NEW POISSON BRACKETS FROM NEW HAMILTONIAN VARIABLES

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ABSTRACT

In the abstract submitted to GR-13 it was proposed to report an unexpected result that the transformation to Ashtekar's variables from the ordinary vierbein Hamiltonian formalism differs from the canonical one[1]. When surface integrals are taken into account the Poisson bracket

$$\{\int N^{ia}(x)A_i^a(x)d^3x, \int M^{jb}(y)A_j^b(y)d^3x\}$$

is nonzero. Therefore the general expression for the Poisson bracket of arbitrary local functionals of Ashtekar's variables should be changed if a boundary is present:

$$\{H_1, H_2\} = \{H_1, H_2\}_{standard} + \{H_1, H_2\}_{boundary}.$$

Unfortunately, that talk was not given, since the support kindly suggested by the Organizing Committee of GR-13 was not enough for the author to reach Cordoba from Moscow. Now one can find something provisional in this. A bit later it became clear that a rigorous treatment of the surface correction to the Poisson bracket in Ashtekar's formalism required that the most general definition of the field theory Poisson bracket should be modified.

The standard bracket uses the Euler-Lagrange variational derivative as a definition of the differential for a local functional. But if we allow the field variations on the boundary to be nonzero, then the functional differential should be given by more general formula involving the so-called higher Eulerian derivatives (in this terminology the standard Euler-Lagrange variational derivative is of order zero). It occures so, that use of a prolonged variational derivative leads to an extension of the standard formula for the field theory Poisson bracket. The new formula is invariant under changes both of dependent and independent variables if such invariance takes place for the functionals involved. It is clear that similar formulas take place for commutators in the quantum field theory where, of course, the problem of operator ordering should be considered in addition.

The new Poisson bracket can be used in studying problems with nontrivial dynamics on a boundary, including free boundary problems. It occures possible to get the Hamiltonian equations on a boundary which are in general different from the bulk equations. Boundary condition imposed by hand is similar to a gauge and could lead to a fixation of a Lagrangian multiplier or to a new constraint.

We apply the new Poisson brackets to construct the finite domain Hamiltonian formalism in the Ashtekar and ADM variables and discuss the treatment of some surface integrals as quasi-local energy-momentum characteristics.

References

1. V.O.Soloviev, "How much canonical are Ashtekar variables?" GR-13 Abstracts (Cordoba, 1992).