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The Hierarchy Problem and New Dimensions at a Millimeter

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Abstract

We propose a new framework for solving the hierarchy problem which does not rely on either supersymmetry or technicolor. In this framework, the gravitational and gauge interactions become united at the weak scale, which we take as the only fundamental short distance scale in nature. The observed weakness of gravity on distances $\gtrsim 1$ mm is due to the existence of $n \geq 2$ new compact spatial dimensions large compared to the weak scale. The Planck scale $M_{Pl} \sim G_N^{-1/2}$ is not a fundamental scale; its enormity is simply a consequence of the large size of the new dimensions. While gravitons can freely propagate in the new dimensions, at sub-weak energies the Standard Model (SM) fields must be localized to a 4-dimensional manifold of weak scale "thickness" in the extra dimensions. This picture leads to a number of striking signals for accelerator and laboratory experiments. For the case of $n = 2$ new dimensions, planned sub-millimeter measurements of gravity may observe the transition from $1/r^2 \to 1/r^4$ Newtonian gravitation. For any number of new dimensions, the LHC and NLC could observe strong quantum gravitational interactions. Furthermore, SM particles can be kicked off our 4 dimensional manifold into the new dimensions, carrying away energy, and leading to an abrupt decrease in events with high transverse momentum $p_T \gtrsim$ TeV. For certain compact manifolds, such particles will keep circling in the extra dimensions, periodically returning, colliding with and depositing energy to our four dimensional vacuum with frequencies of $\sim 10^{12}$ Hz or larger. As a concrete illustration, we construct a model with SM fields localised on the 4-dimensional throat of a vortex in 6 dimensions, with a Pati-Salam gauge symmetry $SU(4) \times SU(2) \times SU(2)$ in the bulk.

1 Introduction

There are at least two seemingly fundamental energy scales in nature, the electroweak scale $m_{EW} \sim 10^3$ GeV and the Planck scale $M_{Pl} = G_N^{-1/2} \sim 10^{18}$ GeV, where gravity becomes as strong as the gauge interactions. Over the last two decades, explaining the smallness and radiative stability of the hierarchy $m_{EW}/M_{Pl} \sim 10^{-17}$ has been one of the greatest driving forces behind the construction of theories beyond the Standard Model (SM) . While many different specific proposals for weak and Planck scale physics have been made, there is a commonly held picture of the basic structure of physics beyond the SM. A new effective field theory (e.g. a softly broken supersymmetric theory or technicolor) is revealed at the weak scale, stabilizing and perhaps explaining the origin of the hierarchy. On the other hand, the physics responsible for making a sensible quantum theory of gravity is revealed only at the Planck scale. The desert between the weak and Planck scales could itself be populated with towers of new effective field theories which can play a number of roles, such as triggering dynamical symmetry breakings or explaining the pattern of fermion masses and mixings.

In this picture, the experimental investigation of weak scale energies is quite exciting, as it is guaranteed to reveal the true mechanism of electroweak symmetry breaking and stabilization of the hierarchy. One can also hope that a detailed measurement of low energy parameters can give valuable clues to the structure of effective field theories at higher energies, perhaps even approaching the Planck scale. Nevertheless, it is fair to say that in this paradigm, the thorough exploration of the weak scale will never give a direct experimental handle on strong gravitational physics.

It is remarkable that such rich theoretical structures have been built on the assumption of the existence of two disparate fundamental energy scales, m_{EW} and M_{Pl}. However, there is an important difference between these scales. While electroweak interactions have been probed at distances $\sim m_{EW}^{-1}$, gravitational forces have not remotely been probed at distances $\sim M_{Pl}^{-1}$: gravity has only been accurately measured in the $\sim 1cm$ range. Our interpretation of M_{Pl} as a fundamental energy scale (where gravitational interactions become strong) is then based on the assumption that gravity is unmodified over the 33 orders of magnitude between where it is measured at ~ 1 cm down to the Planck length $\sim 10^{-33}$ cm. Given the crucial way in which the fundamental role attributed to M_{Pl} affects our current thinking, it is worthwhile questioning this extrapolation and seeking new alternatives to the standard picture of physics beyond the SM.

In fact, given that the fundamental nature of the weak scale is an experimental certainty, we wish to take the philosophy that m_{EW} is the only fundamental short distance scale in nature, even setting the scale for the strength of the gravitational interaction. In this approach, the usual problem with the radiative stability of the weak scale is trivially resolved: the ultraviolet cutoff of the theory is m_{EW}. How can the usual $(1/M_{Pl})$ strength of gravitation arise in such a picture? A very simple idea is to suppose that there are n extra compact spatial dimensions of radius $\sim R$. The Planck scale $M_{Pl(4+n)}$ of this $(4+n)$ dimensional theory is taken to be $\sim m_{EW}$ according to our philosophy. Two test masses of mass m_1, m_2 placed within a distance $r \ll R$ will feel a gravitational potential dictated by Gauss's law in $(4+n)$ dimensions

$$V(r) \sim \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2}} \frac{1}{r^{n+1}}, \; (r \ll R). \tag{1}$$

On the other hand, if the masses are placed at distances $r \gg R$, their gravitational flux lines can not continue to penetrate in the extra dimensions, and the usual $1/r$ potential is obtained,

$$V(r) \sim \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2} R^n} \frac{1}{r}, \; (r \gg R) \tag{2}$$

so our effective 4 dimensional M_{Pl} is

$$M_{Pl}^2 \sim M_{Pl(4+n)}^{2+n} R^n. \tag{3}$$

Putting $M_{Pl(4+n)} \sim m_{EW}$ and demanding that R be chosen to reproduce the observed M_{Pl} yields

$$R \sim 10^{\frac{30}{n}-17} \mathrm{cm} \times \left(\frac{1\mathrm{TeV}}{m_{EW}}\right)^{1+\frac{2}{n}}. \tag{4}$$

For $n=1$, $R \sim 10^{13}$ cm implying deviations from Newtonian gravity over solar system distances, so this case is empirically excluded. For all $n \geq 2$, however, the modification of gravity only becomes noticeable at distances smaller than those currently probed by experiment. The case $n=2$ ($R \sim 100\mu\mathrm{m} - 1$ mm) is particularly exciting, since new experiments will be performed in the very near future, looking for deviations from gravity in precisely this range of distances [11].

While gravity has not been probed at distances smaller than a millimeter, the SM gauge forces have certainly been accurately measured at weak scale distances. Therefore, the SM particles cannot freely propagate in the extra n dimension, but must be localized to a 4 dimensional submanifold. Since we assume that m_{EW} is the only short-distance scale in the theory, our 4-dimensional world should have a "thickness" $\sim m_{EW}^{-1}$ in the extra n dimensions. The only fields propagating in the $(4+n)$ dimensional bulk are the $(4+n)$ dimensional graviton, with couplings suppressed by the $(4+n)$ dimensional Planck mass $\sim m_{EW}$.

As within any extension of the standard model at the weak scale, some mechanism is needed in the theory above m_{EW} to forbid dangerous higher dimension operators (suppressed only by m_{EW}) which lead to proton decay, neutral meson mixing etc. In our case, the theory above m_{EW} is unknown, being whatever gives a sensible quantum theory of gravity in $(4+n)$ dimensions! We therefore simply assume that these dangerous operators are not induced. Any extension of the SM at the weak scale must also not give dangerously large corrections to precision electroweak observables. Again, there could be unknown contributions from the physics above m_{EW}. However, at least the purely gravitational corrections do not introduce any new electroweak breakings beyond the W, Z masses, and therefore should decouple as loop factor $\times(m_{W,Z}/m_{EW})^2$, which is already quite small even for $m_{EW} \sim 1$ TeV.

Summarizing the framework, we are imagining that the space-time is $R^4 \times M_n$ for $n \geq 2$, where M_n is an n dimensional compact manifold of volume R^n, with R given by eq. (4). The $(4+n)$ dimensional Planck mass is $\sim m_{EW}$, the only short-distance scale in the theory. Therefore the gravitational force becomes comparable to the gauge forces at the weak scale. The usual 4 dimensional M_{Pl} is not a fundamental scale at all, rather, the effective 4 dimensional gravity is weakly coupled due to the large size R of the extra dimensions relative to the weak scale. While the graviton is free to propagate in all $(4+n)$ dimensions, the SM fields must be localized on a 4-dimensional submanifold of thickness m_{EW}^{-1} in the extra n dimensions.

Of course, the non-trivial task in any explicit realization of this framework is localization of the SM fields. A number of ideas for such localizations have been proposed in the literature, both in the context of trapping zero modes on topological defects[7] and within string theory. In section 3, we will construct models of the first type, in which there are two extra dimensions and, given a dynamical assumption, the SM fields are localized within the throat of a weak scale vortex in the 6 dimensional theory. We want to stress, however, that this particular construction must be viewed at best as an "existence proof" and there certainly are other possible ways for realizing our proposal, without affecting its most important consequences.

It is interesting that in our framework supersymmetry is no longer needed from the low energy point of view for stabilizing the hierarchy, however, it may still be crucial for the self-consistency of the theory of quantum gravity above the m_{EW} scale; indeed, the theory above m_{EW} may be a superstring theory.

Independently of any specific realization, there are a number of dramatic experimental consequences of our framework. First, as already mentioned, gravity becomes comparable in strength to the gauge interactions at energies $m_{EW} \sim$ TeV. The LHC and NLC would then not only probe the mechanism of electroweak symmetry breaking, they would probe the true quantum theory of gravity!

Second, for the case of 2 extra dimensions, the gravitational force law should change from $1/r^2$ to $1/r^4$ on distances $\sim 100\mu$m-1 mm, and this deviation could be observed in the next few years by the new experiments measuring gravity at sub-millimeter distances[11].

Third, since the SM fields are only localized within m_{EW}^{-1} in the extra n dimensions, in sufficiently hard collisions of energy $E_{esc} \gtrsim m_{EW}$, they can acquire momentum in the extra dimensions and escape from our 4-d world, carrying away energy.* In fact, for energies above the threshold E_{esc}, escape into the extra dimensions is enormously favored by phase space. This implies a sharp upper limit to the transverse momentum which can be seen in 4 dimensions at $p_T = E_{esc}$, which may be seen at the LHC or NLC if the beam energies are high enough to yield collisions with c.o.m. energies greater than E_{esc}.

Notice that while energy can be lost into the extra dimensions, electric charge (or any other unbroken gauge charge) can not be lost. This is because the massless photon is localized in our Universe and an isolated charge can not exist in the region where electric field can not penetrate, so charges can not freely escape into the bulk. In light of this fact, let us examine the fate of a charged particle kicked into the extra dimensions in more detail. On very general grounds (which we will discuss in more detail in section 3), the photon (or any other massless gauge field) can be localized in our Universe, provided it can only propagate in the bulk in the form of a massive state with mass $\sim m_{EW}$, m_{EW}^{-1} setting the penetration depth of the electric flux lines into the extra dimensions. In order for the localized photon to be massless it is necessary that the gauge symmetry be unbroken at least within a distance $\gg m_{EW}^{-1}$ from our four-dimensional surface (otherwise the photon will get mass through the "charge screening", see section 3). As long as this condition is satisfied, the four-dimensional observer will see an unbroken gauge symmetry with the right 4-d Coulomb law. Now, consider a particle with nonzero charge (or any other unbroken gauge quantum number) kicked into the extra dimensions. Due to the conservation of flux, an electric flux tube of the width m_{EW}^{-1} must be stretched between the escaping particle and our Universe. Such a string has a tension $\sim m_{EW}^2$ per unit length. Depending on the energy available in the collision, the charged particle will be either be pulled back to our Universe, or the flux tube will break into pieces with

*Usually in theories with extra compact dimensions of size R, states with momentum in the compact dimensions are interpreted from the 4-dimensional point of view particles of mass $1/R$, but still localized in the 4-d world. This is because the at the energies required to excite these particles, there wavelength and the size of the compact dimension are comparable. In our case the situation is completely different: the particles which can acquire momentum in the extra dimensions have TeV energies, and therefore have wavelengths much smaller than the size of the extra dimensions. Thus, they simply escape into the extra dimensions.

opposite charges at their ends. In either case, charge is conserved in the 4-dimensional world, although energy may be lost in the form of neutral particles propagating in the bulk. Similar conclusions can be reached by considering a soft photon emission process[8].

Once the particles escape into the extra dimensions, they may or may not return to the 4-dimensional world, depending on the shape and/or the topology of the n dimensional compact manifold M_n. In the most interesting case, the particles orbit around the extra dimensions, periodically returning, colliding with and depositing energy to our 4 dimensional space with frequency $R^{-1} \sim 10^{27-30/n}$ Hz. This will lead to continuous "fireworks", which in the case of $n = 2$ can give rise to $\sim$ mm displaced vertices.

2 Phenomenological and Astrophysical Constraints

In our framework physics below a TeV is very simple: It consists of the Standard Model together with a graviton propagating in 4+n dimensions. Equivalently –in four dimensional language–our theory consists of the Standard model together with the graviton and all its Kaluza-Klein (KK) excitations recurring once every $1/R$, per extra dimension n. We shall refer to all of them collectively as the "gravitons", independent of their mass. Since each graviton couples with normal gravitational strength $\sim 1/M_{Pl}$ to matter, its effect on particle physics and astrophysical processes is negligible. Nevertheless, since the multiplicity of gravitons beneath any relevant energy scale E is $(ER)^n$ can be large, the *combined* effect of all the gravitons is not always negligible and may lead to observable effects and constraints. In this section we will very roughly estimate the most stringent of these constraints, mainly to show that our framework is not grossly excluded by current lab and astrophysical bounds. Clearly, a much more detailed study must be done to more precisely determine the constraints on n and m_{EW} in our framework.

Consider any physical process involving the emission of a graviton. The amplitude of this process is proportional to $1/M_{Pl}$ and the rate to $1/M_{Pl}^2$. Consequently, the total combined rate for emitting any one of the available gravitons is

$$\sim \frac{1}{M_{Pl}^2}(\Delta E R)^n \tag{5}$$

where ΔE is the energy available to the graviton and the last term counts the KK gravitons' multiplicity for n extra dimensions. Using eq (3) we can rewrite this as

$$\sim \frac{\Delta E^n}{m_{EW}^{2+n}} \tag{6}$$

Note that the same result can be seen from the $4+n$ dimensional point of view. The m_{EW} suppressions of the couplings of the $4+n$ dimensional graviton are determined by expanding $g_{AB} = \eta_{AB} + h_{AB}/\sqrt{m_{EW}^{2+n}}$, where h_{AB} is the canonically normalised graviton in $4+n$ dimensions. Squaring this amplitude to obtain the rate yields precisely the m_{EW} dependence found above. As a result, the branching ratio for emitting a graviton in any process is

$$\sim (\Delta E/m_{EW})^{2+n} \tag{7}$$

The experimentally most exciting (and most dangerous) case has $m_{EW} \sim$ TeV and $n = 2$. Of course, we must assume that weak-scale suppressed operators giving proton decay, large K-$\bar{K}$ mixing etc. are forbidden. Of the remaining lab constraints, the ones involving the largest energy transfers ΔE (such as Υ and Z decays) are most constrained. The braching ratio for graviton emisson in Ypsilon decays is unobservable $\sim 10^{-8}$. For Z $Z \to X + graviton$ the branching ratio goes up to $\sim 10^{-5}$. Absence of such decay modes puts the strongest laboratory constraints to the scale m_{EW} and/or n. Nevertheless, they are easy to satisfy, in part because of their sensitivity to small changes in the value of m_{EW}. Production of gravitons in very high energy collisions will give the same characteristic signatures as the missing energy searches, except for one difference: the missing energy is now being carried by massless particles.

Next we consider astrophysical constraints. The gravitons are similar to goldstone bosons, axions and neutrinos in at least one respect. They can carry away bulk energy from a star and accelerate its cooling dynamics. For this reason their properties are constrained by the sun, red giants and SN 1987A. The simplest way to estimate these constraints is to translate from the known limits on goldstone particles. The dictionary that allows us to do that follows from eq(6) :

$$1/F^2 \longleftrightarrow \Delta E^n/m_{EW}^{2+n} \tag{8}$$

relating the emission rate of goldstones and gravitons. Here F is the goldstone boson's decay constant. For the sun the available energy ΔE is only a keV. Therefore, even for the maximally dangerous case $m_{EW} = 1$ TeV and $n = 2$, the effective F is 10^{12} GeV, large enough to be totally safe for the sun; the largest F that is probed by the sun is $\sim 10^7$ GeV.

For red giants the available energy is $\sim$100 keV and the effective $F \sim 10^{10}$ GeV. This value is an order of magnitude higher than the lower limit from red giants. Finally we consider the supernova 1987A. There, the maximum available energy per particle is presumed to be between 20 and 70 MeV . Choosing the more favorable 20 MeV we find an effective $F \sim 10^8$ GeV, which is smaller than the lower limit of 10^{10} GeV claimed from SN 1987A. Therefore, the astrophysical theory of SN 1987 A places an interesting constraint on the fundamental scale m_{EW} or/and the number of extra dimensions

n. The constraint is easily satisfied if $n > 2$ or if $m_{EW} > 10$ TeV. Of course, when the number of dimensions gets large enough so that $1/R \gtrsim 100$ MeV, (corresponding to $n \gtrsim 7$), none of the astrophysical bounds apply, since all the relevant temperatures would be too low to produce even the lowest KK excitation of the graviton.

Finally, although accelerators have not achieved collisions in the TeV energy range where the most exotic aspects of the extra dimensions are revealed, one may wonder whether very high energy cosmic rays of energies $\sim 10^{15} - 10^{19}$ eV (which in colliding with protons correspond to c.o.m. energies $\sim$ 1-100 TeV) have already probed such physics. However, the cosmic rays are smoothly accelerated to their high energies without any "hard" interactions, and they have dominantly soft QCD interactions with the protons they collide with. Therefore, there are no significant constraints from very high energy cosmic ray physics on our framework.

Having outlined our general ideas, some dramatic experimental consequences and being reassured that existing data do not significantly constrain the framework, we turn to contructing an explicit model realising our picture, with SM fields localised on the four-dimensional throat of a vortex in 6 dimensions.

3 Construction of a Realistic Model.

In this section we construct a realistic model incorporating the ideas of this paper. As stressed in the introduction, this should be viewed as an example or an "existence proof", since similar constructions are possible in the context of field theory as well as string theory. In particular one can change the structure and dimensionality of the manifold, the localization mechanism, the gauge group and the particle content of the theory without affecting the key ideas of our paper. Furthermore, many of the phenomenological consequences are robust and do not depend on such details.

The space time is 6-dimensional with a signature g_{AB} = (-1,1,1,1,1,1). The two extra dimensions are compactified on a manifold with a radius $R \sim$ 1mm. We will discuss two possible topologies: a two-sphere and a two-torus with the zero inner radius. In both cases the key point is that the observable particles (quarks, leptons , Higgs and gauge bosons) are localized inside a small region of weak-scale size equal to the inverse cutoff length $\sim \Lambda^{-1}$ and can penetrate in the bulk only in form of the heavy modes of mass $\sim \Lambda$. Thus for the energies $< \Lambda$ ordinary matter gets confined to a four-dimensional hypersurface, our universe. The transverse x_5, x_6 dimensions can be probed only through the gravitational force, which is the only long-range interaction in the bulk.

There are several ways to localize the Standard Model particles in our four-dimensional space-time. Here we consider the possibility that localization is dynamical and the ordinary particles are "zero modes" trapped in the core of a four-dimensional vortex. This topological defect, in its ground state, is independent of four coordinates (x_μ) and thus carves-out the four-dimensional hypersurface which constitutes our universe.

Consider first x_5, x_6 to be compactified on a two-sphere. Define a six-dimensional scalar field $\Phi(x_A)$ transforming under some $U(1)_V$ symmetry. We assume that Φ gets a nonzero VEV $\sim \Lambda$ and breaks $U(1)_V$ spontaneously. The vortex configuration is independent of the four coordinates x_μ and can be set up through winding the phase by 2π around the equator of the sphere:

$$\Phi = \phi_{bulk} e^{i\theta} \tag{9}$$

where $2\pi > \theta > 0$ is an azimuthal angle on the sphere and ϕ_{bulk} is the constant expectation value that minimizes a potential energy (modulo the small curvature corrections). Such a configuration obviously implies two zeros of the absolute value Φ on the both sides of the equator, which can be placed at the north and the south poles respectively. These zeros represent the vortex–anti-vortex pair of characteristic thickness $\sim \Lambda^{-1}$. Since this size is much smaller than the separation length $\sim$ 1mm, vortex can be approximated by the Nielsen-Olesen solution [3]

$$\Phi = f(r)e^{i\theta}, \quad f(0) = 0, \quad f(r)|_{r>>\Lambda^{-1}} \rightarrow \phi_{bulk} \tag{10}$$

where $0 < r < 2\pi R$ is a radial coordinate on the sphere, and an anti-vortex corresponds to the change $\theta \rightarrow -\theta, r \rightarrow 2\pi R - r$. If $U(1)_V$ is gauged the magnetic flux will be entering the south pole and coming out from the north one.

Since any closed loop on a two-sphere can be smoothly deformed to a point, vortices on a sphere are not truly stable, and can annihilate with anti-vortices if they encounter one another. However the vortex anti-vortex pair are separated by a millimeter, which is 10^{16} times their size; they therefore are, for all practical purposes, stable. In addition there can be other mechanisms of stabilization if other forces are involved (e.g. repelling charges or currents flowing along the strings, etc.).

Alternatively, compactification on a torus can lead to a truly stable vortex. This is because a torus contains many non-contractible loops, and the phase of Φ winding along such a loop is topologically stable. Such a configuration is obtained from the previously discussed two-sphere by identifying its poles with a single point and subsequently removing this point from the manifold. This manifold is then equivalent to a two-torus with zero inner radius; it can carry topological charge and accommodate a single vortex on it. The magnetic flux goes through the point that was removed from the manifold. An observer looking at the south pole will see the vortex with incoming flux. If he travels towards the north pole along the meridian he will arrive to the same vortex, since the poles have been identified, but will see it as an anti-vortex since he will now be looking at the flux up-side down.

Next, we come to the localization of the standard model particles on a vortex. We discuss the localization of different spins separately.

3.1 Localization of Fermions

Fermions can be trapped on the vortex as "zero modes" [1] because of the index theorem [2]. Consider a pair of six-dimensional left-handed Weyl spinors $\Gamma_7\psi, \xi = \psi, \xi$. These, in terms of the four-dimensional chiral Weyl spinors can be written as

$$\psi = (\psi_L, \psi_R), \;\; \xi = (\xi_L, \xi_R) \tag{11}$$

Assume now that this pair is getting a mass from coupling to the vortex field:

$$h\Phi\psi\xi + \text{h.c.}, \tag{12}$$

where $h \sim \Lambda^{-1}$ has dimensions of inverse mass. The six-dimensional Dirac equation in the vortex background is:

$$\Gamma_A \partial^A \psi^+ = h\phi_{bulk} e^{i\theta}\xi, \tag{13}$$

and similarly for ξ^+. To look for solutions with localized massless fermions we separate variables through the angle-independent anzatz $\psi = \psi(x_\mu)\beta(r)$ and $\xi = \xi(x_\mu)\beta(r)$, where $\mu = 1,..4$, $\beta(r)$ is a radial scalar function in the 2 dimensional compact space of x_5 and x_6. To be zero modes of the four-dimensional Dirac operator, the spinors $\psi(x_\mu)$ and $\xi(x_\mu)$ must satisfy

$$\Gamma_5 e^{i\theta(-i\Gamma_5\Gamma_6)}\psi^+\partial_r\beta(r) = h\phi_{bulk}e^{i\theta}\xi, \tag{14}$$

and similarly for ξ^+. Since ψ^+ and ξ must be eigenvalues of the $(-i\Gamma_5\Gamma_6)$ operator, they automatically have definite four-dimensional chirality (say left for the vortex and right for the anti-vortex). In this case the normalizable wavefunction has the localized radial dependence $\beta(r) = e^{-h\int_0^r f(r')dr'}$. Thus the vortex supports a single four-dimensional massless chiral mode which can be

$$\psi_L + \xi_R^+ \tag{15}$$

In general, as a consequence of the index theorem[1][2], every pair of six-dimensional chiral fermions getting mass from the vortex field, deposits a single zero mode of definite four-dimensional chirality. Thus through the couplings to the vortex field one can reproduce the whole set of the four-dimensional chiral fermions – quarks and leptons– localized on the submanifold. These localized modes can get nonzero masses through the usual Higgs mechanism. Let ψ and ψ' be the six-dimensional chiral spinors (from different pairs) that deposit two different zero modes on the vortex. These zero modes can get masses through the couplings to a scalar field H

$$H\psi\psi', \tag{16}$$

provided it has a nonzero expectation value in the core of the vortex but vanishes in the bulk. The index is unaffected by the existence of such a scalar since it has a zero VEV outside the core.

3.2 Localization of Higgs Scalars

Now let us consider how the Higgs fields with non-zero VEVs can be localized on the vortex. A massive scalar field can be easily localized provided it has a suitable sign coupling to the vortex field in the potential

$$h'|\Phi|^2|H|^2 \tag{17}$$

If $h' > 0$, this contribution is positive in the bulk and zero in the core. Thus H will see the defect as an attractive potential, which for a certain range of parameters can lead to a bound-state solution. We will treat H as the six-dimensional progenitor of the Weinberg-Salam Higgs particle. Then the physically most important case is when H develops a non-zero expectation value in the the defect. This is easily possible, provided an effective mass^2 of the Higgs becomes negative in the throat of the vortex. The simplest prototype potential of this sort is:

$$(h'|\Phi|^2 - m^2)HH^+ + c(HH^+)^2, \quad \text{with} \quad m^2, h', c > 0, \tag{18}$$

where H is a six-dimensional scalar field. We will assume that $h'\Phi^2_{bulk} - m^2 > 0$ and thus H is zero in the bulk. However, it can develop VEV in the vortex core. This does not require any fine tuning and can be seen by examining the stability of the trivial solution with respect to the small excitations $H(x_5, x_6)e^{-i\omega t}$ in the vortex background. The analysis is similar to the one of the superconducting cosmic string [4]. The linearized equation for small excitations is the two dimensional Schrodinger equation

$$-\partial^2_{5,6}H + [h'f - m^2]H = \omega^2 H \tag{19}$$

which certainly has a normalizable boundstate solution with $\omega^2 < 0$ in a range of parameters. Thus H becomes tachionic in the core marking the instability of the trivial solution $H = 0$. As a result H develops an expectation value in the throat of the vortex which decays exponentially for large r. Since H is a Higgs doublet there are the three massless Goldstone modes localized on the vortex . These get eaten up by W and Z bosons through the usual Higgs effect. There is also a localized massive mode, an ordinary Higgs scalar, which corresponds to the small vibrations of the expectation value in the core $H(0) \rightarrow H(0) + h(x_\mu)$. Such a vibration propagates in four-dimensions as the ordinary massive scalar Higgs.

3.3 Localization of Gauge Fields

There are several possible mechanisms for localizing gauge fields on a vortex (or on any other topological defect)† through the coupling to the vortex scalar. In general, a particle localized in such a way will not be massless, unless there is a special reason such as the index theorem for fermions and the Goldstone theorem or supersymmetry for bosons. Here, we propose to localize massless gauge fields by generalizing the four-dimensional confinement mechanism of ref[5] (see also [8]). Before discussing how this mechanism is generalized to our case, it is instructive to understand why the simplest approach fails. Consider the $U(1)_{EM}$ electromagnetism in the presence of a thin vortex along the z-axis. Let ϕ^- be a charged scalar field that develops an expectation value and breaks $U(1)_{EM}$ spontaneously through the Higgs mechanism. Now introducing a cross-coupling with the vortex field

$$(-a|\Phi|^2 + m^2)|\phi^-|^2 + b|\phi^-|^2, \quad a, b, m^2 > 0 \tag{20}$$

and appropriately adjusting parameters (no fine tuning), we can force ϕ^- to vanish in the vortex‡; as a consequence, the tree-level mass of the six-dimensional electric field coupled to it will also vanish. Unfortunately, the four-dimensional photon trapped in this way does not remain massless. It has a massive dispersion relation due to charge screening. This can be understood as follows: Since the charged field ϕ^- condenses in the vacuum, the Universe is superconducting everywhere except in the interior of the thin vortex. Two test charges placed at different points x_μ and x'_μ in the vortex will not interact by Coulomb's law; their electric field polarizes the surrounding medium, and the field lines end on the superconductor. As a result, the electric flux along the vortex dies-off exponentially with (longtitudinal) distance within a characteristic length given by the width of the vortex.

It is clear that for the localized gauge field to be massless the surrounding medium which repels the electric field lines should not contain any charge condensate, otherwise all the field lines can be absorbed by the medium. To construct such an example, consider a thin planar-layer between two infinite superconductors. Two magnetic monopoles located inside the layer interact through a long range magnetic field. This is because the magnetic flux is

† An alternate way to localize massless gauge fields involves D-brane constructions[10]

‡An alternative possibility is that ϕ^- is a vortex field itself, charged under the electromagnetism, just as in the Abrikosov vortices in superconductors. In this case ϕ^- will vanish at the origin for topological reasons.

repelled (or "totally reflected") from the superconductor, since it contains no magnetic charges on which the magnetic field lines can end. Consequently, the magnetic flux is entirely contained inside the layer and, as a result of flux conservation, the field lines spread according to Coulomb's law.

In ref[5] a dual version of this mechanism –in which the superconductor is replaced by a confining medium with monopole condensation– was suggested as a way to obtain massless gauge bosons localized on a sub-manifold. Suppose that away from the vortex $U(1)_{EM}$ becomes a part of a confining group which develops a mass gap $\sim \Lambda$. Then the electric flux lines will be repelled by monopole condensation in the dual Meisner effect; no images are created since there is no charged condensate in the medium.

It is not difficult to construct an explicit four-dimensional prototype model of this sort. It includes an $SU(2)$ Yang-Mills theory with a scalar field χ in the adjoint representation, plus a vortex field Φ, which breaks some additional $U(1)_V$ symmetry and forms the string (for the present discussion it is inessential whether $U(1)_V$ is global or gauged). The Lagrangian has the form ($SU(2)$ indices are suppressed)

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{Tr}G_{\mu\nu}G_{\mu\nu} + (D_\mu\chi)^2 - \lambda(\chi^2)^2 - \chi^2(h|\Phi|^2 - M^2) + \\ + |\partial_\mu\Phi|^2 - \lambda'(|\Phi|^2 - \phi_{bulk}^2)^2 \,, \tag{21}$$

where $G_{\mu\nu}$ is the "gluon" field strength tensor, and $h, \phi_{bulk}^2, M^2, \lambda, \lambda'$ are the positive parameters and we assume $h\phi_{bulk}^2 > M^2$. In a certain range of parameters the absolute minimum of the theory is achieved for $\chi = 0$. In this vacuum, Φ develops the VEV $\langle\Phi\rangle = \phi_{bulk}$ and forms the vortex . Although χ is zero in the vacuum, it can acquire an expectation value inside the vortex, where its mass^2 becomes negative, just like in the example with the Higgs doublet considered above. In this case $SU(2)$ is broken to $U(1)_{EM}$ on the string, but is restored outside. Inside the string, two out of the three gluons acquire large masses of order of M. The third gluon becomes a photon. Two degrees of freedom in the χ field are eaten up by the Higgs mechanism, the remaining degree of freedom is neutral. The massless degree of freedom in the effective 1 + 1 dimensional theory on the string is a photon. It is massless, since the $U(1)_{EM}$ gauge symmetry is unbroken everywhere. On the other hand outside the vortex the photon becomes a member of the nonabelian gauge theory, which confines and develops a mass gap. Thus the photon can only escape from the string in the form of a composite heavy "glueball" with a mass of the order of Λ which we take to be the UV cutoff $\sim m_{EW}$. This guarantees that at low energies the massless photon will be trapped on the string. The theory inside the string is in the abelian 1 + 1 dimensional"Coulomb" phase.

How is this mechanism generalized to our six-dimensional case? Of course, we do not know how confinement works in a higher dimensional theory. Nev-

ertheless, we believe and will postulate that the higher dimensional theory in the bulk will posses a mass gap $\sim \Lambda$ provided that:
1) Outside the vortex the standard model gauge group, in particular electromagnetism and strong interactions, are extended into a larger nonabelian gauge theory.
2) There is no light (with a mass below the cut-off scale Λ) matter in the bulk enforced by general principles, such as Goldstone's theorem.
3) The tree-level gauge coupling blows up away from the vortex [8].
The latter condition can be satisfied e.g. if the value of the gauge coupling is set by an expectation value of the higgs field (or any function of it) which vanishes away from the vortex. For instance, in the previous four-dimensional toy model such a coupling is

$$\Lambda^{-2}\mathrm{Tr}\chi^2\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} \tag{22}$$

In summary, we presented one possible way for localizing particles in our four-dimensional space-time. There are other possibilities within both field and string theory –such as D-brane constructions– for accomplishing the same goal.

3.4 A Realistic Theory

In this section we assemble the above ingredients to construct a prototype model incorporating the ideas of this paper. We embed the Standard Model in the Pati-Salam group $G = SU(4)\otimes SU(2)_R\otimes SU(2)_L$ which is the unbroken gauge group in the bulk. In addition, we introduce a $U(1)_V$ factor and a singlet scalar field Φ charged under it. Φ develops an expectation value and forms a vortex of thickness $\sim \Lambda^{-1}$ in the compact 2-D submanifold spanned by x_5, x_6. The interior of the vortex is our 4-dimensional space-time with all the light matter confined to it. The only light particle propagating in the bulk is the six-dimensional graviton.

The gauge group is spontaneously broken to $SU(3)\otimes U(1)_{EM}$ inside the vortex, by a set of six-dimensional scalar fields $\chi = (15.1.1)$, $\chi' = (4.2.1)$ and $H = (1.2.2)$ which develop nonzero VEVs only in the core of the vortex due to their interactions with the Φ field. We assume a soft hierarchy $\chi' \sim \chi \sim \Lambda \sim 10H \sim m_{EW}$. The crucial assumption is that in the bulk the gauge group is strongly coupled and develops a mass gap of the order of the cut-off. This, together with the fact that $SU(3)\otimes U(1)_{EM}$ is unbroken everywhere guarantees that the gluons and the photon are massless and trapped in our four-dimensional manifold. $W^{\pm}$ and a Z bosons are localized as massive states.

The matter fermions are assumed to originate from the following six-dimensional chiral spinors per generation:

$$Q = (4,1,2), \bar{Q} = (\bar{4},1,2), Q_c = (\bar{4},2,1), \bar{Q}_c = (4,2,1), \tag{23}$$

which get their bulk masses through the coupling to the vortex field

$$h\Phi Q\bar{Q} + h'\Phi^* Q_c\bar{Q}_c \tag{24}$$

(where h and h' are parameters of the inverse cut-off size). The index theorem ensures that each pair deposits a single chiral zero mode which can be chosen as $Q_L + \bar{Q}_R^+$ and $Q_{cL}^+ + \bar{Q}_{cR}$. These states get their masses through the couplings to the Higgs doublet field which condenses in the core of the vortex

$$gHQQ_c + \bar{g}H\bar{Q}\bar{Q}_c \tag{25}$$

To avoid unacceptable flavor violations,the couplings in eqs(24) should be flavor-universal. This can be guaranteed by some flavor symmetry. Flavor violations must come from the ordinary Yukawa couplings (see eq(25) to be under control.

The theory presented here has rich and exotic phenomenology, thanks to the existence of the extra dimensions. At energies above the cutoff of a $\Lambda \sim$ TeV there is a plethora of particles which can quite freely migrate and allow us to look into the extra dimensions. Furthermore, naturalness requires that the migration into the extra dimensions cannot be postponed much beyond the TeV scale.

4 Summary and Outlook

The conventional paradigm for High Energy Physics –which dates back to at least 1974– postulates that there are two fundamental scales , the weak interaction and the Planck scale. The large disparity between these scales has been the major force driving most attempts to go beyond the Standard Model, such as supersymmetry and technicolor. In this paper we propose an alternate framework in which gravity and the gauge forces are united at the weak scale. As a consequence, gravity lives in more than four dimensions at macroscopic distances –leading to potentially measurable deviations from Newton's inverse square law at sub-mm distances. The LHC and NLC are now even more interesting machines. In addition to their traditional role of probing the electroweak scale, they are quantum-gravity machines, which can also look into extra dimensions of space via exotic phenomena such as apparent violations of energy, sharp high-p_T cutoffs and the disappearance and reappearance of particles from extra dimensions.

The framework that we are proposing changes the way we think about some fundamental issues in particle physics and cosmology. The first and most obvious change in particle physics occurs in our view of the hierarchy problem. Postulating that the cutoff is at the weak scale nullifies the usual argument about ultraviolet sensitivity, since the weak scale now becomes the ultraviolet! The new hierarchy that we now have to face, in the six dimensional case, is that between the millimeter and the weak scales. This hierarchy is stable in the sense that small changes of parameters have small effects on the physics –so there is no fine tuning problem. There is also no issue of radiatively destabilizing the mm scale by physics at the weak cutoff. In this respect, our proposal shares the same "set it and forget it" philosophy of the original proposal supersymmetric standard model [12]. An important and favorable difference is that the mm scale is not a Lagrangean parameter that needs to be stabilized by a symmetry, such as supersymmetry. It is a parameter characterizing a solution, the size of the two extra dimensions. It is not uncommon to have solutions much larger than Lagrangean parameters; the world around us abounds with solutions that are much larger than the electron 's Compton-wavelength. A related secondary question is whether the magnitude of the mm scale may be calculated in a theory whose fundamental length is the weak scale. We have not addressed this question which is imbedded in the higher dimensional theory. It is amusing to note that if there are many new dimensions, their size –given by eq (4)– approaches the weak scale and there is no large hierarchy.

Finally we come to the early universe. The most solid aspect of early cosmology, namely primordial nucleosynthesis, remains intact in our framework. The reason is simple: The energy per particle during nucleosynthesis is at most a few MeV, too small to significantly excite gravitons. Furthermore, the horizon size is much larger than a mm so that the expansion of the universe is given by the usual 4-dimensional Robertson-Walker equations. Issues concerning very early cosmology, such as inflation and baryogenesis may change. This, however, is not necessary since there may be just enough space to accommodate weak-scale inflation and baryogenesis.

In summary, there are many new interesting issues that emerge in our framework. Our old ideas about unification, inflation, naturallness, the hierarchy problem and the need for supersymmetry are abandoned, together with the successful supersymmetric prediction of coupling constant unification [12]. Instead, we gain a fresh framework which allows us to look at old problems in new ways. Lagrangean parameters become parameters of solutions and the phenomena that await us at LHC, NLC and beyond are even more exciting and unforseen.

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