Correlations and Critical Behavior in Gluodynamics

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- "The A^2 Asymmetry and Gluon Propagators in Lattice $SU(3)$ Gluodynamics at $T \simeq T_c$,"
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  In collaboration with V.Bornyakov, V.Goy, V.Mitrjushkin.
- Talk given at the online conference
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  (Univ. of Stavanger, Norway, August 2nd-6th 2021)
- Sent for the Proceedings to EPJ Web of Conferences (vConf2021).
  In collaboration with V.Bornyakov, V.Goy, E.Kozlovsky, V.Mitrjushkin.

Our goal: to find critical behavior of the gluon propagators.

Outline

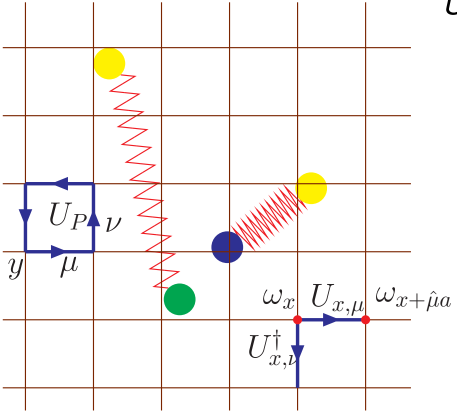
Average value of a dynamical variable $\mathcal{F}$ at temperature T can be expressed in terms of the path integral:

$$\langle \mathcal{F} \rangle = \sum_n \langle n| \, e^{-H/T} \, \hat{\mathcal{F}} \, |n\rangle = \int_{\varphi(0,\vec{x})=\varphi(1/T,\vec{x})} D\varphi e^{-S_E[\varphi]} \, \mathcal{F}[\varphi] \, .$$

We consider

φ	gauge field on $\mathbb{R}^3 \times \left[0, \frac{1}{T}\right]$	$A_\mu^b(t_E, \vec{x})$
$\mathcal{F}$:	1. Products of gauge fields	$A_\mu^b(t_E, \vec{x}) A_\nu^c(0, \vec{0})$
	2. Asymmetry	$\left\langle A_4^a A_4^a - \frac{1}{3} \sum_{i=1}^{3} A_i^a A_i^a \right\rangle$
	3. Polyakov loop	$P \exp\left(iga \int_0^{1/T} A_4(t_E) dt_E \right)$

and evaluate the parh integral numerically using lattice regularization.



Perturbation theory *etc*:

$$U_{x,\mu} = \exp\left(-\imath g a A_\mu(x)\right) \in SU(N_c)$$

We use:

$$A_\mu(x) = \left.\frac{U_\mu(x) - U_\mu^\dagger(x)}{2\imath a g}\right|_{traceless}$$

$$S_E = \frac{2N_c}{g^2}\sum_P\left(1 - \frac{1}{N_c}\mathrm{Re}\,\mathrm{Tr}\,U_P\right)$$

$$U_{x,\mu} \longrightarrow \omega_x^\dagger U_{x,\mu}\omega_{x+\hat{\mu}a}$$

Gauge fixing

Gauge transformations:

$$U_{x\mu} \overset{\omega}{\mapsto} U^{g}_{x\mu} = \omega^{\dagger}_{x} U_{x\mu} \omega_{x+\mu}, \qquad \omega_x \in SU(N_c)\ .$$

Vector potentials:

$$\mathbf{A}_{x\mu} = \frac{1}{2i}\Big(U_{x\mu} - U^{\dagger}_{x\mu}\Big)_{\text{traceless}} \equiv A^{a}_{x,\mu} T^{a}\ , \tag{1}$$

The lattice Landau gauge condition

$$(\partial \mathbf{A})_x = \sum_{\mu=1}^{4} (\mathbf{A}_{x\mu} - \mathbf{A}_{x-\hat{\mu};\mu}) = 0 \tag{2}$$

represents a stationarity condition for the gauge-fixing functional

$$F_U(\omega) = \frac{1}{4V} \sum_{x\mu} \frac{1}{N_c}\, \mathrm{Re}\,\mathrm{Tr}\ U^{\omega}_{x\mu}\ , \tag{3}$$

with respect to gauge transformations g_x .

We simulate an ensemble of gauge-field configurations with the weight

$$\exp\left(-S_E[U]\right)$$

so that the average values of $\mathcal{F}$ can be determined by the formula

$$\langle \mathcal{F} \rangle = \frac{1}{N} \sum_{j=1}^{N} \mathcal{F}_n,$$

and fix Landau gauge $\partial_\mu A_\mu = 0$ by finding the minimum of the functional

$$\min_\omega F_U(\omega) = \frac{1}{2 N_c N_s^3 N_t} \min_\omega \sum_{x,\mu} \mathrm{Re}\,\mathrm{Tr}\, U^\omega_{x,\mu},$$

Polyakov loop

$$\mathcal{L}(\vec{x}) = P \exp\left(iga \int_{(\vec{x},0)}^{(\vec{x},1/T)} A_0^c(\vec{x},\tau)\Gamma^c d\tau \right)$$

$$\langle \mathcal{L}(\vec{x}) \rangle = \frac{\int_T DA \, \exp\left(-\, S_E(A)\right) \mathcal{L}(\vec{x})}{\int_T DA \, \exp\left(-\, S_E(A)\right)}$$

$$\mathcal{P} = \frac{1}{VN_c} Tr \int d\vec{x} \langle \mathcal{L}(\vec{x}) \rangle$$

is related to the free energy of a quark as follows:

$$\exp\left(- \frac{F_q(V,T)}{T} \right) = \mathcal{P}$$

McLerran, Svetitsky, 1981

$$e^{-F_{q\bar{q}}/T} \simeq \sum_s \langle s|\psi(0,\vec{r}) e^{-\hat{H}/T} \psi^\dagger(0,\vec{r}')|s\rangle =$$

$$= \sum_s \langle s| e^{-\hat{H}/T} \psi\left(\frac{1}{T},\vec{r}\right) \psi^\dagger(0,\vec{r}')|s\rangle \;;$$

$$\psi(t,\vec{r}) = T \exp\left(iga \int_{(\vec{r},0)}^{(\vec{r},t)} A_4^c(\vec{r},\tau)\Gamma^c d\tau \right) \psi(t,\vec{r})$$

$$e^{-F_{q\bar{q}}/T} \simeq \mathrm{Tr}\left(e^{-\hat{H}/T} \mathcal{L}(\vec{r}) \mathcal{L}^\dagger(\vec{r}') \right)$$

$$|\vec{r} - \vec{r}| \to \infty \qquad\qquad e^{-F_{q\bar{q}}/T} \to |\mathcal{P}|^2$$

$$A_\mu \to A_\mu^\Lambda = (\Lambda Z)^\dagger A_\mu (\Lambda Z) + \frac{i}{g}\,(\Lambda Z)^\dagger \partial_\mu (\Lambda Z).$$

$$A_\mu \to A_\mu^\Lambda = \Lambda^\dagger A_\mu \Lambda + \frac{i}{g}\,\Lambda^\dagger \partial_\mu \Lambda.$$

In the case of $SU(2)$

$$Z \in \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$$

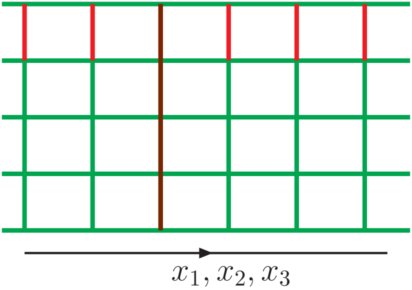
Gauge transformation is the same on both sides!

Center of the SU(3) group

$$Z \in \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} \frac{-1 \pm \imath\sqrt{3}}{2} & 0 & 0 \\ 0 & \frac{-1 \pm \imath\sqrt{3}}{2} & 0 \\ 0 & 0 & \frac{-1 \pm \imath\sqrt{3}}{2} \end{pmatrix} \right\}$$

Center transformations change the Polyakov loop

$$\mathcal{P} \to Z\mathcal{P}$$



Center symmetry:

$\mathbb{Z}_2 : U_{x,\mu} \to - U_{x,\mu}$

$T > T_c$
spontaneous breaking of $\mathbb{Z}_2$
signals transition
to deconfinement:

Free energy
of a free quark
becomes finite

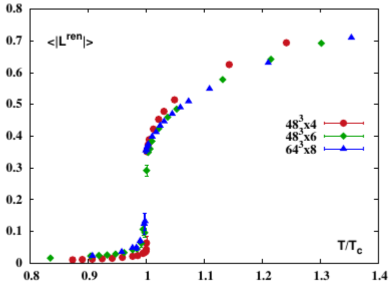
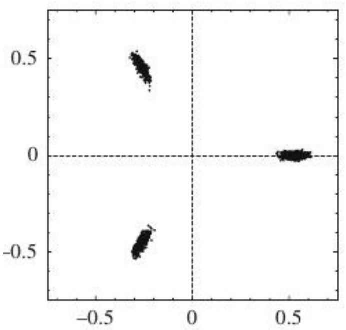
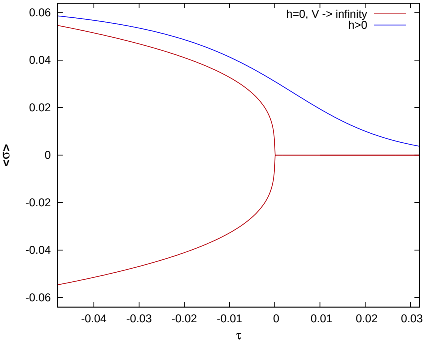


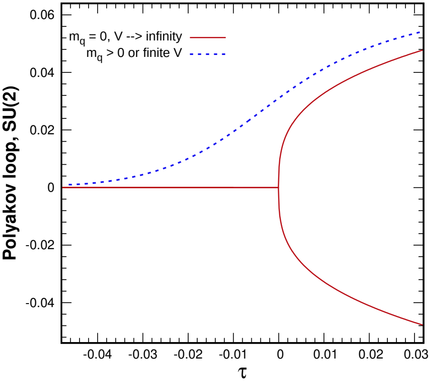
FIG. 1: The modulus of renormalized Polyakov loop $\langle |L^{\rm ren}| \rangle$ obtained in $SU(3)$ lattice gauge theory.

First order phase transition in SU(3)



Z_3 sectors of the Polyakov loop in the SU(3) theory



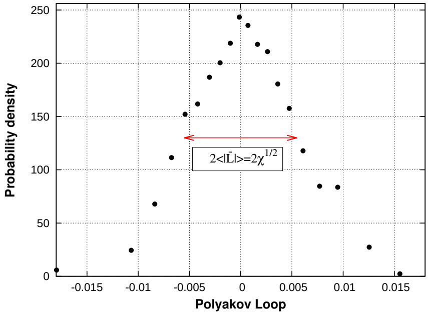


Magnetization
in the Ising model

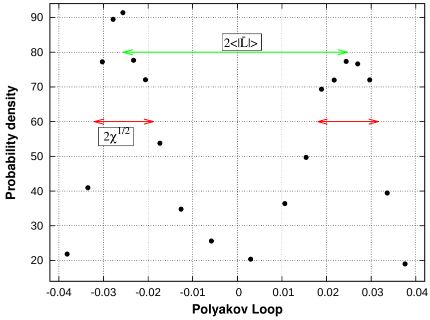
Polyakov loop
in the SU(2) theory

$$\tau = \frac{T - T_c}{T_c}$$

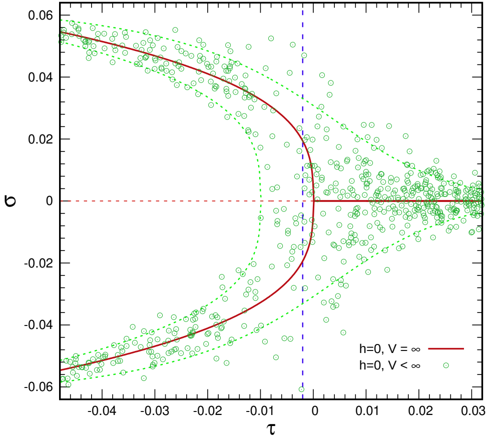
Distributions of configurations in the Polyakov loop



$T < T_c$



$T > T_c$

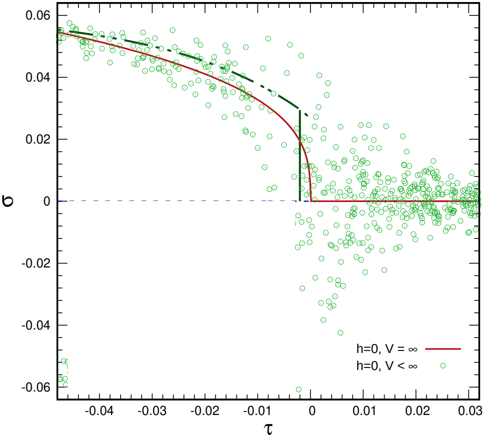


Our attention
should be focused on

temperature and volume
dependence

of the distribution
of configurations
in magnetization

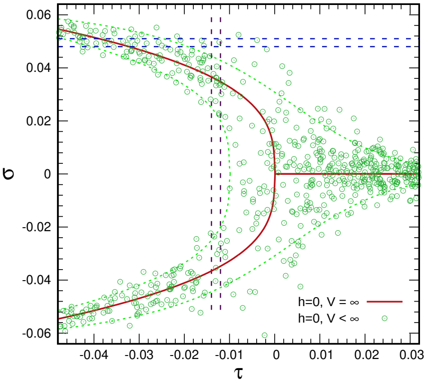
(or in the Polyakov loop)



Magnetization
in a finite volume:
faulty computation pattern

Voluntaristic exclusion
of negative magnetizations
at $T < T_{fake}$
results in fake discontinuity
of the average spin

Distributions in a finite volume



Distribution of configurations in the magnetization σ and in a quantity Q correlated with σ involves information on temperature dependence of Q.

Scale fixing (Lomonosov, Avogadro, Perrin, Gross ....)

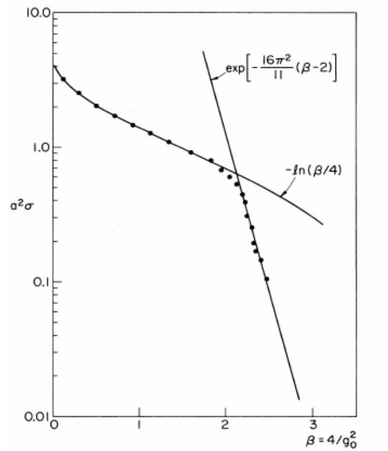


FIG. 34. SU(2) string tension Monte Carlo data vs coupling, $\beta=4/g^2$.

String tension σ: $V(r)=\sigma r$,

$$W \sim e^{-tV(r)}$$

$$W \sim e^{-(\sigma a^2) n_t n_r}$$

$$\ln(\sigma a^2) = f(\beta) = -\frac{6\pi^2}{11}\beta + \frac{102}{121}\ln\beta +$$

Sommer parameter r_0:

$$r_0^2 F(r_0) = 1.65$$

$$\beta = \frac{2N_c}{g^2}$$

$N_s^3 \times N_t$ lattice with pure glue Wilson action.
$SU(2)$: $N_t = 8$, $N_s = 32, 48, 72$, scale setting [Karsch et al. 1992]:

$$\ln(a\sqrt{\sigma}) = -\frac{3\pi^2}{11}\beta + \frac{51}{121}\ \ln\beta + 0.296 + \frac{4.25}{\beta}\ ,$$

$\beta_c = 2.5104$, $\sqrt{\sigma} = 0.44$ GeV, $T_c = 297$ MeV.
$SU(3)$: $N_t = 8$, $N_s = 24$, scale setting [Sommer, Necco 2004]:

$$\ln\frac{a}{r_0} = -1.6804 - 1.7331(\beta - 6) + 0.7849(\beta - 6)^2 - 0.4428(\beta - 6)^3\ ,$$

the Sommer parameter $r_0 = 0.5$ fm, $\sqrt{\sigma} = 0.47$ GeV.
$\beta_c = 6.06$ and $\dfrac{T_c}{\sqrt{\sigma}} = 0.63 \quad \Longrightarrow\ T_c = 294$ MeV

- $SU(2)$ $a_c = 0.084$ fm, $L = 2.6 \div 6.6$ fm

β	2.508	2.510	2.512	2.513	2.515	2.518
τ	-0.008	-0.001	0.005	0.008	0.015	0.025

- $SU(3)$ $a_c = 0.083$ fm, $L = 2.0$ fm

β	6.000	6.044	6.075	6.122
τ	-0.096	-0.026	0.025	0.104

We use variable $\tau = \dfrac{T - T_c}{T_c}$

Definition of the longitudinal (L) and transverse (T) propagators:

$$D^{ab}_{\mu\nu}(p) = \delta_{ab}\left(P^T_{\mu\nu}(p)D_T(p) + P^L_{\mu\nu}(p)D_L(p)\right)\,,$$

where $P^{T;L}_{\mu\nu}(p)$ - orthogonal transverse (longitudinal) projectors are defined at $p = (\vec{p} \neq 0;\ p_4 = 0)$ as follows

$$P^T_{ij}(p) = \left(\delta_{ij} - \frac{p_i p_j}{\vec{p}^2}\right), \quad P^T_{\mu 4}(p) = 0\,; \tag{4}$$

$$P^L_{44}(p) = 1\,; \quad P^L_{\mu i}(p) = 0\,. \tag{5}$$

Two scalar propagators - longitudinal $D_L(p)$ and transverse $D_T(p)$ - are given by

$$D_L(p) = \frac{1}{3} \sum_{a=1}^{N_c^2-1} \langle A_0^a(p) A_0^a(-p) \rangle$$

$$D_T(p) = \left[\begin{array}{ll} \dfrac{1}{2(N_c^2-1)} \displaystyle\sum_{a=1}^{N_c^2-1} \sum_{i=1}^{3} \langle A_i^a(p) A_i^a(-p) \rangle & p \neq 0 \\ \dfrac{1}{3(N_c^2-1)} \displaystyle\sum_{a=1}^{N_c^2-1} \sum_{i=1}^{3} \langle A_i^a(p) A_i^a(-p) \rangle & p = 0 \end{array} \right.$$

The Chromo-Electric-Magnetic Asymmetry

$$\begin{aligned} \langle A_E^2 \rangle &= g^2 \langle A_4^a(x) A_4^a(x) \rangle, \\ \langle A_M^2 \rangle &= g^2 \langle A_i^a(x) A_i^a(x) \rangle. \end{aligned} \tag{6}$$

The quantity of particular interest is the chromoelectric-chromomagnetic asymmetry

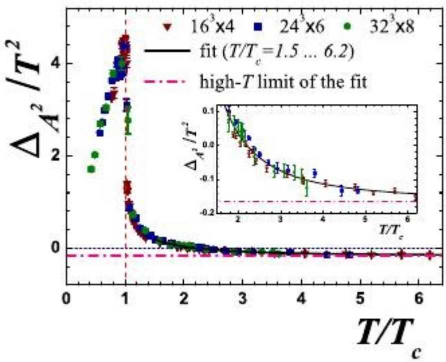
$$\mathcal{A} = \frac{\langle A_E^2 \rangle - \frac{1}{3} \langle A_M^2 \rangle}{T^2} \ . \tag{7}$$

which can be expressed in terms of the propagators,

$$\mathcal{A} = \frac{2N_c N_t (N_c^2 - 1)}{\beta a^2 N_s^3} \left[D_L(0) - D_T(0) + \sum_{p \neq 0} \left(\frac{3|\vec{p}|^2 - p_4^2}{3p^2} D_L(p) - \frac{2}{3} D_T(p) \right) \right]$$

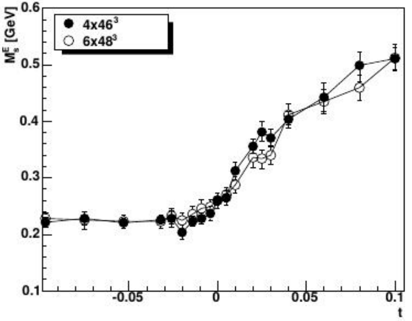
We work in the Landau gauge $\partial_\mu A_\mu^a = 0$

What has been done before? SU(2)



Chernodub, Ilgenfritz 2008

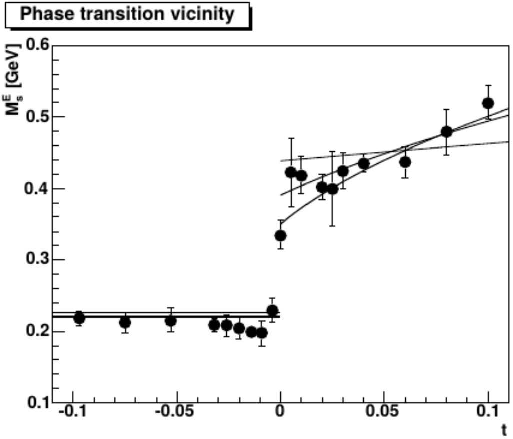
$\mathcal{A}$



Maas, Pawlowski von Smekal, Spielmann 2011

$$m_e = \frac{1}{\sqrt{D_L(0)}}$$

What has been done before?

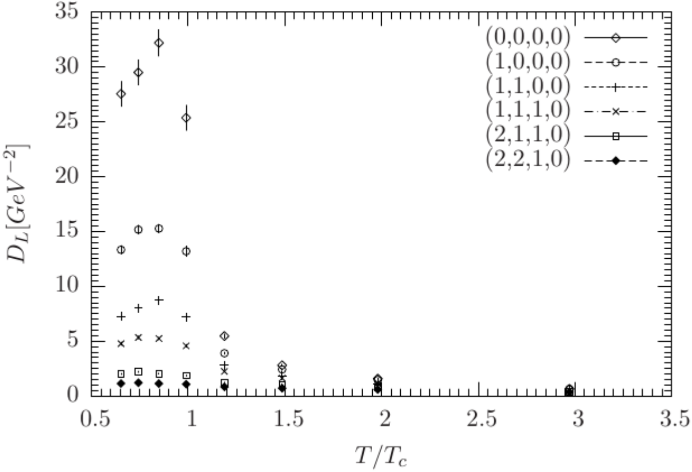


SU(3)

$$m_e = \frac{1}{\sqrt{D_L(0)}}$$

A.Maas et al., 2011

What has been done before? SU(3)



$D_L(p_n)$ as a function of the temperature, R.Aouane et al., 2011

In 2018 we argued that

- correlation between the asymmetry and the Polyakov loop
- universasality hypothesis

implies coincidence of the critical exponents
of magnetization in the 3D Ising model
and of $\mathcal{A}$ and $D_L(0)$ in SU(2) gauge theory

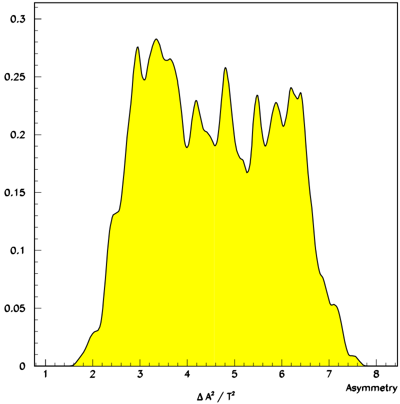
If $\langle\mathcal{A}\rangle(\mathcal{P})$ is a smooth function:

$$\mathcal{A} = \mathcal{A}_0 + B_{\mathcal{A}}\tau^{\beta_{\mathcal{A}}} + \overline{o}(\tau^{\beta_{\mathcal{A}}})$$

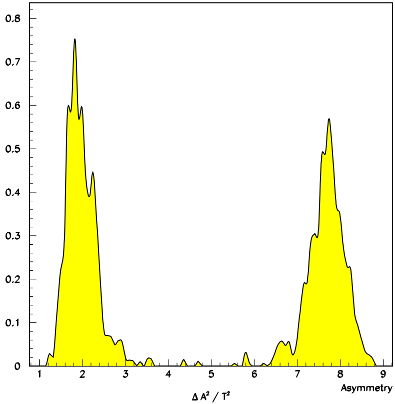
then

$$\beta_{\mathcal{A}} = \beta = 0.326419(3),$$
$$B_{\mathcal{A}} = A_1 B = -54.02(24)$$

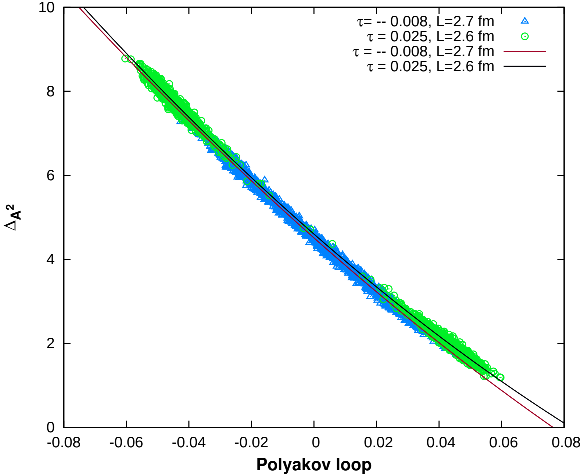
Distributions of configurations in the asymmetry



$T < T_c$

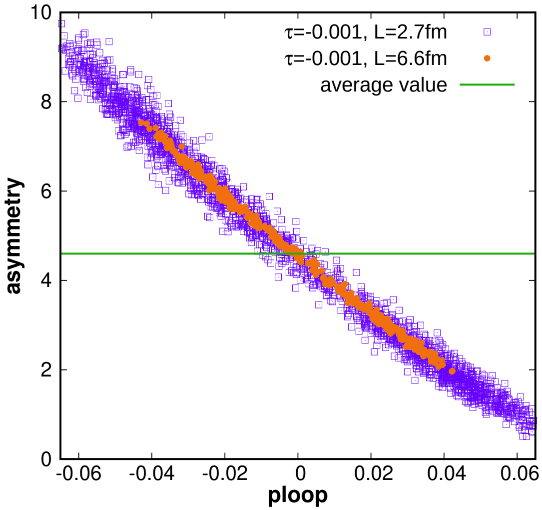


$T > T_c$

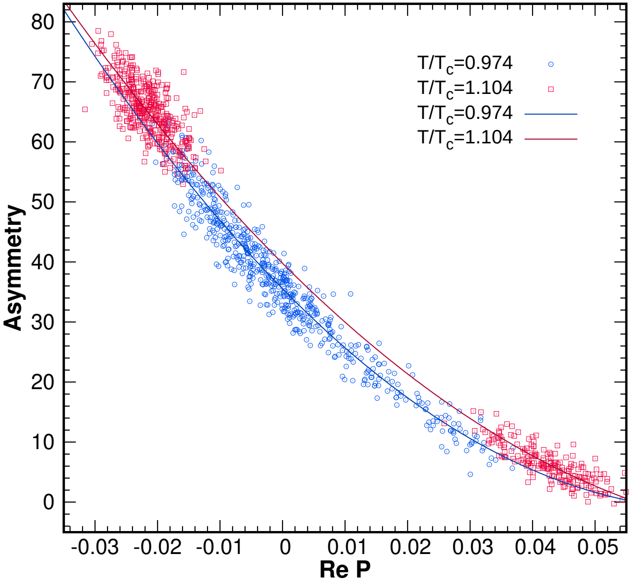


Correlation on the scatter plot

SU(2)

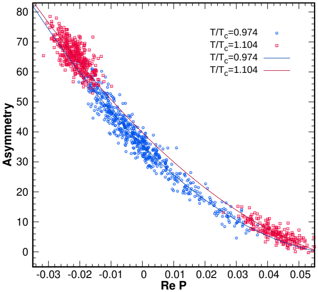


Dependence
of the scatter plot
on the lattice volume



Correlation on the scatter plot

SU(3)



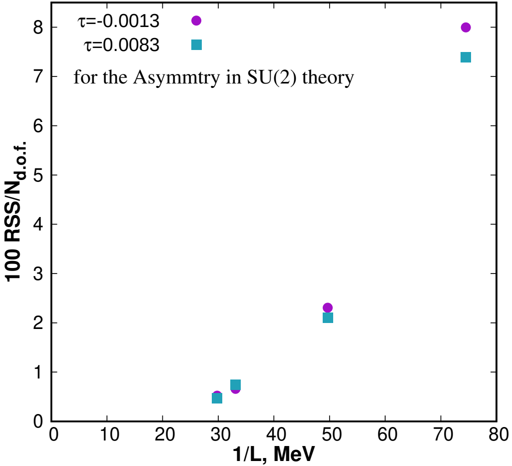
We consider conditional CDF $F(\mathcal{A}|\mathcal{P})$ of the asymmetry at a given value of Polyakov loop and the conditional average

$$E(\mathcal{A}|\mathcal{P}) = \int \frac{dF(\mathcal{A}|\mathcal{P})}{d\mathcal{A}}\, \mathcal{A} d\mathcal{A} \qquad (8)$$

It can be fitted by the formula

$$E(\mathcal{A}|\mathcal{P}) \simeq \mathbf{A_0} + \mathbf{A_1}\, \mathrm{Re}\,\mathcal{P} + \mathbf{A_2}\left(\mathrm{Re}\,\mathcal{P}\right)^2 \qquad (9)$$

assuming its independence of $\mathrm{Re}\,\mathcal{P}$

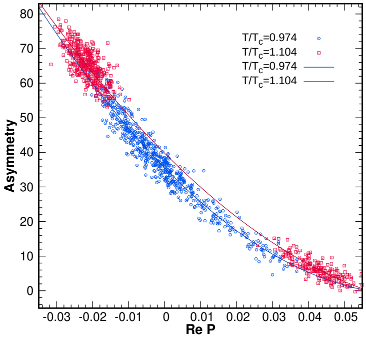


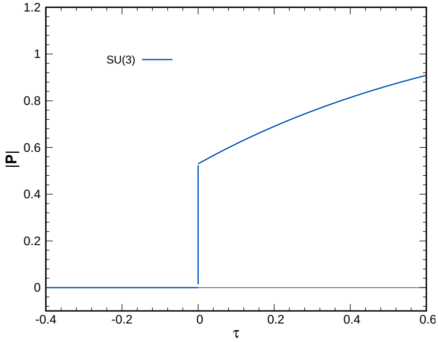
$$RSS_{tot} = \sum_{j=1}^{N_{data}} \Big(\mathcal{A}_j - \langle\mathcal{A}\rangle\Big)^2$$

Fraction of variance unexplained

$$r = \frac{RSS}{RSS_{tot}}$$

$$RSS = \sum_{j=1}^{N_{data}} \Big[\mathcal{A}_j - \mathbf{A_0} + \mathbf{A_1}\,\mathrm{Re}\,\mathcal{P}_j + \mathbf{A_2}\big(\mathrm{Re}\,\mathcal{P}_j\big)^2\Big]^2$$

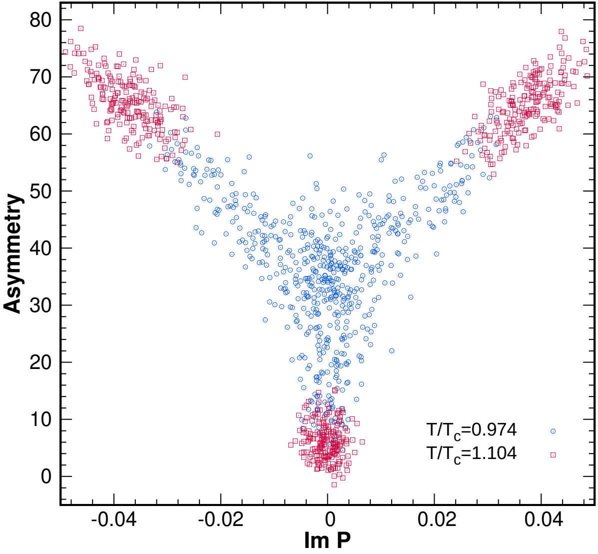




Smooth dependence of $\mathcal{A}$ on $\mathcal{P}$ & jump of $|\mathcal{P}|$ at $\{\tau = 0, V \to \infty\}$

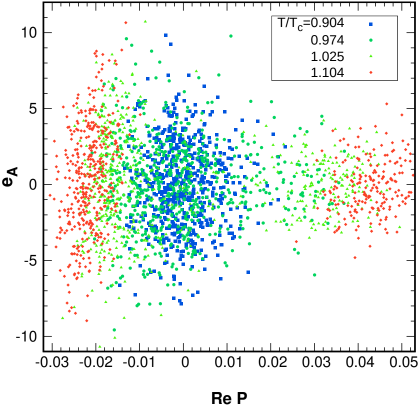
$\Longrightarrow$ a jump of $\mathcal{A}$ at the transition

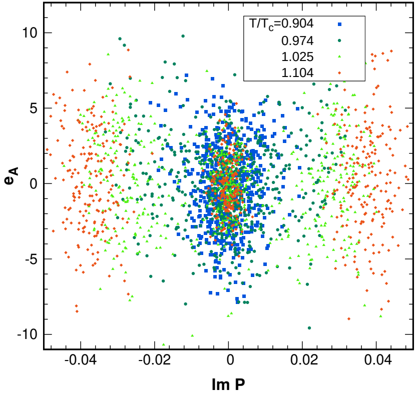
when $V \to \infty$



SU(3)

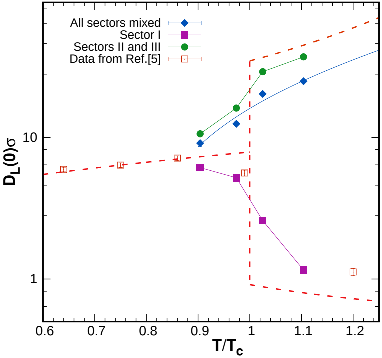
This scatter plot is readily explained by correlation between $\mathcal{A}$ and Re $\mathcal{P}$ only

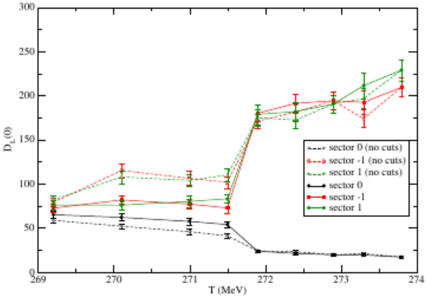




Residuals $\quad e_A(n) = \mathcal{A}_n - \mathbf{A_0} - \mathbf{A_1}\,\mathrm{Re}\,\mathcal{P}_n - \mathbf{A_2}(\mathrm{Re}\,\mathcal{P}_n)^2$

show correlation with neither $\mathrm{Re}\,\mathcal{P}$ nor $\mathrm{Im}\,\mathcal{P}$

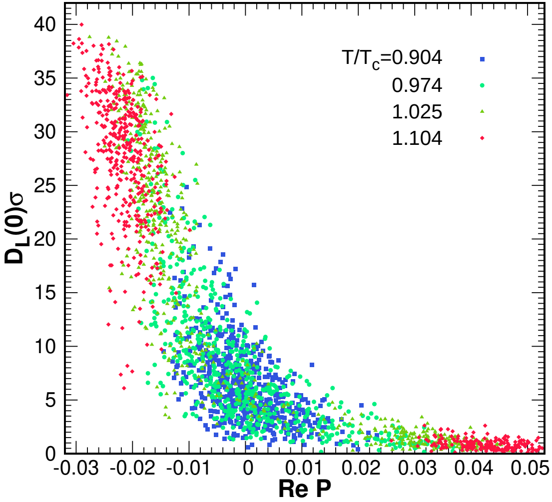




(b) $72^3 \times 8$ lattices.

Our results;
dashed line - predicted behavior at the phase transition

O.Oliveira, P.Silva 2016


T/T$_c$=0.904
0.974
1.025
1.104
$D_L(0)\sigma$
Re P
40
35
30
25
20
15
10
5
0
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05

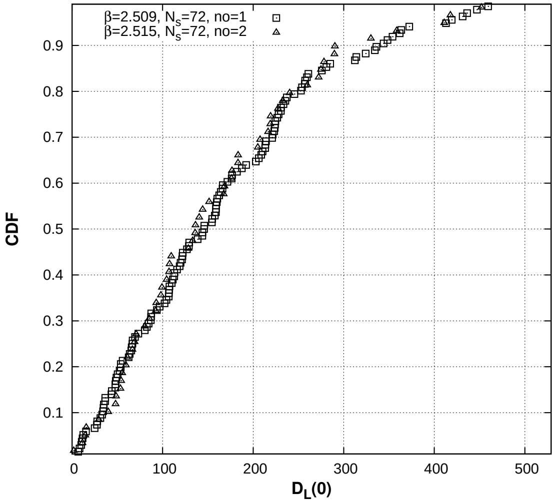
Homoscedasticity

- Independence of the variance of the conditional distribution on the predictor (independence of the variance of $(D_L(0)|\mathcal{P})$) on $\mathrm{Re}\,\mathcal{P}$).

Homoscedasticity is severely broken

Non-Gaussian behavior in the $SU(3)$ case:

The Kolmogorov-Smirnov test for the $D_L(0)$ distribution at $-0.005 < \mathrm{Re}\,\mathcal{P} < 0.005$ indicates that
the probability that it is Gaussian is less than 0.002.



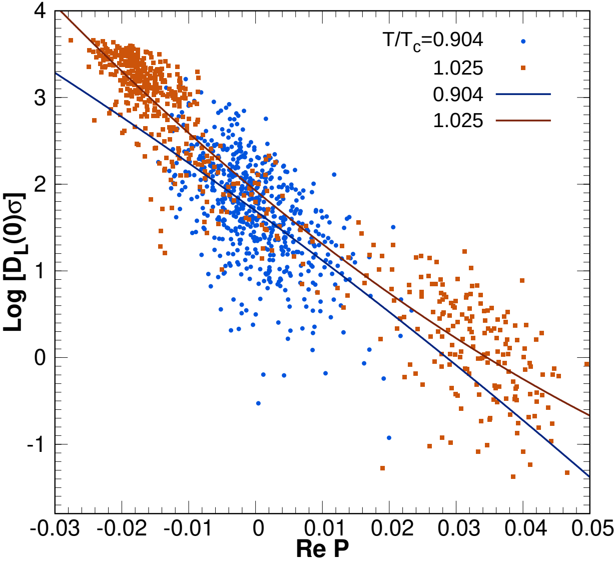
$SU(2)$ theory non-Gaussian distribution:

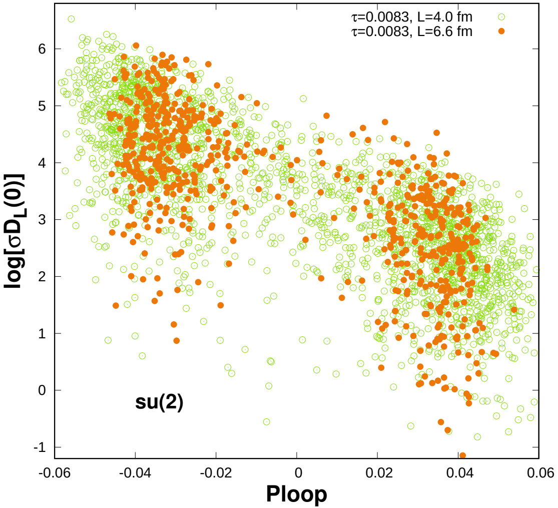
$-0.030 < \mathcal{P} < -0.025$;

$L = 6$ fm;

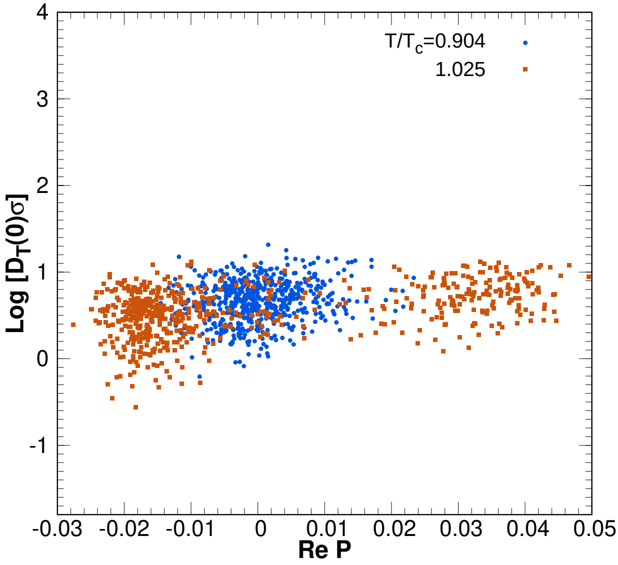
$1 \rightarrow \tau = -0.0045$;

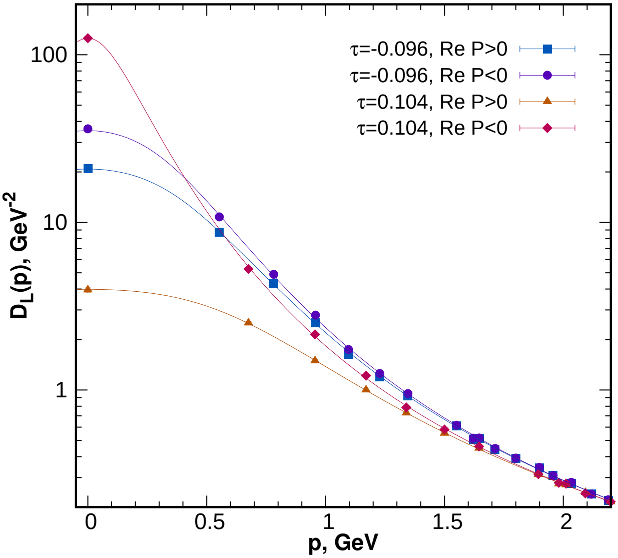
$2 \rightarrow \tau = 0.0148$


T/T_c=0.904
1.025
0.904
1.025
Log [D_L(0)σ]
Re P
4
3
2
1
0
-1
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05

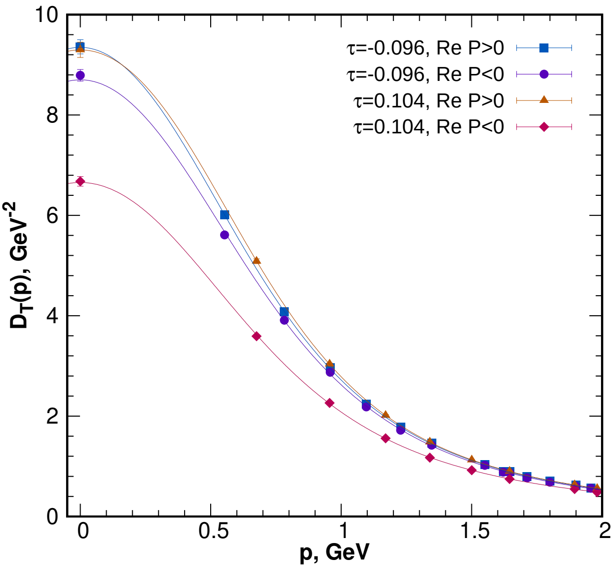


(In)dependence
of the scatter plot
on the lattice volume
for the propagators


T/T_c=0.904
1.025
Log [D_T(0)σ]
Re P
4
3
2
1
0
-1
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05



τ=-0.096, Re P>0
τ=-0.096, Re P<0
τ=0.104, Re P>0
τ=0.104, Re P<0
100
10
1
0
0.5
1
1.5
2
p, GeV
$D_L(p)$, GeV^{-2}


τ=-0.096, Re P>0
τ=-0.096, Re P<0
τ=0.104, Re P>0
τ=0.104, Re P<0
$D_T(p)$, GeV^{-2}
p, GeV
0
2
4
6
8
10
0
0.5
1
1.5
2

Correlation length ξ and the screening mass

A popular defnition of screening masses [Maas 2011]:

$$\mathcal{M}_E^2 = \frac{1}{D_L(0)}\,, \qquad \mathcal{M}_M^2 = \frac{1}{D_T(0)}. \tag{10}$$

It depends on renormalization and is very sensitive to finite-volume effects and its relation to screening is not clear.

The screening concept itself stems from considering Yukawa-type potentials

$$V = \frac{g^2 e^{-\,m|\vec{x}|}}{4\pi|\vec{x}|} \to \text{Fourier Transform} \to \tilde{V} = \frac{g^2}{|\vec{p}|^2 + m^2}$$

The relation

$$\tilde{V}_{E,M}(\vec{p}) = g^2 D_{L,T}(p_0 = 0, \vec{p})$$

is valid

- in nonrelativistic approximation
  (for an interaction of static sources or currents)
- if one-particle exchange dominates.

Screening is an adequate concept concerning the shape of the potential provided that $V(|\vec{x}|)$

- is a monotonous function
- decreases rapidly as $|\vec{x}| \to \infty$

The above conditions should be considered in view of the following definition:

$$\xi^2 = \frac{1}{2}\frac{\int dx_4 d\vec{x}\tilde{D}(x_4,\vec{x})|\vec{x}|^2}{\int dx_4 d\vec{x}\tilde{D}(x_4,\vec{x})} =$$

$$= -\frac{1}{2D(0,\vec{0})}\sum_{i=1}^{3}\left(\frac{d}{dp_i}\right)^2\Big|_{\vec{p}=0} D(0,\vec{p}) \ .$$

Then the screening mass

$$M = \frac{1}{\xi}$$

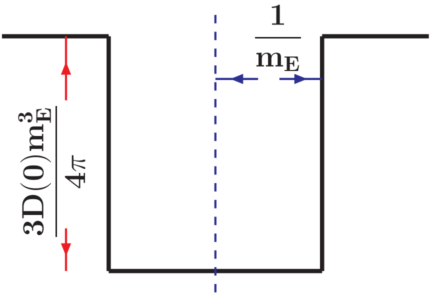
Since

$$\xi^2 \sim \int d\vec{x}\,\tilde{V}(\vec{x})|\vec{x}|^2 \,,$$

- The correlation length does exist only when the integral in the above formula converges ($V < \dfrac{c}{|\vec{x}|^{5+\epsilon}}$).
- Small ξ implies small radius of action of the potential provided that $V(\vec{x})$ does not oscillate.

When $m_E \to \infty$, one-gluon exchange dominates in the interaction of static color charges and, therefore,

$$\tilde{V}_{E,M}(\vec{p}) = \int d\vec{x}\, V(\vec{x}) = g^2 D_{L,T}(0,\vec{p})$$

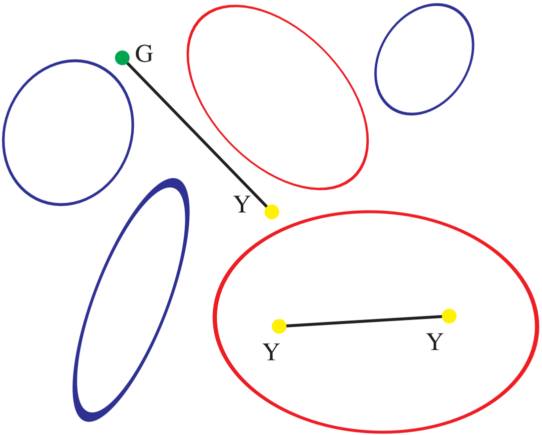


The parameters
of the potential well
are determined
by the screening mass and
zero-momentum value
of the propagator.

τ	m_E^2 $\mathrm{Re}\,\mathcal{P} > 0$	m_E^2 $\mathrm{Re}\,\mathcal{P} < 0$	m_M^2 $\mathrm{Re}\,\mathcal{P} > 0$	m_M^2 $\mathrm{Re}\,\mathcal{P} < 0$
-0.096	0.373(31)	0.214(31)	0.638(34)	0.642(39)
-0.026	0.445(71)	0.136(11)	0.609(24)	0.586(32)
0.025	0.523(56)	0.0498(38)	0.672(37)	0.565(18)
0.104	0.95(20)	0.0272(11)	0.664(43)	0.611(8)

Таблица: Values of the chromoelectric and chromomagnetic screening masses (in GeV^2) in different Polyakov-loop sectors. No difference between sectors (*II*) and (*III*) has been found, they are referred to as "$\mathrm{Re}\,\mathcal{P} < 0$".

Bubbles of glue in the deconfinement phase



Interaction
of color charges
in the bubbles with
$\mathrm{Im}\mathcal{P} \neq 0$
differs from that
in the conventional
deconfinement phase

$$\arg(\mathcal{P}) = \frac{2\pi}{3}$$

$$\arg(\mathcal{P}) = -\frac{2\pi}{3}$$

Conclusions

- Both the asymmetry $\mathcal{A}$ and the zero-momentum longitudinal propagator $D_L(0)$ have a significant correlation with the real part of the Polyakov loop $\mathcal{P}$.
- We determined critical behavior of $\mathcal{A}$ and $D_L(0)$ in the infinite-volume limit. No discontinuities at a finite volume can take place.
- Chromoelectric interactions relative to chromomagnetic are weakly suppressed and short-range in the sector $\mathrm{Re}\mathcal{P} > 0$ and moderately suppressed and long-range in each sector with $\mathrm{Re}\mathcal{P} < 0$.

To be studied: Dependence of the conditional variance of $D_L(0)$ and $D_{L,T}(0)$ and $D_{L,T}(p_{min})$ on lattice volume in the SU(3) case