Критическая метагравитация: экранировка космологической постоянной и возникновение массивно-гравитонных темных компонент Вселенной

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АННОТАЦИЯ

В общих рамках модифицированной гравитации - метагравитации - строится теория критической метагравитации, согласованно объединяющая Вейль-поперечную гравитацию и хиггсовский механизм для гравитации. Показывается, что эта теория может дать решение проблемы космологической постоянной с появлением массивных тензорного и скалярного гравитонов в качестве темных компонент Вселенной.

Cosmological Constant (CC) problem(s)

1. Naturalness problem
2. Coincidence problem
3. Quantum stability problem

Building blocks

1. Weyl transverse gravity
   K.-I. Izawa, Prog. Theor. Phys. **93**, 615 (1995)

2. Higgs mechanism for gravity
   A. H. Chamseddine and V. Muhanov, JHEP 1008:011 (2010)

3. Quartet-metric gravity/metagravity
   YFP, EPJC **76**, 215 (2016)

Contents

1. Generic metagravity
    1.1 Effective texture
    1.2 Emergent structure

2. Critical metagravity
    Full nonlinear theory
    Weak-field limit

1. GENERIC METAGRAVITY

Effective action

Basic action:

$$S[g_{\mu\nu}, X^a] = \int \mathcal{L}(g_{\mu\nu}, X^a)\, d^d x \tag{1}$$

Metametric

$$\omega_{\mu\nu} \equiv \partial_\mu X^a \partial_\nu X^b \eta_{ab}$$

Scalar graviton:

$$\sigma = \ln(\sqrt{-g}/\sqrt{-\omega})$$

Effective metric (disformal):

$$\tilde{g}_{\mu\nu} = k_g(\sigma) g_{\mu\nu} + k_\omega(\sigma)\omega_{\mu\nu}$$

Effective action:

$$S[g_{\mu\nu}, X^a] = \int \tilde{L}(\tilde{g}_{\mu\nu}, \omega_{\mu\nu}, \sigma)\sqrt{-\tilde{g}}\, d^d x \tag{2}$$

Meta-Higgs field:

$$\tilde{\omega}^\mu_\nu = \tilde{g}^{\mu\lambda}\omega_{\lambda\nu}$$

Gauge symmetry

GDiff gauge symmetry

$$\tilde{D}_\xi \tilde{g}_{\mu\nu} = \tilde{g}_{\mu\lambda}\tilde{\nabla}_\nu \xi^\lambda + \tilde{g}_{\nu\lambda}\tilde{\nabla}_\mu \xi^\lambda$$

Weyl rescaling

$$\Delta_\zeta g_{\mu\nu} = \zeta g_{\mu\nu}$$
$$\Delta_\zeta X^a = 0$$
$$\Delta_\zeta \omega_{\mu\nu} = \Delta_\zeta \omega = 0$$
$$\Delta_\zeta \sigma = 1/2\, g^{\mu\nu}\Delta_\zeta g_{\mu\nu} = 2\zeta$$

Conformal effective metric: $k_\omega = 1,\ k_g = e^{\bar{\kappa}},\ \bar{g}_{\mu\nu} = e^{\bar{\kappa}(\sigma)} g_{\mu\nu}$

$$\begin{aligned} \Delta_\zeta \bar{g}_{\mu\nu} &= \zeta(1+2\bar{\kappa}')\bar{g}_{\mu\nu} \\ \Delta_\zeta \sqrt{-\bar{g}}/\sqrt{-\bar{g}} &= 1/2\, \bar{g}^{\mu\nu}\Delta_\zeta \bar{g}_{\mu\nu} = 2\zeta(1+2\bar{\kappa}') \end{aligned}$$

Meta-Higgs field: $\bar{g}^\mu_\nu = \bar{g}^{\mu\lambda}\omega_{\lambda\nu}$

$$\Delta_\zeta \bar{\omega}^\mu_\nu = \omega_{\nu\lambda}\Delta_\zeta \bar{g}^{\lambda\mu} = -\zeta(1+2\bar{\kappa}')\bar{\omega}^\mu_\nu$$

WGDiff gauge symmetry: $\bar{\kappa}' = -1/2$

CC screening

Generic Field Equations (FEs):

$$
\begin{aligned}
\frac{\delta \bar{L}}{\delta \bar{g}^{\mu\nu}} - \frac{1}{4}\frac{\delta \bar{L}}{\delta \bar{g}^{\kappa\lambda}}\bar{g}^{\kappa\lambda}\bar{g}_{\mu\nu} &= 0, \\
(1+2\bar{\kappa}')\Big(\frac{1}{2}\frac{\delta \bar{L}}{\delta \bar{g}^{\kappa\lambda}}\bar{g}^{\kappa\lambda} - \bar{L}\Big) - \frac{\delta \bar{L}}{\delta \sigma} &= 0, \\
\bar{\nabla}_\lambda\Big[\frac{\delta \bar{L}}{\delta \omega_{\kappa\lambda}} X_{\kappa a} - \frac{1}{2}\Big(\frac{1}{2}\frac{\delta \bar{L}}{\delta \bar{g}^{\rho\sigma}}\bar{g}^{\rho\sigma} - \bar{L}\Big) X^{-1\lambda}{}_{a}\Big] &= 0
\end{aligned}
$$

Decomposition:

$$
\bar{L} \equiv \bar{L}_v + \bar{L}_G(\bar{g}_{\mu\nu}, \omega_{\mu\nu}, \sigma), \quad \bar{L}_v \equiv -M_{\rm Pl}^2 \bar{\Lambda}
$$

Three sectors: transversal/traceless, longitudinal/trace and metascalar

Critical metagravity: $\bar{\kappa}' = -1/2$

Tensor-scalar partition model

Tensor-scalar-matter:

$$\bar{L}_G \equiv \bar{L}_t + \bar{L}_{sm} = \bar{L}_g(\bar{g}_{\mu\nu}) + \bar{L}_\omega(\bar{g}_{\mu\nu}, \omega_{\mu\nu}) + \bar{L}_{sm}(\bar{g}_{\mu\nu}, \sigma, \phi^I)$$

Metric-meta-Higgs:

$$\bar{L}_g = -\frac{1}{2} M_{\rm Pl}^2 R(\bar{g}_{\mu\nu}), \quad \bar{L}_\omega = -\bar{V}_\omega(\bar{\omega}^\mu_\nu)$$

Scalar:

$$\bar{L}_s = \frac{1}{2} M_s^2 k_s(\sigma) \bar{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V_s(\sigma)$$

Unreduced FEs

Generic FEs:

$$\begin{aligned}\bar{R}_{\mu\nu} - \frac{1}{4}\bar{R}\bar{g}_{\mu\nu} - \frac{1}{M_{\rm Pl}^2}(\bar{T}_{\mu\nu} - \frac{1}{4}\bar{T}\bar{g}_{\mu\nu}) &= 0, \\ \frac{1}{4}(1+2\bar{\kappa}')(\bar{R}+4\bar{\Lambda}+\frac{1}{M_{\rm Pl}^2}\bar{T}) - \frac{1}{M_{\rm Pl}^2}\frac{\delta \bar{L}_{sm}}{\delta\sigma} &= 0, \\ \bar{\nabla}_\lambda\Big(\frac{1}{M_{\rm Pl}^2}\bar{g}^{\lambda\rho}\frac{\partial \bar{V}_\omega}{\partial\bar{\omega}^\rho_\kappa}X_{\kappa a} + \frac{1}{8}(\bar{R}+4\bar{\Lambda}+\frac{1}{M_{\rm Pl}^2}\bar{T})X^{-1\lambda}{}_a\Big) &= 0, \\ \delta\bar{L}_{sm}/\delta\phi^I &= 0\end{aligned}$$

Energy-momentum tensor:

$$\bar{T}_{\mu\nu} = \bar{T}_{\omega\mu\nu} + \bar{T}_{sm\mu\nu}, \quad \bar{T} = \bar{g}^{\mu\nu}\bar{T}_{\mu\nu} = \bar{T}_\omega + \bar{T}_{sm}$$

Meta-Higgs:

$$\bar{T}_{\omega\mu\nu} \equiv -\Big(\frac{\partial \bar{V}_\omega}{\partial\bar{\omega}^\mu_\lambda}\bar{g}_{\rho\nu} + \frac{\partial \bar{V}_\omega}{\partial\bar{\omega}^\nu_\lambda}\bar{g}_{\rho\mu}\Big)\bar{\omega}^\rho_\lambda + \bar{V}_\omega\bar{g}_{\mu\nu} \tag{0}$$

Scalar-matter:

$$\bar{T}_{sm\mu\nu} \equiv \frac{2}{\sqrt{-\bar{g}}}\frac{\partial(\sqrt{-\bar{g}}\bar{L}_{sm})}{\partial\bar{g}^{\mu\nu}} = 2\frac{\partial\bar{L}_{sm}}{\partial\bar{g}^{\mu\nu}} - \bar{L}_{sm}\bar{g}_{\mu\nu}, \tag{1}$$

Covariant conservation condition

Reduced Bianchi identity:

$$-\frac{1}{2}\partial_\mu\left(\bar{\kappa}'(\bar{R}+4\bar{\Lambda}+\frac{1}{M_{\rm Pl}^2}\bar{T})\right)=\frac{1}{M_{\rm Pl}^2}\bar{\nabla}^\lambda\bar{\Theta}_{\lambda\mu}$$

Total effective energy-momentum tensor:

$$\bar{\Theta}_{\mu\nu}\equiv\bar{T}_{\mu\nu}+\Delta\bar{T}_{\boldsymbol{sm}\mu\nu}=\bar{T}_{\omega\mu\nu}+\bar{T}_{\boldsymbol{sm}\mu\nu}+\Delta\bar{T}_{\boldsymbol{sm}\mu\nu}\equiv\bar{T}_{\omega\mu\nu}+\bar{\Theta}_{\boldsymbol{sm}\mu\nu}$$

Additional scalar contribution:

$$\Delta\bar{T}_{\boldsymbol{sm}\mu\nu}\equiv-\frac{\delta\bar{L}_{\boldsymbol{sm}}}{\delta\sigma}\bar{g}_{\mu\nu}$$

Covariant conservation condition:

$$\bar{\nabla}^\lambda\bar{\Theta}_{\lambda\mu}=0$$

Reduced case

Constraint:

$$\frac{1}{2}\bar{\kappa}'(\bar{R} + 4\bar{\Lambda} + \frac{1}{M_{\rm Pl}^2}\bar{T}) = \bar{\Lambda}_0 - \bar{\Lambda}$$

Reduced FEs:

$$\begin{aligned}
\bar{R}_{\mu\nu} - \frac{1}{4}\bar{R}\bar{g}_{\mu\nu} - \frac{1}{M_{\rm Pl}^2}(\bar{T}_{\mu\nu} - \frac{1}{4}\bar{T}\bar{g}_{\mu\nu}) &= 0, \\
\frac{1}{4}(\bar{R} + 4\bar{\Lambda}_0 + \frac{1}{M_{\rm Pl}^2}\bar{T}) - \frac{1}{M_{\rm Pl}^2}\frac{\delta \bar{L}_{sm}}{\delta\sigma} &= 0, \\
\bar{\nabla}_\lambda\left(\frac{1}{M_{\rm Pl}^2}\bar{g}^{\lambda\rho}\frac{\partial \bar{V}_\omega}{\partial \bar{\omega}^\rho_\kappa}X_{\kappa a} + \frac{1}{4}(\bar{\Lambda} - \bar{\Lambda}_0 + \frac{1}{M_{\rm Pl}^2}\frac{\delta \bar{L}_{sm}}{\delta\sigma})X^{-1}{}^\lambda_a\right) &= 0
\end{aligned}$$

GR-like form:

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{R}\bar{g}_{\mu\nu} - \bar{\Lambda}_0\bar{g}_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}\bar{\Theta}_{\mu\nu}$$

2. CRITICAL METAGRAVITY (CM)

Unreduced case

Critical index: $\hat{\kappa}' = -1/2$

$$\hat{g}_{\mu\nu} = e^{-\sigma/2} g_{\mu\nu} = (\omega/g)^{1/4} g_{\mu\nu}, \quad \hat{g} = \omega$$

FEs:

$$\hat{R}_{\mu\nu} - \frac{1}{4}\hat{R}\hat{g}_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}(\hat{T}_{\mu\nu} - \frac{1}{4}\hat{T}\hat{g}_{\mu\nu}),$$

$$\hat{\nabla}_\lambda \left[\frac{1}{M_{\rm Pl}^2} \hat{g}^{\lambda\rho} \frac{\partial \hat{V}_\omega}{\partial \hat{\omega}^\rho_\kappa} X_{\kappa a} + \frac{1}{8}\left(\hat{R} + 4\hat{\Lambda} + \frac{1}{M_{\rm Pl}^2}\hat{T}\right) X^{-1}{}^{\lambda}_{a} \right] = 0,$$

$$\frac{\delta \hat{L}_{sm}}{\delta \sigma} = \frac{\delta \hat{L}_{sm}}{\delta \phi^I} = 0$$

$$\hat{T}_{\mu\nu} \equiv \hat{T}_{\omega\mu\nu} + \hat{T}_{sm\mu\nu}, \quad \hat{T} = g^{\mu\nu}\hat{T}_{\mu\nu}$$

Reduced case

Covariant conservation condition:

$$\partial_\mu(\hat{R} + \hat{T}/M_{\rm Pl}^2) = 4\hat{\nabla}^\lambda \hat{T}_{\lambda\mu} = 0,$$

Constraint:

$$\hat{R} + 4\hat{\Lambda}_0 + \hat{T}/M_{\rm Pl}^2 = 0,$$

FEs:

$$\begin{aligned}
\hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} - \hat{\Lambda}_0\hat{g}_{\mu\nu} - \frac{1}{M_{\rm Pl}^2}(\hat{T}_{\omega\mu\nu} + \hat{T}_{sm\mu\nu}) &= 0, \\
\hat{\nabla}_\lambda\left(\frac{1}{M_{\rm Pl}^2}\hat{g}^{\lambda\rho}\frac{\partial\hat{V}_\omega}{\partial\hat{\omega}^\rho_\kappa}X_{\kappa a} + \frac{1}{4}(\hat{\Lambda} - \hat{\Lambda}_0)X^{-1}{}^\lambda_a\right) &= 0
\end{aligned}$$

$$\delta\hat{L}_{sm}/\delta\sigma = \delta\hat{L}_{sm}/\delta\phi^I = 0$$

Weak-field limit: flat background

Basic fields:

$$g_{\mu\nu} \equiv g_{\star\mu\nu} + h_{\mu\nu}, \quad X^a \equiv X^a_\star + \chi^a$$

Effective fields:

$$\begin{aligned} \sigma &\equiv \sigma_\star + s, \\ \hat{g}_{\mu\nu} = e^{-\sigma/2} g_{\mu\nu} &\equiv \hat{g}_{\star\mu\nu} + \hat{h}_{\mu\nu}, \\ \hat{\omega}^\mu_\nu = \hat{g}^{\mu\lambda}\omega_{\lambda\nu} &\equiv \hat{\omega}_\star{}^\mu_\nu - \hat{f}^\mu_\nu \end{aligned}$$

Meta-coordinates: $X^\alpha = \delta^\alpha_a X^a_\star$

Flat background:

$$\omega_{\star\alpha\beta} = g_{\star\alpha\beta} = \hat{g}_{\star\alpha\beta} = \eta_{\alpha\beta}, \quad \hat{\omega}_\star{}^\alpha_\beta = \hat{\omega}_\star^{-1}{}^\alpha_\beta = \delta^\alpha_\beta, \quad \sigma_\star = 0$$

Linear approximation (LA)

Basic fields:

$$
\begin{aligned}
g^{\pm 1}_{\alpha\beta} &= \eta_{\alpha\beta} \pm h_{\alpha\beta}, \\
X^{\pm 1}{}^{\beta}_{\alpha} &= \delta^{\beta}_{\alpha} \pm \partial_\alpha \chi^\beta, \\
\omega^{\pm 1}_{\alpha\beta} &= \eta_{\alpha\beta} \pm (\partial_\alpha \chi_\beta + \partial_\beta \chi_\alpha)
\end{aligned}
$$

Effective fields:

$$
\hat{g}^{\pm 1}_{\alpha\beta} = \eta_{\alpha\beta} \pm \hat{h}_{\alpha\beta}, \quad \hat{\omega}^{\pm 1}{}^{\alpha}_{\beta} \equiv \delta^{\alpha}_{\beta} \mp \hat{f}^{\alpha}_{\beta}
$$

$$
\begin{aligned}
\hat{h}_{\alpha\beta} &\equiv h_{\alpha\beta} - 1/4(h - 2\partial\chi)\eta_{\alpha\beta}, \quad \hat{h} = 2\partial\chi \\
\eta_{\alpha\gamma}\hat{f}^{\gamma}_{\beta} \equiv \hat{f}_{\alpha\beta} &= \hat{h}_{\alpha\beta} - (\partial_\alpha \chi_\beta + \partial_\beta \chi_\alpha), \quad \hat{f} = 0
\end{aligned}
$$

WGDiff gauge symmetry

Basic fields:

$$\begin{aligned} \chi^\alpha &\to \chi^\alpha + \xi^\alpha, \\ \omega_{\alpha\beta} &\to \omega_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha, \\ h_{\alpha\beta} &\to h_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha + \zeta \eta_{\alpha\beta} \end{aligned}$$

Effective fields:

$$\begin{aligned} \hat{h}_{\alpha\beta} &\to \hat{h}_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha, \\ \hat{f}_{\alpha\beta} &\to \hat{f}_{\alpha\beta}, \quad s \to s + 2\zeta \end{aligned}$$

Unitary gauge: $\xi_\alpha = 0$

$$\hat{h}_{\alpha\beta} = \hat{f}_{\alpha\beta} \tag{2}$$

Meta-Higgs field

Meta-Higgs potential:

$$\hat{V}_\omega = v_0 + \frac{1}{2} v_2 (\hat{\omega}^\kappa_\lambda - \frac{1}{4}\hat{\omega}^\rho_\rho \delta^\kappa_\lambda)(\hat{\omega}^\lambda_\kappa - \frac{1}{4}\hat{\omega}^\rho_\rho \delta^\lambda_\kappa)$$

LA:

$$\hat{V}_\omega = \frac{1}{2} v_2 \hat{f}^\alpha_\beta \hat{f}^\beta_\alpha$$

Energy-momentum tensor:

$$\hat{T}_{\omega\alpha\beta} = 2 v_2 \hat{f}_{\alpha\beta}, \quad \hat{T}_\omega \equiv \hat{T}_{\omega\alpha\beta}\eta^{\alpha\beta} = 0$$

FEs

Tensor Ricci:

$$\hat{R}_{\alpha\beta} = \frac{1}{2}(\partial_\alpha \partial_\gamma \hat{f}^\gamma_\beta + \partial_\beta \partial_\gamma \hat{f}^\gamma_\alpha - \partial^2 \hat{f}_{\alpha\beta})$$

Ricci scalar:

$$\hat{R} \equiv \hat{R}_{\alpha\beta}\eta^{\alpha\beta} = \partial_\gamma \partial_\delta \hat{f}^{\gamma\delta}$$

FEs:

$$\begin{aligned} \partial_\alpha \partial_\gamma \hat{f}^\gamma_\beta + \partial_\beta \partial_\gamma \hat{f}^\gamma_\alpha - \partial^2 f_{\alpha\beta} - \frac{1}{2}\partial_\gamma \partial_\delta \hat{f}^{\gamma\delta}\eta_{\alpha\beta} &= m_g^2 \hat{f}_{\alpha\beta}, \\ \partial_\alpha (m_g^2 \hat{f}^\alpha_\beta - \frac{1}{2}\partial_\gamma \partial_\delta \hat{f}^{\gamma\delta}\delta^\alpha_\beta) &= 0 \end{aligned}$$

$$v_2 \equiv 1/4\, M_{\mathrm{Pl}}^2 m_g^2$$

Conservation condition:

$$\partial_\gamma \hat{f}^{\gamma\alpha} = 0$$

Constraint:

$$\hat{\Lambda}_0 = -1/4 \partial_\delta \partial_\gamma \hat{f}^{\delta\gamma} = 0$$

Massive gravitons

Tensor graviton (10-4-1=5):

$$(\partial^2 + m_g^2)\hat{f}_{\alpha\beta} = 0, \quad \partial^\beta \hat{f}_{\alpha\beta} = 0, \quad \eta^{\alpha\beta}\hat{f}_{\alpha\beta} = 0, \quad \text{à la F-P}$$

Massless limit: $m_g \to 0$
Residual on-mass-shell gauge symmetry(5-3=2):

$$\hat{f}_{\alpha\beta} \to \hat{f}_{\alpha\beta} - (\partial_\alpha \hat{\varphi}_\beta + \partial_\beta \hat{\varphi}_\alpha), \quad \partial^2 \hat{\varphi}_\alpha = 0, \quad \partial\hat{\varphi} = 0$$

Scalar graviton (5+1=6=2+3+1):

$$(\partial^2 + m_s^2)s = 0$$

CM critical points

Lagrangian:

$$\hat{L} = \hat{L}(\hat{g}_{\mu\nu}, \hat{\omega}^{\mu}_{\nu}, \sigma; \hat{\Lambda}, m_g, m_s)$$

Restrictions:

$$\hat{g} = \omega, \quad \hat{\omega} = 1$$

FEs:

$$\begin{aligned} \hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} - \hat{\Lambda}_0\hat{g}_{\mu\nu} - \frac{1}{M_{\rm Pl}^2}(\hat{T}_{\omega\,\mu\nu} + \hat{T}_{sm\mu\nu}) &= 0, \\ \hat{\nabla}_\lambda\Big(\frac{1}{M_{\rm Pl}^2}\hat{g}^{\lambda\rho}\frac{\partial\hat{V}_\omega}{\partial\hat{\omega}^\rho_\kappa}X_{\kappa a} + \frac{1}{4}(\hat{\Lambda} - \hat{\Lambda}_0)X^{-1\lambda}{}_a\Big) &= 0 \end{aligned}$$

Requirements to the next-to-GR theory:

1. Improve theoretical consistency of GR
   - 1.1 CC problem:
     Naturalness problem
     Coincidence problem
     Radiative stability problem
   - 1.2 Consistent massive tensor graviton
2. Reproduce GR in a limit
3. Predict new phenomena beyond GR: DE, DM, etc.

Resume: CM vs. GR

GR: $(g_{\mu\nu} \mid \mathit{GDiff})$

Generic metagravity: $(g_{\mu\nu}, X^a \mid \mathit{GDiff})$
$4 = 3(\text{tensor}) + 1(\text{ghost})$

CM: $(\hat{g}_{\mu\nu}, X^a \mid \mathit{WGDiff})$

1. $4 = 3(\text{tensor}) + 1(\text{gauge})$
2. Explicit Weyl-scale symmetry violation:
   $4 = 3(\text{tensor}) + 1(\text{scalar})$

CM: next-to-GR EFT of gravity as

platform for beyond-GR, DE, DM, etc.

Спасибо за внимание!