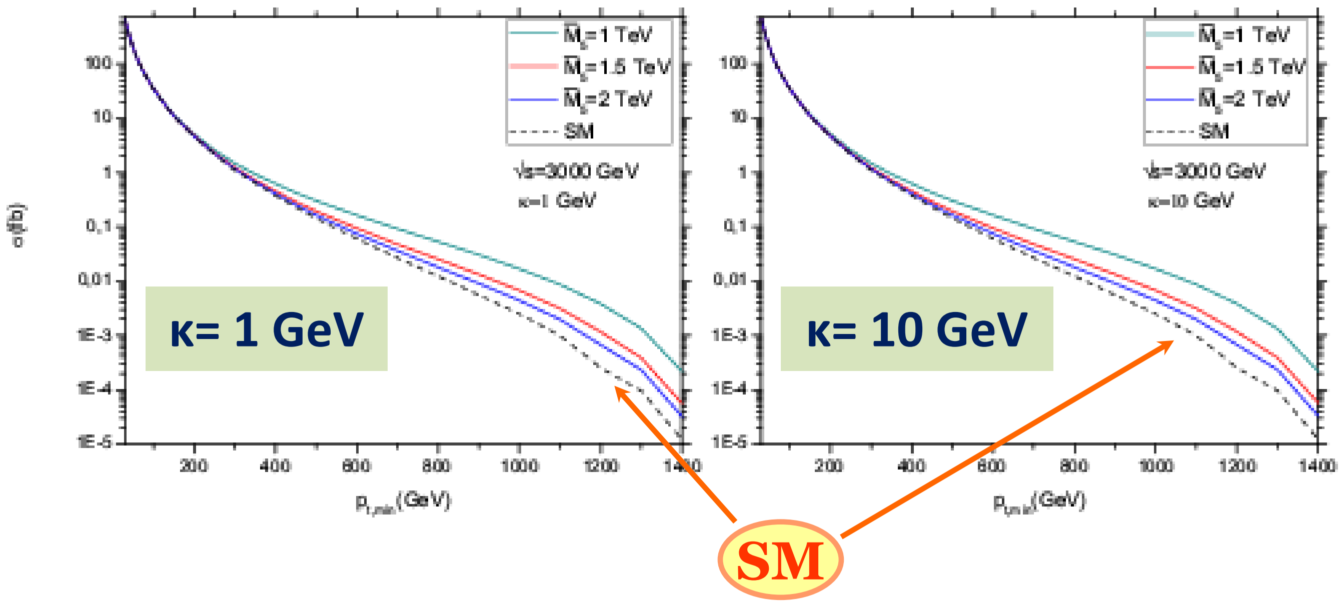
Total cross section for the process e+e- → e+ γγ e- → e'+ $\mu^+\mu^-$ e'- in the ADD model with the HLZ convention as a function of $p_{t,min}$ for √s = 3000 GeV and scale cutoff M_s =3.5 TeV.

m_1 = mass of the lightest graviton, $\beta = \kappa/M_{Pl}$



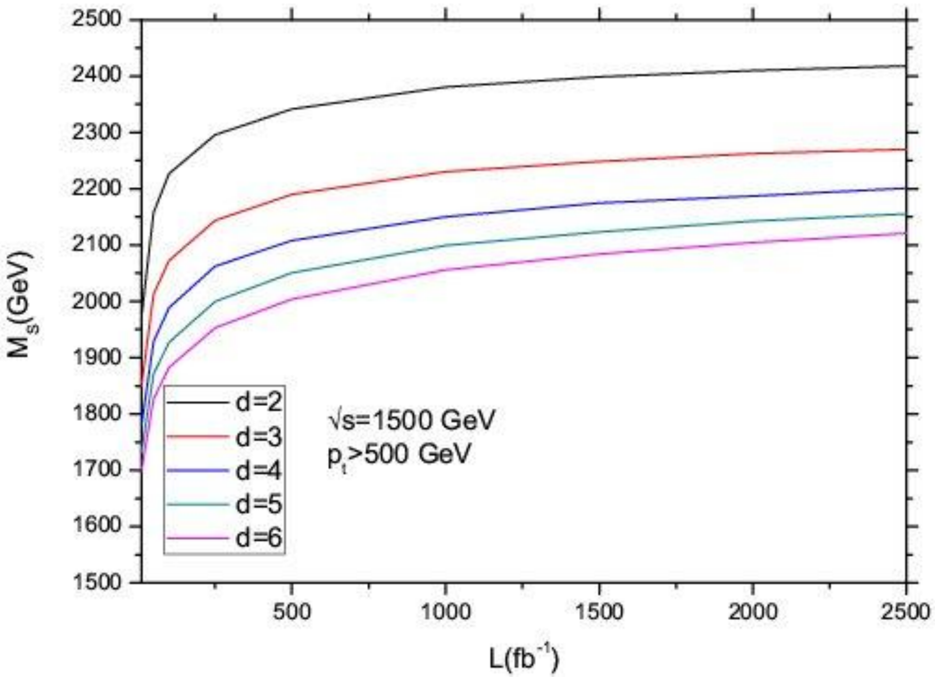
Total cross section for the process e+e- → e+ γγ e- → e'+ $\mu^+\mu^-$ e'- in the RS model as a function of mass of the lightest KK resonance for $\sqrt{s}$ = 1500 GeV and different values of β.

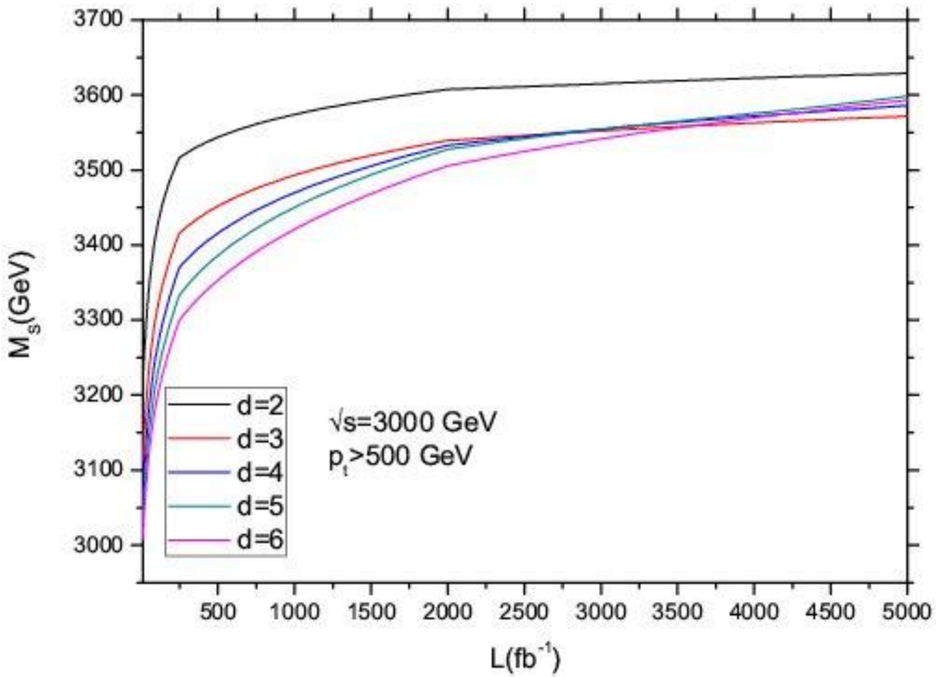
Weak dependence on parameter κ



Total cross section for the process e+e- → e+γγe- → e'+ $\mu^+\mu^-$ e'- in the RSSC model as a function of $p_{t,min}$ for √s = 3000 GeV and different values of M_5 and κ.

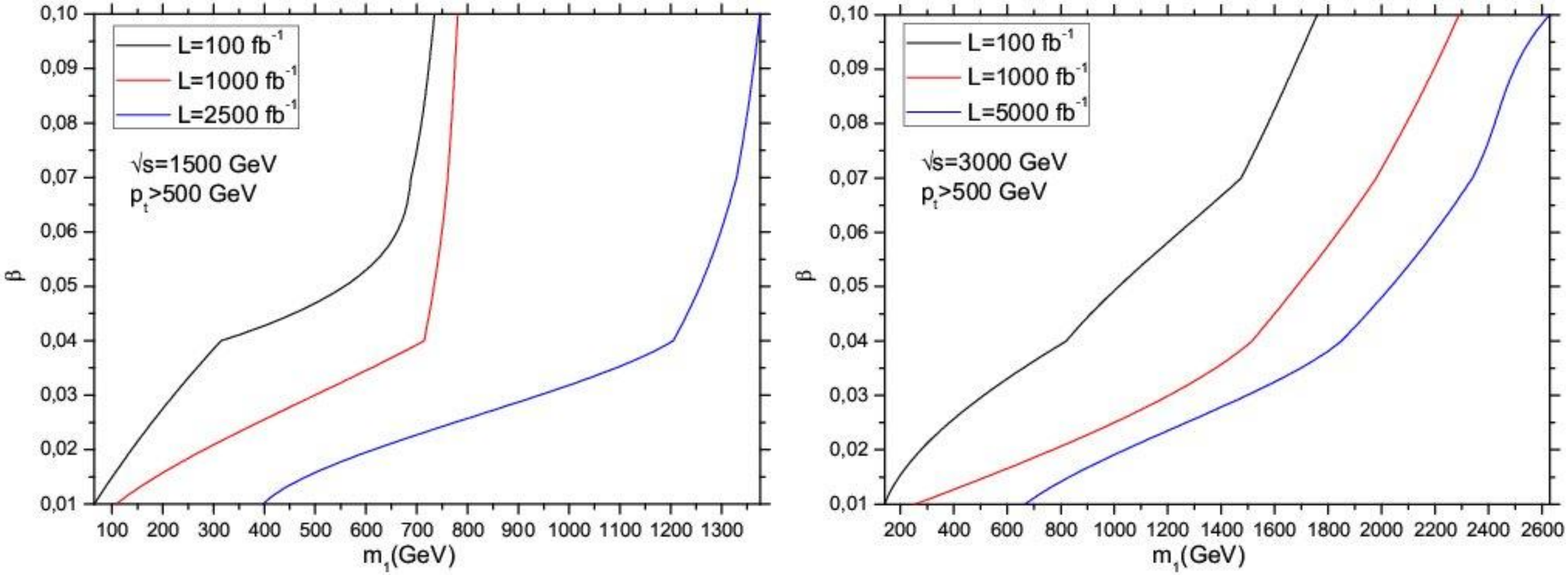
d = number of EDs



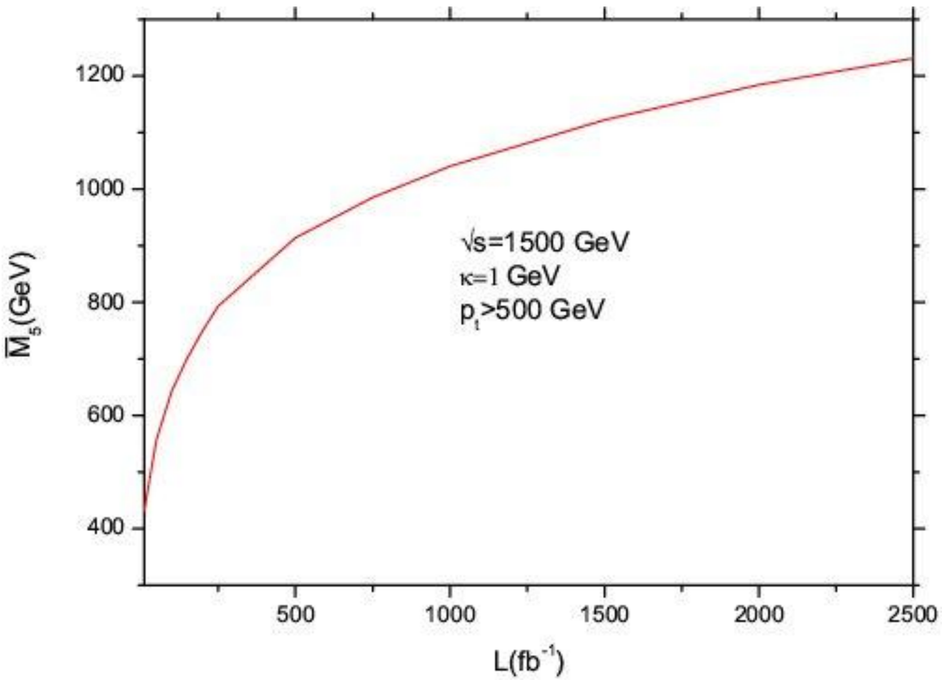


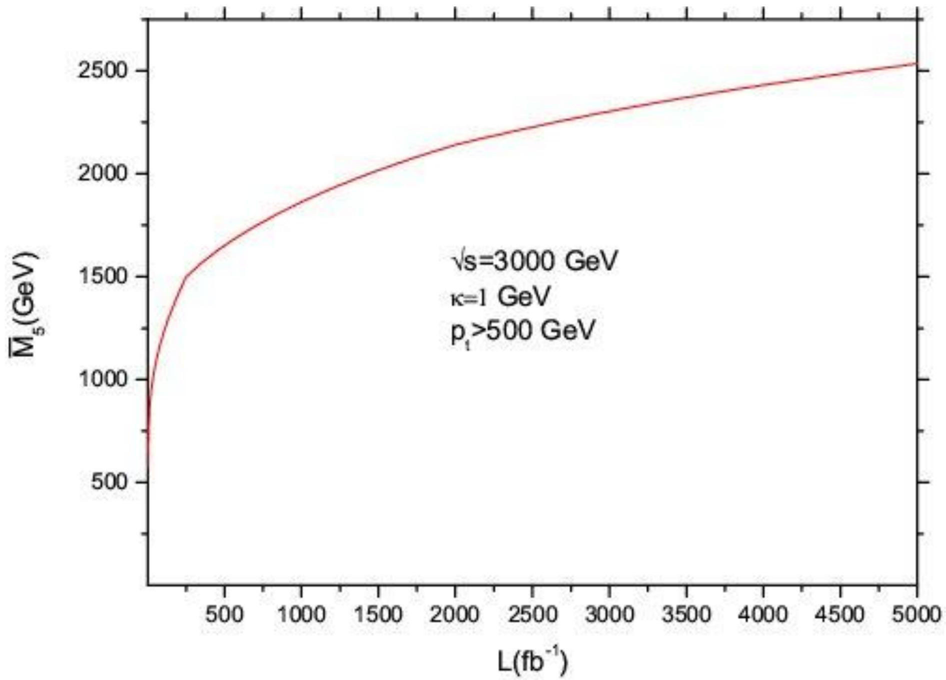
95% C.L. CLIC search bounds in the ADD model with the HLZ convention for √s = 1500 GeV and 3000 GeV, p_t > 500 GeV as a function of integrated LHC luminosity L

m_1 = mass of the lightest graviton, $\beta = \kappa/M_{Pl}$



95% C.L. CLIC exclusion regions for the parameters m_1, β in the RS model for $\sqrt{s}$ = 1500 GeV and 3000 GeV as a function of integrated LHC luminosity L





95% C.L. CLIC search bounds in the RSSC model for
√s = 1500 GeV and 3000 GeV, p_t > 500 GeV
as a function of integrated LHC luminosity *L*

Много говорить не буду, а то опять чего-нибудь скажу (ЭВЧ)

Заключение

- Начальные γγ-состояния подпроцессов естественно приводят к эксклюзивному рождению с «неповреждёнными» протонами (электронами) на LHC (CLIC).
- Вычислены дифференциальное и полное сечения процесса $pp \to p\gamma\gamma p \to p'\gamma\gamma p'$ при энергии 14 ТэВ.
- С достоверностью 95% оценены значения 5-ти мерной массы Планка M_5, доступные для обнаружения на LHC в данном процессе: $M_5 = 1.37(1.74)$ ТэВ для $L=300(3000)$ фб$^{-1}$.

Заключение (продолжение)

- Данные ограничения не зависят от другого параметра модели κ, определяющего кривизну пространства-времени, при условии $\kappa \ll M_5$.
- Изучен индуцированный фотонами процесс рождения мюонной пары $e^+e^- \to e^+\gamma\gamma e^- \to e'^+\mu^+\mu^- e'^-$ на проектируемом коллайдере CLIC.
- Вычислены сечения рассеяния для энергий пучка 750 ГэВ и 1500 ГэВ и интегральных светимостей вплоть до 2500 фб$^{-1}$ и 5000 фб$^{-1}$, соответственно.
- Получены ограничения на параметры моделей с дополнительными измерениями, которые могут быть достигнуты в указанном процессе.

План дальнейшей работы

Вычисление сечений виртуального рождения аксионо-подобных частиц a (ALPs) на коллайдере CLIC в процессе, индуцированном фотонами

(e+e- → e+γγe- → e+ a e- → e+γγe-)

Спасибо за внимание!

(тов.Сухов) Вопросы есть? Вопросов нет.

Back-up slides

Measured light-by-light cross section (CMS, 2019):

σ_{exp} = 122 ± 46 (stat) ± 29 (syst) ± 4 (theo)

SM prediction (d'Enterria & Sillveria, 2013):

σ_{theo} = 138 ± 14 nb

Experimental studies of photon-induced reactions

Tevatron, CDF Collaboration, 2009:

$pp \to p\gamma\gamma p \to pl^+l^-p$ ($l = e,\mu$)

LHC (7 TeV), CMS Collaboration, 2012:

$pp \to p\gamma\gamma p \to pl^+l^-p$ ($l = e,\mu$)

LHC (7 TeV), ATLAS Collaboration, 2014:

$pp \to p\gamma\gamma p \to pl^+l^-p$ ($l = e,\mu$)

LHC (8 TeV), ATLAS Collaboration, 2016:

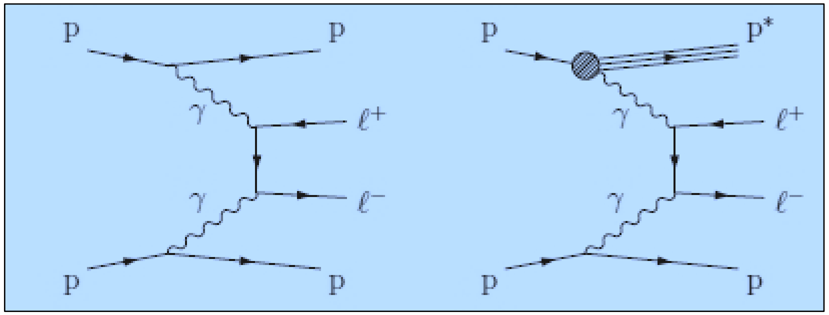
$pp \to p\gamma\gamma p \to pl^+l^-p$ ($l = e,\mu$)

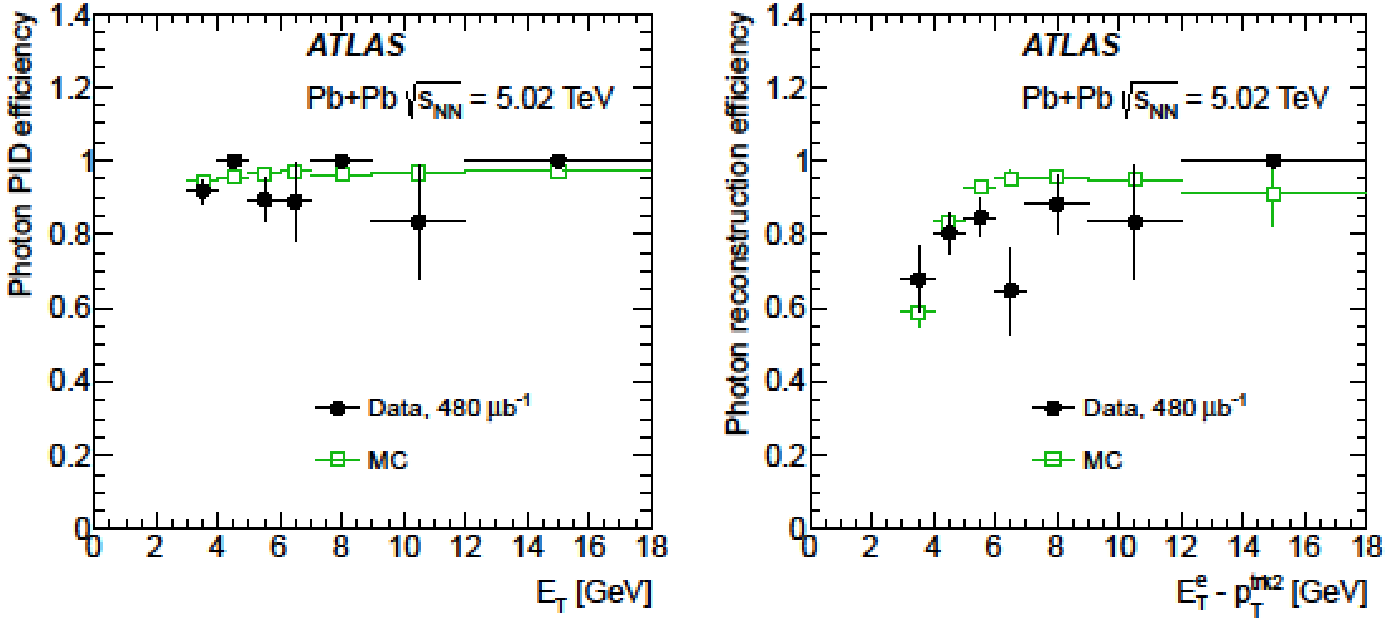
First observation of proton-tagging $\gamma\gamma$ collision:
LHC (13 TeV), CMS-TOTEM Collaboration, 2018

$pp \to p\gamma\gamma p^* \to pl^+l^-p^*$

$m(l^+l) > 110$ GeV

12 $\mu^+\mu^-$, 8 e^+e^-





Photon identification (a) and reconstruction efficiency (b)

Cut / Process	Signal (full)	Signal with (without) f.f (EFT)	Excl.	DPE	DY, di-jet + pile up	$\gamma\gamma$ + pile up
$[0.015 < \xi_{1,2} < 0.15$, $p_{T1,(2)} > 200, (100)$ GeV]	65	18 (187)	0.13	0.2	1.6	2968
$m_{\gamma\gamma} > 600$ GeV	64	17 (186)	0.10	0	0.2	1023
$[p_{T2}/p_{T1} > 0.95$, $\lvert\Delta\phi\rvert > \pi - 0.01]$	64	17 (186)	0.10	0	0	80.2
$\sqrt{\xi_1\xi_2 s} = m_{\gamma\gamma} \pm 3\%$	61	16 (175)	0.09	0	0	2.8
$\lvert y_{\gamma\gamma} - y_{pp}\rvert < 0.03$	60	12 (169)	0.09	0	0	0

Number of signal events and background events

(Fichet et. al, 2015)

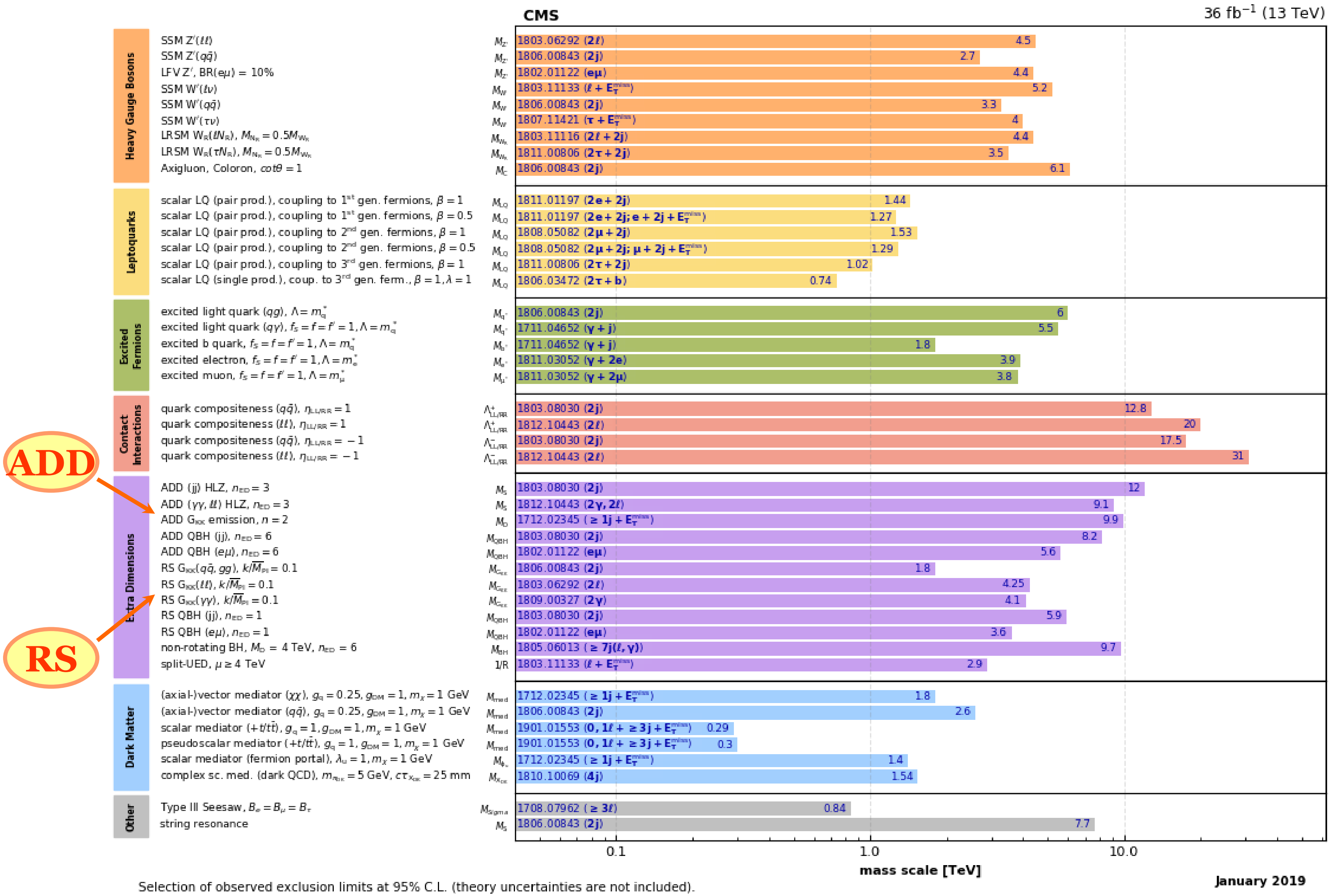
Statistical significance

$$S = \sqrt{2\left[\left(N_S + N_B\right)\ln\left(1 + N_S / N_B\right) - N_S\right]}$$

N_S (N_B) - number of signal (background) events

$$S \approx \frac{N_S}{\sqrt{N_B}}, \quad N_S << N_B$$

Overview of CMS EXO results



Original Randall-Sundrum solution

(Randall & Sundrum, 1999)

$$\sigma_{\mathrm{RS}}(y) = \kappa \,|\, y \,|$$

$$\Lambda_{\mathrm{RS}} = -24 M_5^3 \kappa^2, \quad (\Lambda_1)_{\mathrm{RS}} = -(\Lambda_2)_{\mathrm{RS}} = 24 M_5^3 \kappa$$

$$\rightarrow \quad \sigma'_{\mathrm{RS}}(y) = \kappa \varepsilon(y) \qquad \sigma''_{\mathrm{RS}}(y) = 2\kappa \delta(y)$$

The RS solution:

- does not **explicitly** reproduce the jump on TeV brane (at $y=\pi r_c$)
- is **not symmetric** with respect to both branes (located at $y=0$ and $y=\pi r_c$)
- does not include a **constant** term

Explicit account of periodicity and Z_2-symmetry

Solution for the warp function in variable $x = y / r_c$ *(A.K., 2015)*

$$\sigma(y) = \frac{\kappa r_c}{2}\left[\left|\mathrm{Arccos}(\cos x)\right| - \left|\mathrm{Arccos}(\cos x) - \pi\right|\right] + \frac{\pi\kappa r_c}{2} - C$$

Arccos(z) is principal value of inverse cosine

$$0 \le \mathrm{Arccos}(z) \le \pi, \quad -1 \le z \le 1$$

$$\mathrm{Arccos}(\cos x) = \begin{cases} x - 2n\pi, & 2n\pi \le x \le (2n+1)\pi \\ -x + 2(n+1)\pi, & (2n+1)\pi \le x \le 2(n+1)\pi \end{cases}$$

(see, for instance, Gradshteyn &Ryzhik)

In particular, $\sigma(y) = \kappa y$ for $0 \le y \le \pi r_c$

Orbifold symmetries:

$$\sigma(y+2\pi r_c)=\sigma(y)$$ (periodicity)

$$\sigma(-y)=\sigma(y)$$ (Z_2 symmetry)

1-st derivative of σ(y): $(y \neq \pi n r_c\,,\ n = 0, \pm 1, \pm 2, \ldots)$

$$\sigma'(y)=\kappa\,\mathrm{sign}\left[\sin(y/r_c)\right]$$

$$\sigma'(-y)=-\sigma'(y)$$

2-nd derivative of σ(y):

$$\sigma''(y)=\kappa\sum_{n=-\infty}^{\infty}\left[\delta\left(y+2\pi n r_c\right)-\delta\left(y-\pi r_c+2\pi n r_c\right)\right]$$

$$\sigma''(-y)=\sigma''(y)$$

III. C = κπr$_c$ /2

$$\sigma(0) = -\sigma(\pi r_c) = \kappa\pi r_c / 2$$

"symmetric" scheme

$$\rightarrow \quad M_{\mathrm{Pl}}^2 \simeq \frac{2M_5^3}{\kappa}\sinh(2\pi\kappa r_c)$$

Masses of gravitons $m_n \simeq x_n \kappa \exp(-\kappa\pi r_c / 2)$

Let $M_5 = 2\cdot 10^9\,\mathrm{GeV},\ \kappa = 10^4\ \mathrm{GeV}$

$$\rightarrow \quad m_n \simeq 3.7\, x_n\, (\mathrm{MeV})$$

(A.K., 2015)

Almost continuous spectrum of KK gravitons

RSSC model vs.ADD model

RSSC model is not equivalent to the ADD model with one flat ED of size R=(πκ)$^{-1}$ up to κ ≈10^{-18} eV

Hierarchy relation for small κ

$$M_{\mathrm{Pl}}^2 \cong \frac{M_5^3}{\kappa}\left[\exp(2\pi\kappa r_c)-1\right] \xrightarrow{2\pi\kappa r_c \ll 1} M_5^3(2\pi r_c)$$

But the inequality $2\pi\kappa r_c \ll 1$ **means that**

$$\kappa \ll \frac{M_5^3}{M_{\mathrm{Pl}}^2} \approx 0.17\cdot 10^{-18}\left(\frac{M_5}{1\,\mathrm{TeV}}\right)^3 \mathrm{eV}$$

Recall that $m_n \cong z_n \kappa$

$$\sum_{n=1}^{\infty} \frac{1}{z_n^2 - z^2} = \frac{1}{2z} \frac{J_{\nu+1}(z)}{J_\nu(z)}, \quad J_\nu(z_n) = 0$$

$$S(s) \approx -\frac{1}{4\bar{M}_5^3 \sqrt{s}} \frac{\sin 2A + i \sinh 2\varepsilon}{\cos^2 A + \sinh^2 \varepsilon}$$

(A.K, 2006)

where $A = \frac{\sqrt{s}}{\kappa}, \ \varepsilon = \frac{\eta}{2}\left(\frac{\sqrt{s}}{\bar{M}_5}\right)^3$

Margarita:

'…most of all I'm struck that there's room for all this.'

Koroviev:

`The most uncomplicated thing of all! For someone well acquainted with the fifth dimension, it costs nothing to expand space to the desired proportions. I'll say more, respected lady - to devil knows what proportions!

M. Bulgakov. The Master and Margarita. Chapter 22. By Candlelight.