

# Central elastic scattering

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# Overview

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- 4 Central elastic scattering
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# Abstract

We comment on phase selection of the scattering amplitude, emphasizing that the elastic overlap function should have a central impact parameter profile at high energies and highlighting the role of the reflective scattering mode at the LHC energies. Emerging problems with the use of peripheral impact parameter dependence of the elastic overlap function are explicitly indicated. Their solution is an elimination of the phases connected to peripheral form of the elastic overlap function. Contrary, we adhere to a relative peripheral form of the *inelastic* overlap function with an additional new feature of a maximum at nonzero value of the impact parameter at the highest energies. Phenomenologically, the dynamics of hadron scattering is motivated by a hadron structure with a hard central core presence.

$$\rho(s) \sim \frac{d \ln \sigma_{tot}(s)}{d \ln s}. \quad (1)$$

- TOTEM@LHC 0.14→0.1 Maximal Odderon ?

- TOTEM@LHC+D0@FNAL:

**New major discovery: ODDERON**

**STRONG INTERACTIONS NEWS:**

Odderon discovered

“This result probes the deepest features of quantum chromodynamics, notably that gluons interact between themselves and that an odd number of gluons are able to be ‘colourless’, thus shielding the strong interaction,” says TOTEM spokesperson Simone Giani of CERN. “A notable feature of this work is that the results are produced by joining the LHC and Tevatron data at different energies.”

<https://cerncourier.com/a/odderon-discovered/>  
March 9

<https://indico.cern.ch/event/955960/contributions/4246688/attachments/2202676/3726009/fwdphys2021.pdf>

- Maximal Odderon contradicts to the black disk limit saturation:  
J. Finkelshtein, H.M. Fried, C.-I. Tan, Phys. Lett. B 232 (1989) 257.
- Maximal Odderon contradicts to the unitarity saturation:  
S.M. T., Phys. Lett. B 682 (2009) 40.
- But, Maximal Odderon is consistent with QFT constraints (exponential unitarization used):  
P. Gauron, L. Lukaszuk, B. Nicolescu, Phys. Lett. B 294 (1992) 298.

$$\text{Im}F(s, t = 0) \sim s \ln^2 s,$$

$$\text{Re}F(s, t = 0) \sim s \ln^2 s$$

$$\text{Re}F(s, b) = ???$$

$$\arg F(s, t) = ???$$

## Introduction. Role of the scattering amplitude phase

$$f(s, b) = u(s, b)/[1 + u(s, b)]. \quad (2)$$

$$u(s, b) = f(s, b)/[1 - f(s, b)]. \quad (3)$$

$$f(s, b) \simeq g(s) \exp(-\mu b)$$

and

$$u(s, b)/f(s, b) \rightarrow 1 \quad (4)$$

Energy dependence at  $s \rightarrow \infty$  and fixed  $b$ . Assuming saturation of the unitarity limit  $f \rightarrow 1$  at  $s \rightarrow \infty$ , we obtain  $u(s, b) \rightarrow \infty$ .

$$u(s, b) = \sigma_{el}(s, b)/\sigma_{inel}(s, b). \quad (5)$$

$$X(s) \equiv \sigma_{el}(s)/\sigma_{tot}(s) \rightarrow \text{phase sensitive.}$$

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## Saturation

Black disc

$$\sigma_{tot}(s) \sim \ln^2 s, \quad (6)$$

$$\sigma_{inel}(s) \sim \ln^2 s, \quad X(s) \rightarrow 1/2, \quad \sigma_{sd}(s)/\sigma_{inel}(s) \rightarrow 0, \quad (7)$$

Unitarity

$$\sigma_{tot}(s) \sim \ln^2 s, \quad (8)$$

$$\sigma_{inel}(s) \sim \ln s, \quad X(s) \rightarrow 1, \quad \sigma_{sd}(s)/\sigma_{inel}(s) = \text{const.} \leq 1/2. \quad (9)$$

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**FMO model:**

$$X(s) \rightarrow \text{const.} =? (\leq 1?), \quad |f(s, b)| \leq 1 =? \quad (10)$$

$$s \rightarrow \infty$$

# Problems with a peripheral form of the elastic overlap function

- $\rho$  and phase  $\arg F(s, t)$ :

$$\arg F(s, t) = \frac{\pi}{2} - \arctan \rho.$$

- $$S_l(s) = \kappa_l(s) \exp[2i\delta_l(s)] \quad (11)$$

$$h_{l,el}(s) \equiv |f_l(s)|^2$$

$$\text{Im}f_l(s) = h_{l,el}(s) + h_{l,inel}(s). \quad (12)$$

- $$\kappa_l^2(s) = 1 - 4h_{l,inel}(s). \quad (13)$$

- $$S_l = 1 + 2if_l$$

Impact parameter  $b = 2l/\sqrt{s}$ .



# Problems with a peripheral form of the elastic overlap function

- $\text{Im}f_l \rightarrow 1$ – vanishing real part of the scattering amplitude, i.e.  $\text{Re}f_l \rightarrow 0$ .

$$\text{Im}f_l(s)[1 - \text{Im}f_l(s)] = [\text{Re}f_l(s)]^2 + h_{l,inel}(s). \quad (14)$$

- $l \gg 1$ ,  $s \gg 4m_\pi^2$  and large  $2l/\sqrt{s}$ , i.e. the region of peripheral interactions at high energies.  $\text{Im}f_l(s) \ll 1$ :

$$\text{Im}f_l(s) \simeq [\text{Re}f_l(s)]^2 + h_{l,inel}(s). \quad (15)$$

$$[\text{Re}f_l(s)]^2 \leq \text{Im}f_l(s) \quad (16)$$

- Froissart–Gribov

$$f_l(s) \simeq \omega(s) \exp\left(-\mu \frac{2l}{\sqrt{s}}\right), \quad (17)$$

$\omega$  is a complex function and  $\mu$  – position of the lowest singularity in the  $t$ -channel.

- assumption:

elastic scattering dominant in a peripheral region and the contribution of the inelastic states can be neglected in Eq. (15):

$$\text{Im} f_l(s) \simeq [\text{Re} f_l(s)]^2. \quad (18)$$

$$\text{Im} f_l(s) = h_{l,el}(s) \quad (19)$$

conflict with Eq. (17)



$$\text{Im}f_l(s) \simeq h_{l,inel}(s), \quad (20)$$



$$h_{l,inel}(s) \simeq \text{Im} \omega(s) \exp\left(-\mu \frac{2l}{\sqrt{s}}\right) \quad (21)$$

$$[\text{Re}f_l(s)]^2 \ll h_{l,inel}(s) \quad (22)$$

and, consequently,

$$[\text{Re}f_l(s)]^2 \ll \text{Im}f_l(s) \quad (23)$$

- shadow nature of the elastic scattering
- semiclassical scattering picture of hadrons.

# Central elastic scattering

- Large- $b$  behavior of the scattering amplitude can be obtained from the relation similar to the Froissart–Gribov projection formula.
- At the LHC energy  $\sqrt{s} = 13$  TeV, the unitarity relation at the impact parameters range  $0 \leq b \simeq 0.4$  fm gives

$$[\text{Im}f(s, b) - 1/2]^2 + [\text{Re}f(s, b)]^2 \simeq 0. \quad (24)$$

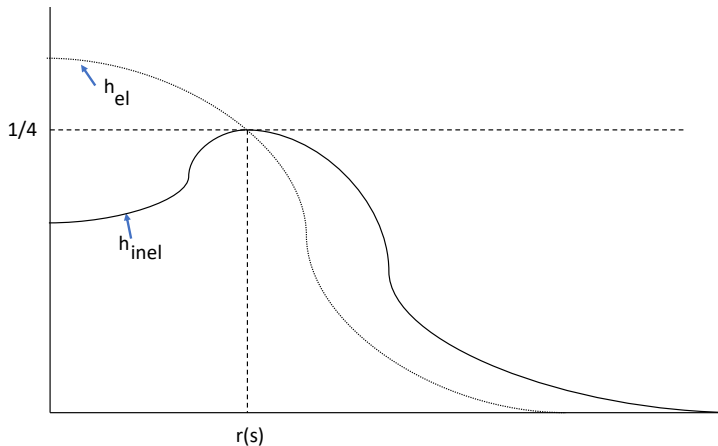
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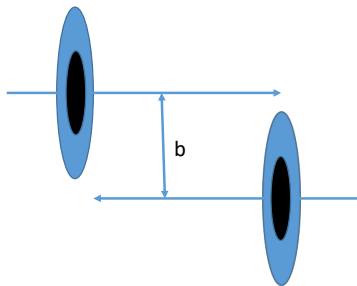
$$[f(s, b) - 1/2]^2 = 1/4 - h_{inel}(s, b) = \kappa^2(s, b)/4. \quad (25)$$

- A central profile of the elastic overlap function is encoded into the relation

$$\frac{\partial h_{el}}{\partial b} = \left( \frac{1 - S}{S} \right) \frac{\partial h_{inel}}{\partial b}. \quad (26)$$

# Central elastic scattering





$$[\text{Ref}(s, b)]^2 \ll h_{inel}(s, b) \quad (27)$$

is valid at large values of  $b$ , i.e. the limiting behavior of the ratio

$$h_{inel}(s, b)/\text{Im}f(s, b) \rightarrow 1 \quad (28)$$

Presence of the central proton's core could be associated with a chiral symmetry spontaneous breaking mechanism when the two scales  $\Lambda_{QCD}$  and  $\Lambda_\chi$  relevant for the color confinement and spontaneous chiral symmetry breaking are different