HOW MUCH CANONICAL ARE ASHTEKAR VARIABLES ?

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ABSTRACT

We show that in general the transformation to the Ashtekar variables from the ordinary vierbein Hamiltonian formalism differs from the canonical one. When surface integrals are taken into account the Poisson bracket

$$\{ \int N^{ia}(x) A^a_i(x) d^3x, \int M^{jb}(y) A^b_j(y) d^3x\}$$

is not zero[1]. Therefore the expressions for the Poisson brackets should be changed if a boundary is present:

$$\{H_1, H_2\} = \{H_1, H_2\}_{old} + \{H_1, H_2\}_{correction},$$

where

$$\{H_1, H_2\}_{correction} = \int d^3x \int d^3y \frac{\delta H_1}{\delta A^a_i(x)} \frac{\delta H_2}{\delta A^b_j(y)} \{ A^a_i(x), A^b_j(y)\} =$$

$$= \frac{i}{4} \oint \frac{dS_k}{E} \epsilon^{acd} (\delta^k_j \delta^{bd} E_{ic} - E_{ib} E_{jc} E^{kd}) \left[\frac{\delta H_1}{\delta A^a_i} \frac{\delta H_2}{\delta A^b_j} - (1 \leftrightarrow 2) \right].$$

Poincaré algebra at spatial infinity in asymptotically flat spacetime is an example of crucial importance of these corrections. In the ordinary canonical formalism, the Poincaré group generators are expressed in terms of surface integrals taken over a two-dimensional sphere at spatial infinity.[2] These surface integrals play the two roles : they guarantee differentiability of the Hamiltonian at the prescribed boundary conditions and they also enter the Poisson brackets algebra

$$\{ H(N^\alpha), H(M^\beta)\} \approx H([N, M]^\gamma),$$

when every vector field represents coordinate transformation from the asymptotic Poincaré group. The criterion of differentiability that is usually exploited can give surface terms only up to the phase space constants. But the straightforward calculation of all the surface terms in the Poisson brackets[3] has no such shortcoming and gives us many advantages, for example, a possibility to search for new boundary conditions or to evaluate central charges. Without our redefinition of the Poisson brackets the realization of the algebra of Poincaré group generators in the Ashtekar's formalism for the Schwarzschild solution meets serious difficulties.[4]

References

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