Correlations and Critical Behavior in Gluodynamics, Part II

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- “The A^2 Asymmetry and Gluon Propagators in Lattice $SU(3)$ Gluodynamics at $T \simeq T_c$,”
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  In collaboration with V.Bornyakov, V.Goy, V.Mitrjushkin.
- Talk given at the online conference
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- Sent for the Proceedings to EPJ Web of Conferences (vConf2021).
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- V. G. Bornyakov and R. N. Rogalyov,
  “Gluons in Two-Color QCD at High Baryon Density,”
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  Published Sep 10, 2021. WOS:000702302400001,

Our goal: to find critical behavior of the gluon propagators.

Outline

Known critical behavior of $\mathcal{P}$

&

Correlation between D_L and $\mathcal{P}$

$$\Longrightarrow$$

Known critical behavior of $\mathcal{P}$

$$A_\mu \to A_\mu^\Lambda = (\Lambda Z)^\dagger A_\mu (\Lambda Z) + \frac{i}{g} (\Lambda Z)^\dagger \partial_\mu (\Lambda Z).$$

$$A_\mu \to A_\mu^\Lambda = \Lambda^\dagger A_\mu \Lambda + \frac{i}{g} \Lambda^\dagger \partial_\mu \Lambda.$$

In the case of $SU(2)$

$$Z \in \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$$

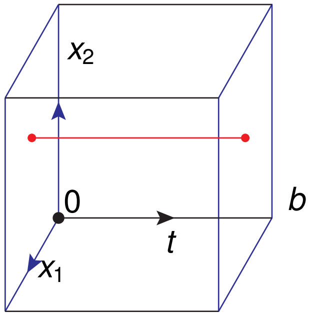
Gauge transformation is the same on both sides!

Center of the SU(3) group

$$Z \in \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} \frac{-1 \pm \imath\sqrt{3}}{2} & 0 & 0 \\ 0 & \frac{-1 \pm \imath\sqrt{3}}{2} & 0 \\ 0 & 0 & \frac{-1 \pm \imath\sqrt{3}}{2} \end{pmatrix} \right\}$$

Center transformations change the Polyakov loop

$$\mathcal{P} \to Z\mathcal{P}$$



We extend the gauge group by nonperiodic gauge transformations:

$$\Lambda(x_1, x_2, b) = Z\Lambda(x_1, x_2, 0) \text{ etc.}$$

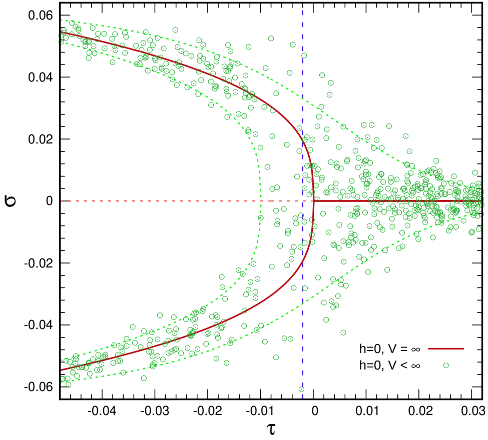
$$\mathcal{L} = T\exp\left(ig\int_0^b A_t(x_1, x_2, t)dt\right)$$

$$\mathcal{L}(x_1, x_2) \longrightarrow \mathcal{L}(x_1, x_2)Z$$

An example of such gauge transformation:

$$\Lambda = \exp\left(\frac{2\,\imath\pi t\lambda_8}{\sqrt{3}b}\right), \qquad \text{where} \qquad \lambda_8 = \frac{1}{\sqrt{3}}diag(1, 1, -2)$$

$$Z = \mathrm{diag}\left(\exp\left(\tfrac{2\imath\pi}{3}\right), \exp\left(\tfrac{2\imath\pi}{3}\right), \exp\left(-\tfrac{4\imath\pi}{3}\right)\right).$$

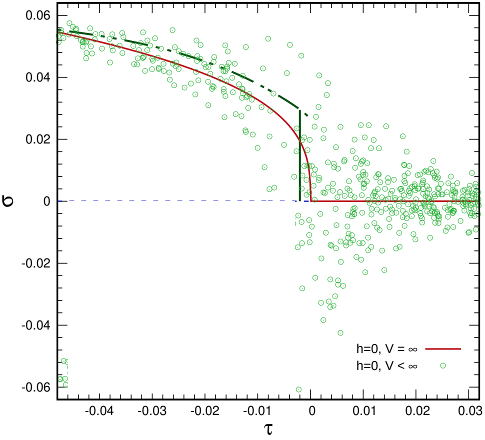


Our attention
should be focused on

temperature and volume
dependence

of the distribution
of configurations
in magnetization

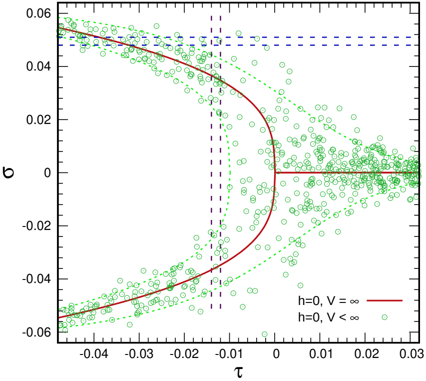
(or in the Polyakov loop)



Magnetization
in a finite volume:
faulty computation pattern

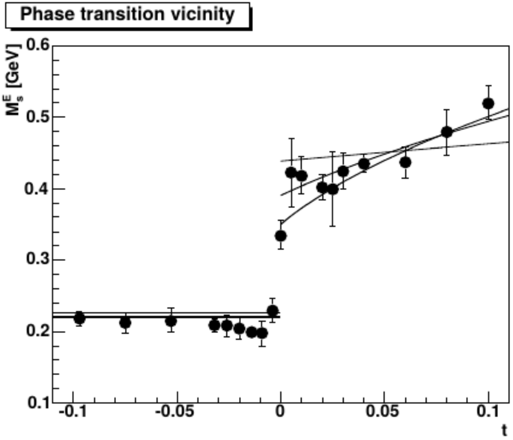
Voluntaristic exclusion
of negative magnetizations
at $T < T_{fake}$
results in fake discontinuity
of the average spin

Distributions in a finite volume



Distribution of configurations in the magnetization σ and in a quantity Q correlated with σ involves information on temperature dependence of Q.

What has been done before?

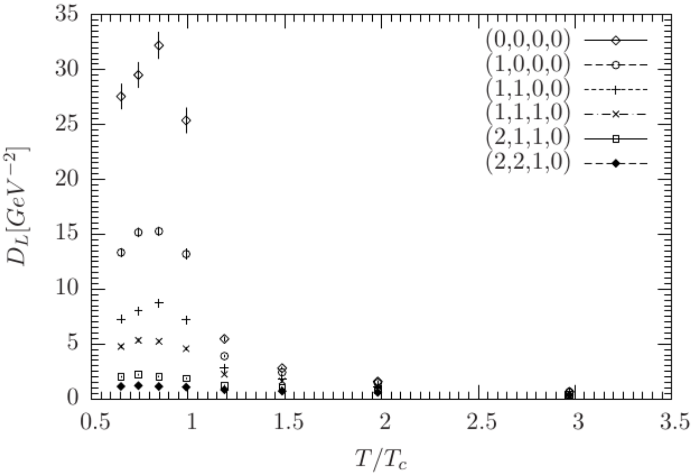


SU(3)

$$m_e = \frac{1}{\sqrt{D_L(0)}}$$

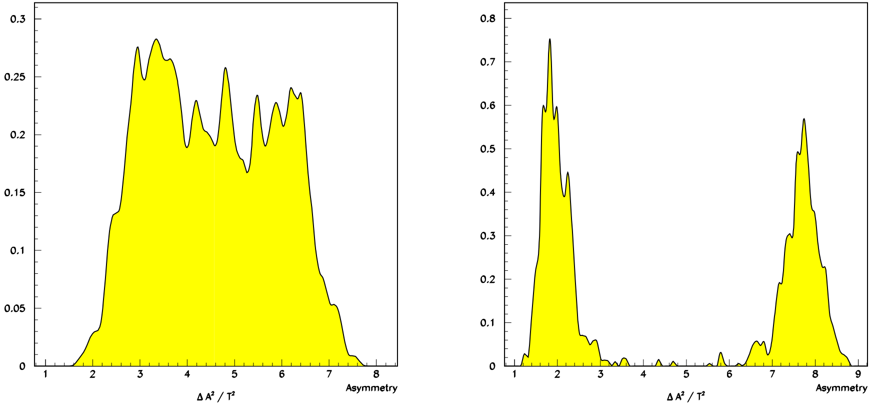
A.Maas et al., 2011

What has been done before? SU(3)

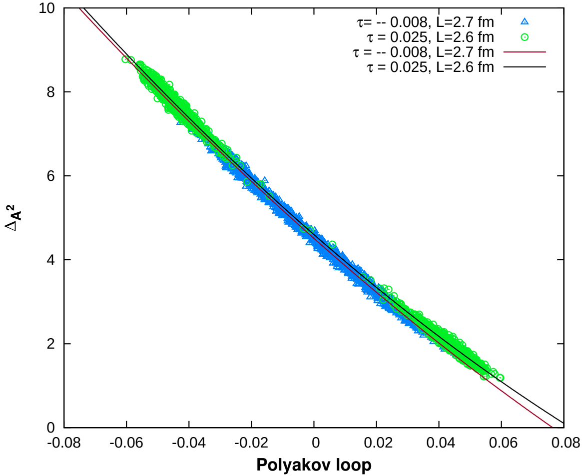


$D_L(p_n)$ as a function of the temperature, R.Aouane et al., 2011

Distributions of configurations in the asymmetry

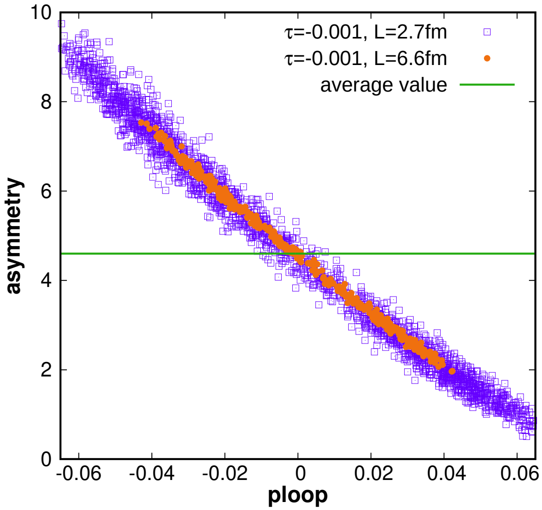


$T < T_c$ $T > T_c$



Correlation on the scatter plot

SU(2)



Dependence
of the scatter plot
on the lattice volume

In 2018 we argued that

- correlation between the asymmetry and the Polyakov loop
- universasality hypothesis

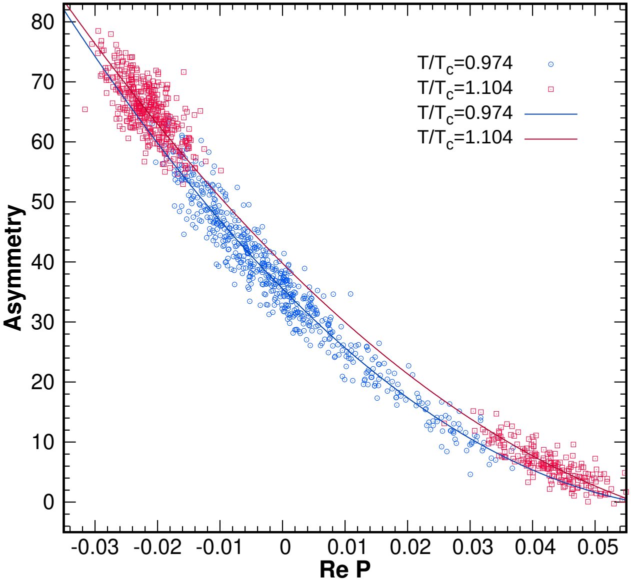
implies coincidence of the critical exponents
of magnetization in the 3D Ising model
and of $\mathcal{A}$ and $D_L(0)$ in SU(2) gauge theory

If $\langle \mathcal{A} \rangle(\mathcal{P})$ is a smooth function:

$$\mathcal{A} = \mathcal{A}_0 + B_{\mathcal{A}} \tau^{\beta_{\mathcal{A}}} + \overline{o}(\tau^{\beta_{\mathcal{A}}})$$

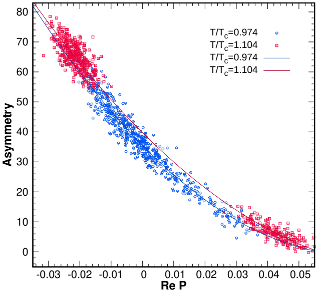
then

$$\beta_{\mathcal{A}} = \beta = 0.326419(3),$$
$$B_{\mathcal{A}} = A_1 B = -54.02(24)$$



Correlation on the scatter plot

SU(3)



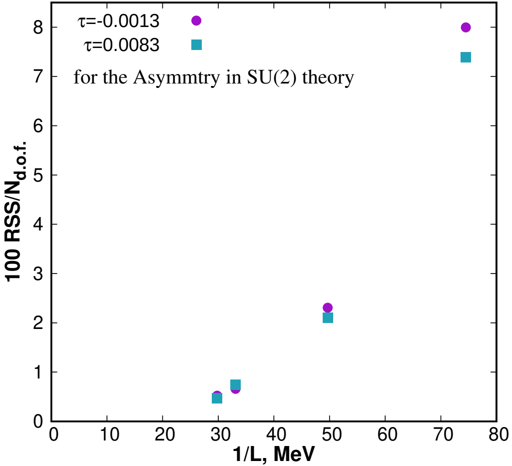
We consider conditional CDF $F(\mathcal{A}|\mathcal{P})$ of the asymmetry at a given value of Polyakov loop and the conditional average

$$E(\mathcal{A}|\mathcal{P}) = \int \frac{dF(\mathcal{A}|\mathcal{P})}{d\mathcal{A}}\, \mathcal{A} d\mathcal{A} \qquad (1)$$

It can be fitted by the formula

$$E(\mathcal{A}|\mathcal{P}) \simeq \mathbf{A_0} + \mathbf{A_1}\,\mathrm{Re}\,\mathcal{P} + \mathbf{A_2}\left(\mathrm{Re}\,\mathcal{P}\right)^2 \qquad (2)$$

assuming its independence of $\mathrm{Im}\,\mathcal{P}$



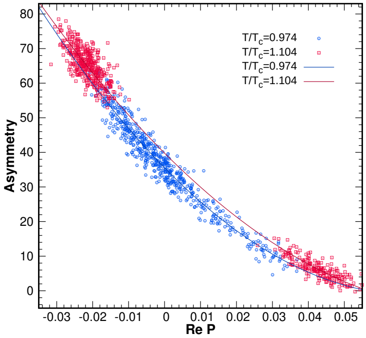
Measure of correlation

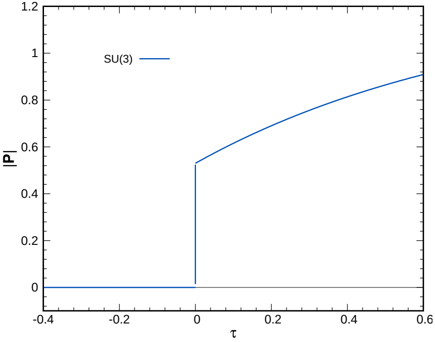
$$RSS_{tot} = \sum_{j=1}^{N_{data}} \Big(\mathcal{A}_j - \langle \mathcal{A} \rangle \Big)^2$$

Fraction of variance unexplained

$$r = \frac{RSS}{RSS_{tot}}$$

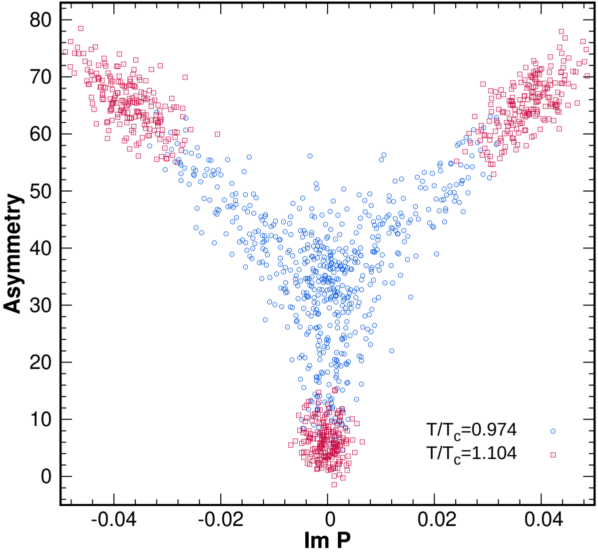
$$RSS = \sum_{j=1}^{N_{data}} \Big[\mathcal{A}_j - \mathbf{A_0} + \mathbf{A_1} \,\mathrm{Re}\, \mathcal{P}_j + \mathbf{A_2} \big(\mathrm{Re}\, \mathcal{P}_j \big)^2 \Big]^2$$





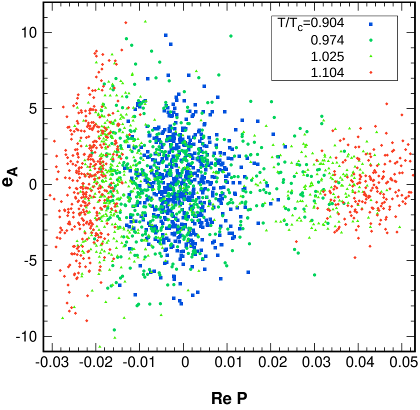
Smooth dependence of $\mathcal{A}$ on $\mathcal{P}$ & jump of $|\mathcal{P}|$ at $\{\tau = 0, V \to \infty\}$

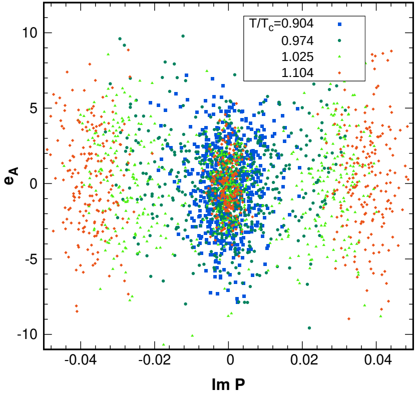
$\Longrightarrow$ a jump of $\mathcal{A}$ at the transition
when $V \to \infty$



SU(3)

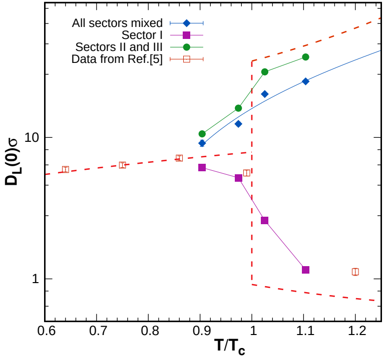
This scatter plot
is readily explained
by correlation
between $\mathcal{A}$ and Re $\mathcal{P}$
only

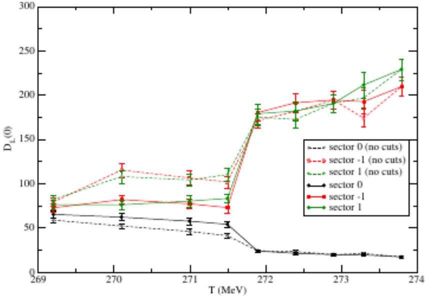




Residuals $$e_A(n) = \mathcal{A}_n - \mathbf{A_0} - \mathbf{A_1}\,\mathrm{Re}\,\mathcal{P}_n - \mathbf{A_2}(\mathrm{Re}\,\mathcal{P}_n)^2$$

show correlation with neither $\mathrm{Re}\,\mathcal{P}$ nor $\mathrm{Im}\,\mathcal{P}$

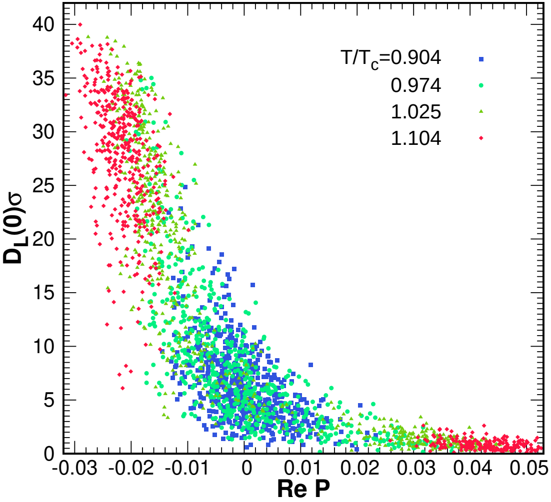




(b) $72^3 \times 8$ lattices.

Our results;
dashed line - predicted behavior at the phase transition

O.Oliveira, P.Silva 2016


T/T_c=0.904
0.974
1.025
1.104
D_L(0)σ
Re P
40
35
30
25
20
15
10
5
0
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05

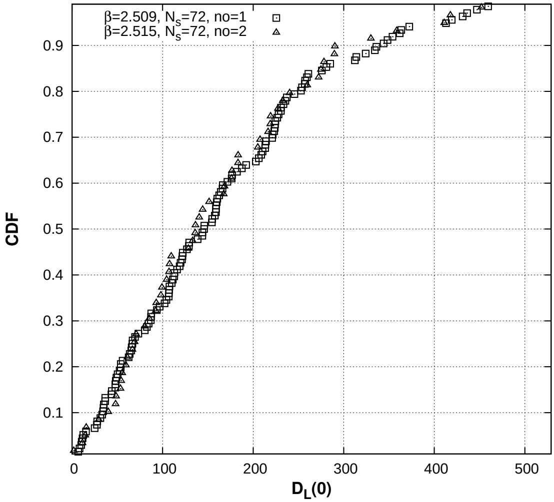
Homoscedasticity

- Independence of the variance of the conditional distribution on the predictor (independence of the variance of $(D_L(0)|\mathcal{P})$ on $\mathrm{Re}\,\mathcal{P}$).

Homoscedasticity is severely broken

Non-Gaussian behavior in the $SU(3)$ case:

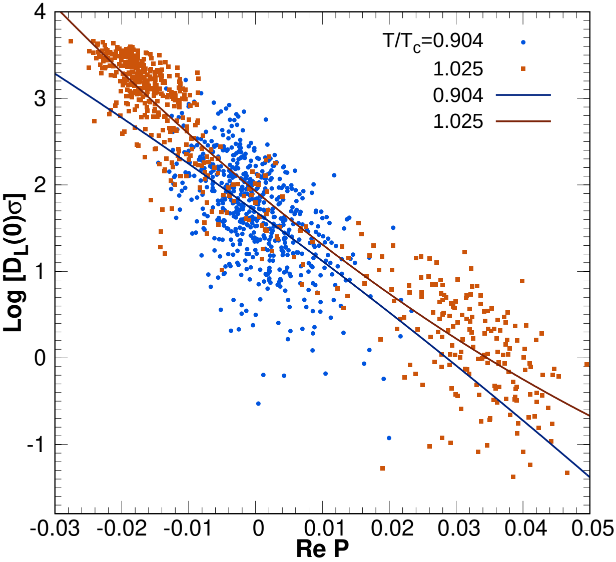
The Kolmogorov-Smirnov test for the $D_L(0)$ distribution at $-0.005 < \mathrm{Re}\,\mathcal{P} < 0.005$ indicates that
the probability that it is Gaussian is less than 0.002.



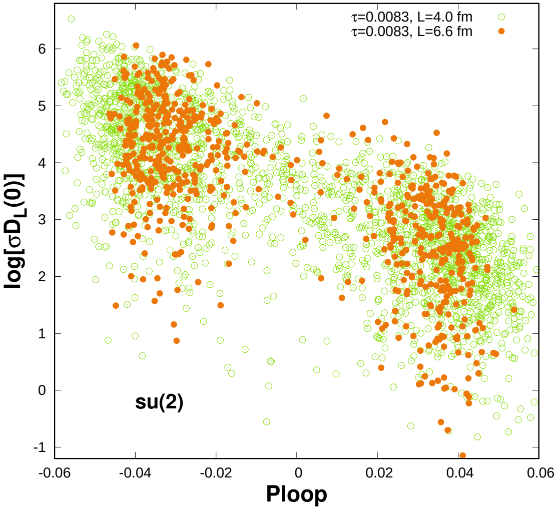
$SU(2)$ theory non-Gaussian distribution:

$-0.030 < \mathcal{P} < -0.025$;
$L = 6$ fm;

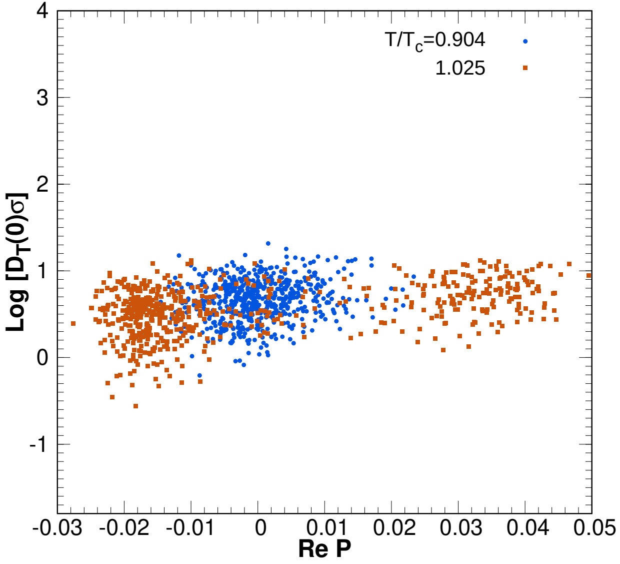
$1 \to \tau = -0.0045$;
$2 \ \to \tau = 0.0148$



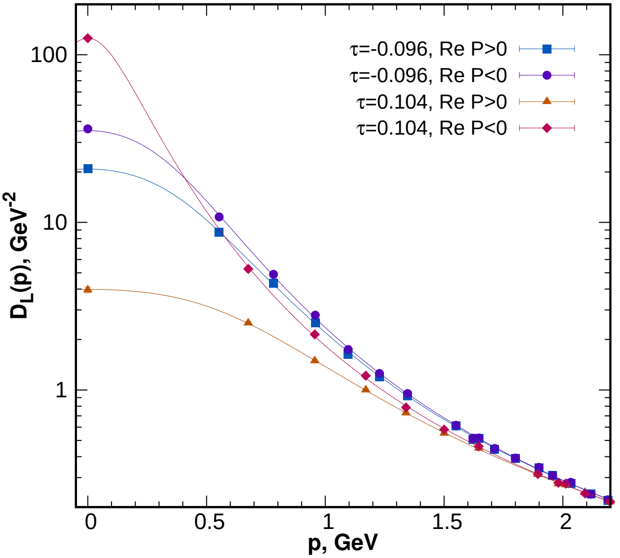
T/T_c=0.904
1.025
0.904
1.025
Log [D_L(0)σ]
Re P
4
3
2
1
0
-1
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05



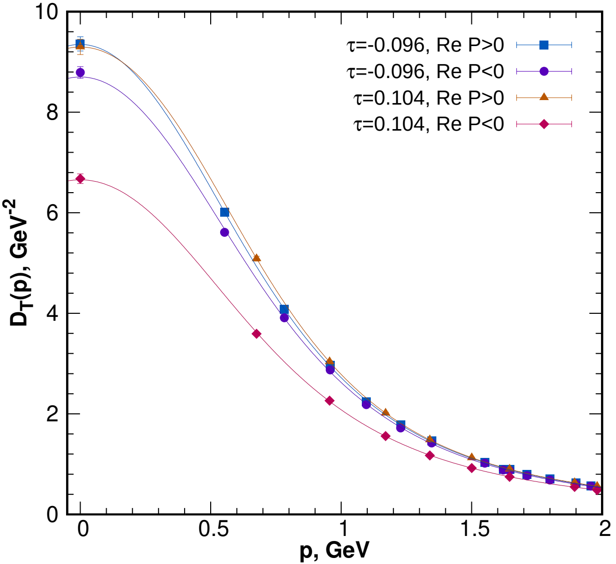
(In)dependence of the scatter plot on the lattice volume for the propagators



T/T$_c$=0.904
1.025
Log [D$_T$(0)σ]
Re P
4
3
2
1
0
-1
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05



τ=-0.096, Re P>0
τ=-0.096, Re P<0
τ=0.104, Re P>0
τ=0.104, Re P<0
100
10
1
0
0.5
1
1.5
2
p, GeV
$D_L(p)$, GeV^{-2}


τ=-0.096, Re P>0
τ=-0.096, Re P<0
τ=0.104, Re P>0
τ=0.104, Re P<0
$D_T(p)$, GeV^{-2}
p, GeV
0
2
4
6
8
10
0
0.5
1
1.5
2

Correlation length ξ and the screening mass

A popular defnition of screening masses [Maas 2011]:

$$\mathcal{M}_E^2 = \frac{1}{D_L(0)}\,, \qquad \mathcal{M}_M^2 = \frac{1}{D_T(0)}. \tag{3}$$

It depends on renormalization and is very sensitive to finite-volume effects and its relation to screening is not clear.

The screening concept itself stems from considering Yukawa-type potentials

$$V = \frac{g^2 e^{-\,m|\vec{x}|}}{4\pi|\vec{x}|} \rightarrow \text{Fourier Transform} \rightarrow \tilde{V} = \frac{g^2}{|\vec{p}|^2 + m^2}$$

The relation

$$\tilde{V}_{E,M}(\vec{p}) = g^2 D_{L,T}(p_0 = 0, \vec{p})$$

is valid

- in nonrelativistic approximation
  (for an interaction of static sources or currents)
- if one-particle exchange dominates.

Screening is an adequate concept concerning the shape of the potential provided that $V(|\vec{x}|)$

- is a monotonous function
- decreases rapidly as $|\vec{x}| \to \infty$

The above conditions should be considered in view of the following definition:

$$\xi^2 = \frac{1}{2}\frac{\int dx_4 d\vec{x}\tilde{D}(x_4,\vec{x})|\vec{x}|^2}{\int dx_4 d\vec{x}\tilde{D}(x_4,\vec{x})} =$$

$$= -\frac{1}{2D(0,\vec{0})}\sum_{i=1}^{3}\left(\frac{d}{dp_i}\right)^2\Big|_{\vec{p}=0} D(0,\vec{p})\ .$$

Then the screening mass

$$M = \frac{1}{\xi}$$

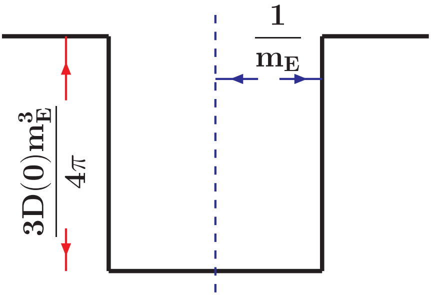
Since

$$\xi^2 \sim \int d\vec{x} V(\vec{x}) |\vec{x}|^2 \ ,$$

- The correlation length does exist only when the integral in the above formula converges ($V < \dfrac{c}{|\vec{x}|^{5+\epsilon}}$).
- Small ξ implies small radius of action of the potential provided that $V(\vec{x})$ does not oscillate.

When $m_E \to \infty$, one-gluon exchange dominates in the interaction of static color charges and, therefore,

$$\tilde{V}_{E,M}(\vec{p}) = \int d\vec{x}\, V(\vec{x}) = g^2 D_{L,T}(0,\vec{p})$$



The parameters
of the potential well
are determined
by the screening mass and
zero-momentum value
of the propagator.

τ	m_E^2 $\mathrm{Re}\,\mathcal{P} > 0$	m_E^2 $\mathrm{Re}\,\mathcal{P} < 0$	m_M^2 $\mathrm{Re}\,\mathcal{P} > 0$	m_M^2 $\mathrm{Re}\,\mathcal{P} < 0$
-0.096	0.373(31)	0.214(31)	0.638(34)	0.642(39)
-0.026	0.445(71)	0.136(11)	0.609(24)	0.586(32)
0.025	0.523(56)	0.0498(38)	0.672(37)	0.565(18)
0.104	0.95(20)	0.0272(11)	0.664(43)	0.611(8)

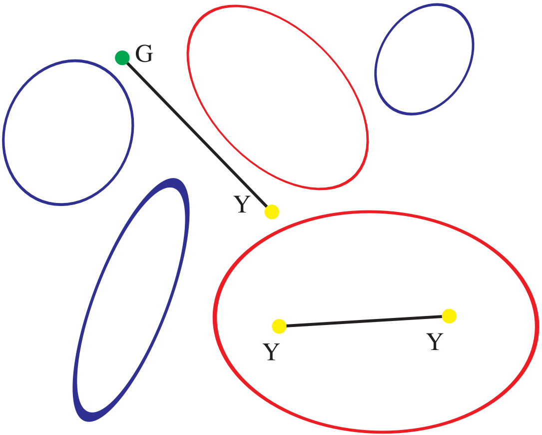
Таблица: Values of the chromoelectric and chromomagnetic screening masses (in GeV^2) in different Polyakov-loop sectors. No difference between sectors (*II*) and (*III*) has been found, they are referred to as "$\mathrm{Re}\,\mathcal{P} < 0$".

Screening in dense quark matter

	$\mu_q < 850$ MeV	$\mu_q > 850$ MeV
$T < 300$ MeV	$m_E \simeq 0.7$ GeV $m_M \simeq 0.7$ GeV	$m_E \simeq 1.8 \div 2.4$ GeV $m_M \simeq 0.4 \div 1.1$ GeV
$T = 560$ MeV	$m_E \simeq 1.6 \div 1.8$ GeV $m_M \simeq 1.2 \div 1.3$ GeV	$m_E \simeq 2.4 \div 3.0$ GeV $m_M \simeq 1.3 \div 1.5$ GeV

Таблица: Dependence of the screening masses on the quark chemical potential and temperature. It should be emphasized that, at $T = 0$ and $\mu_q > 850$ MeV, the magnetic mass decreases from $m_M \simeq 700$ MeV to $m_M \simeq 400$ MeV.

Bubbles of glue in the deconfinement phase



Interaction
of color charges
in the bubbles with
$\mathrm{Im}\mathcal{P} \neq 0$
differs from that
in the conventional
deconfinement phase

$$\arg(\mathcal{P}) = \frac{2\pi}{3}$$
$$\arg(\mathcal{P}) = -\frac{2\pi}{3}$$

Conclusions

- Both the asymmetry $\mathcal{A}$ and the zero-momentum longitudinal propagator $D_L(0)$ have a significant correlation with the real part of the Polyakov loop $\mathcal{P}$.
- We determined critical behavior of $\mathcal{A}$ and $D_L(0)$ in the infinite-volume limit. No discontinuities at a finite volume can take place.
- Chromoelectric interactions relative to chromomagnetic are weakly suppressed and short-range in the sector $\mathrm{Re}\mathcal{P} > 0$ and moderately suppressed and long-range in each sector with $\mathrm{Re}\mathcal{P} < 0$.

To be studied: Dependence of the conditional variance of $D_L(0)$ and $D_{L,T}(0)$ and $D_{L,T}(p_{min})$ on lattice volume in the SU(3) case